

# The 17 Worlds of Planar Ants

Goal. Get you hooked!

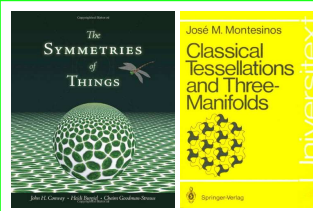
Video, handout, links at  $\omega$ /



**Abstract.** Back in early 2000, I got my first digital camera and set out to take pictures of my kids and of symmetric patterns in the plane ( $\omega$ /Tilings). There are exactly 17 of those, no more, no less. It is an addicting challenge to walk around looking at buildings, brick walls, people's ties, fabrics, what's not, and to try figure out which of the 17 is each one.



Lou Kauffman's Tie



### Books.

- J. H. Conway, H. Burgiel, and C. Goodman-Strauss, *The Symmetries of Things*, CRC Press, 2008.
- J. M. Montesinos, *Classical Tessellations and Three-Manifolds*, Springer-Verlag, 1987.

- What would history look like if we were living on Venus?
- What do the ants on Lou Kauffman's tie think?

### The Renaissance Story

$\omega$ /Longtin



### The Venus Story



$\omega$ /DW

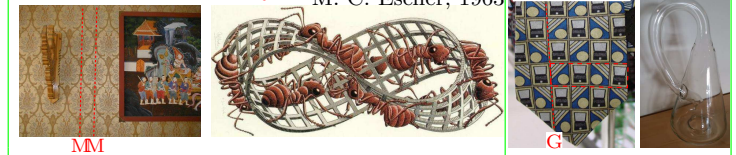
### The Lake Merritt Story



### The Racha Cafe Story

M. C. Escher, 1963

### Tie@Fry's

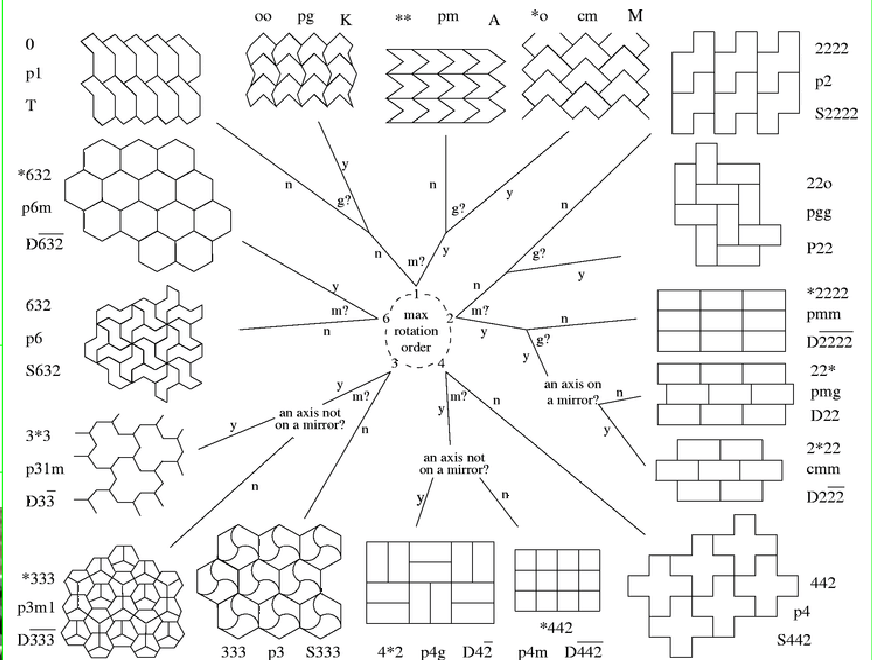


$\omega$ /Sanderson

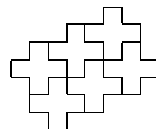
**Claim.** Exactly 10 "features" are possible. They are **M**, **G**, **2**, **3**, **4**, **6**,  $\bar{2}$ ,  $\bar{3}$ ,  $\bar{4}$ , and  $\bar{6}$ .

## Brian Sanderson's Pattern Recognition Algorithm

Is the maximum rotation order 1,2,3,4 or 6? Is there a mirror (m)? Is there an indecomposable glide reflection (g)? Is there a rotation axis on a mirror? Is there a rotation axis not on a mirror?



Note: Every pattern is identified according to three systems of notation, as in the example below:



442: The Conway-Thurston notation, as used in my [tilings page](#).

p4: The International Union of Crystallography notation.

S442: The Montesinos notation, as in his book [Classical Tessellations and Three Manifolds](#)

**Theorem.** There are exactly 17 "tilings" of the plane:  $\emptyset=0$ , **MM**=\*\*, **MG**=\*o, **GG**=oo, **2222**=2222, **333**=333, **442**=442, **632**=632,  $\bar{2}\bar{2}\bar{2}\bar{2}$ =\*2222,  $\bar{3}\bar{3}\bar{3}$ =\*333,  $\bar{4}\bar{4}\bar{2}$ =\*442,  $\bar{6}\bar{3}\bar{2}$ =\*632,  $\bar{4}\bar{2}$ =4\*2,  $\bar{3}\bar{3}$ =3\*3,  $\bar{2}\bar{2}\bar{2}$ =2\*22,  $\bar{2}\bar{2}\bar{M}$ =22\*,  $\bar{2}\bar{2}\bar{G}$ =22o. **18??**



**The 230 Worlds of Spatial Monkeys** (The 219 worlds of Monkeys that Can't Tell their Left from their Right)

$\omega$ /Crys,  $\omega$ /CFHT

