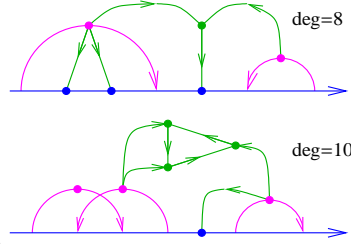
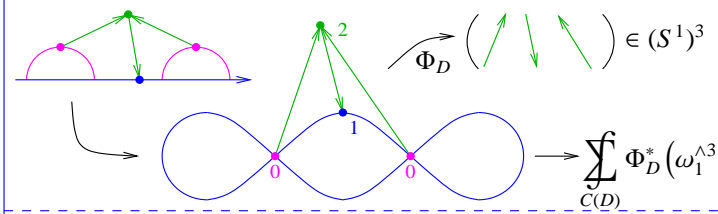


**Feynman Diagrams and a Lower Bound on  $(\mathcal{K}_0/\mathcal{K}_{n+1})^*$ .**

**Feynman Diagrams.** A blue “skeleton line” at the bottom. A magenta “arrow diagram” (directed pairing of skeleton points) on top, with a magenta dot at the middle of each arrow. A green directed graph on top, with 2-in 1-out antisymmetric green vertices, with arbitrary number of green edges starting at the magenta dots, and with some green edges terminating at distinct blue skeleton points. The degree is the total valency of the magenta dots.



**Configuration Space Integrals.**

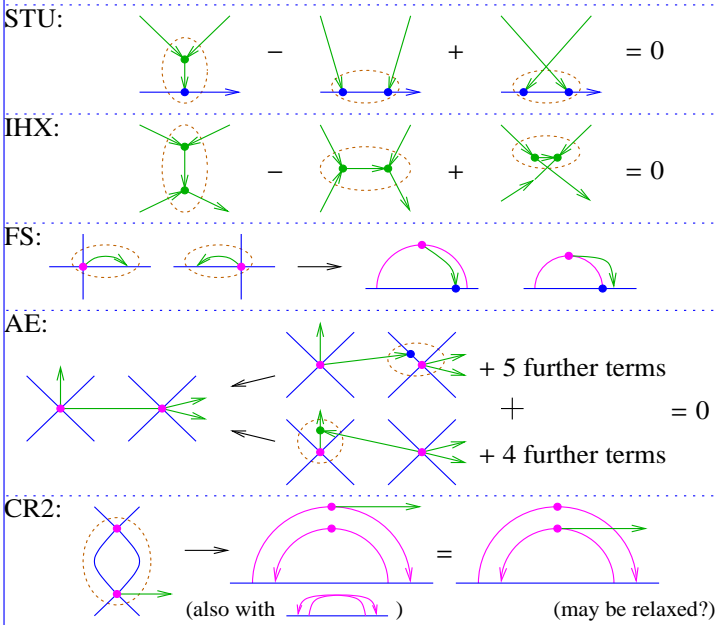


**The “Partition Function” Z.**

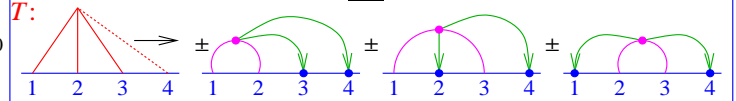
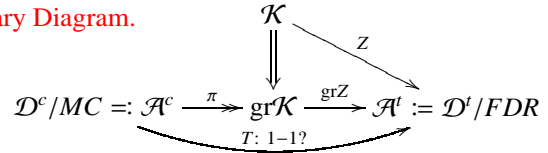
$$K \mapsto Z(K) := \sum_{\text{Feynman diagrams } C(D)} \Phi_D^*(\omega_1^{\wedge e(D)}) \in \mathcal{A}^t := \langle D \rangle / (\partial\text{-relations}).$$

**Theorem (90%).** Z is an invariant of doodles.

**$\partial$ -relations.** STU, IHX, Foot Swap (FS), Arrow Exchange (AE), and Combinatorial R2 (CR2):



**Summary Diagram.**



**An unfinished project!**

- Nothing is written up.
- We don't know if T is injective (meaning, if our upper and lower bounds agree).
- We don't know if all of  $\mathcal{A}^t$  is necessary — it is very possible that it is enough to restrict to the green-less part of  $\mathcal{A}^t$  — to “Gauss Diagram Formulas”.
- We haven't clarified the relationship with Merkov's [Me].
- A few further configuration space integrals can be written beyond those that we have used. We don't know what to do with those, if anything.
- We don't know the relationship, if any, with algebra.
- We don't know the relationship, if any, with quantum field theory.
- We don't know how to do similar things with 2-knots.

**References.** The root, of course, is [Ar]. Further references on doodles include [Kh, FT, Me, Ta, Va1, Va2]. On Goussarov finite-type: [Go, BN].

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