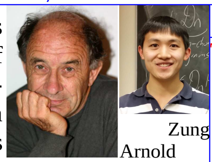
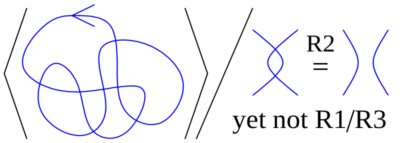
Finite Type Invariants of Doodles, 1

Abstract. I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals". An unfinished project!


Arnold Zung

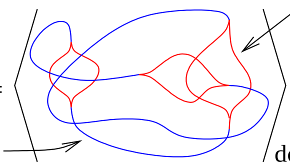
Doodles. $\mathcal{K} = \mathcal{K}_0 = \mathbb{Q}\langle$  $\rangle / ($ $\overset{R2}{=}$ $)$ yet not R1/R3

Easy to classify!

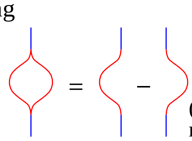
Prior Art. Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness" St invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.


Merkov and Vassiliev

Goussarov Finite-Type.

$\mathcal{K}_n =$ 

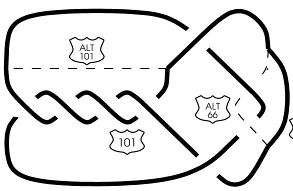
doodles and detours (dnd's)


(equal rotation numbers)

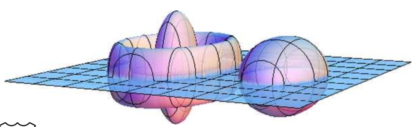

Goussarov

Def. V is of type n if it vanishes on $\mathcal{K}_{n+1}$. $(\mathcal{K}_0/\mathcal{K}_{n+1})^\star \leftrightsquigarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

Knots in 3D.



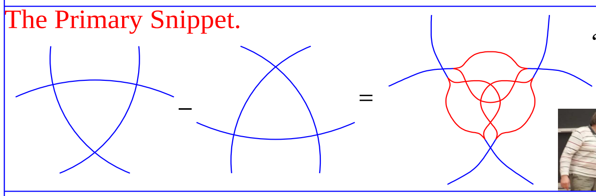
2-Knots in 4D.



The reason I care!

Goals. • Describe $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$ using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if $\mathcal{A}_n$ has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's St and the Merkov invariants integrate in.

The Primary Snippet.



(Arnold's "strangeness" is type 3)

Goryunov, here

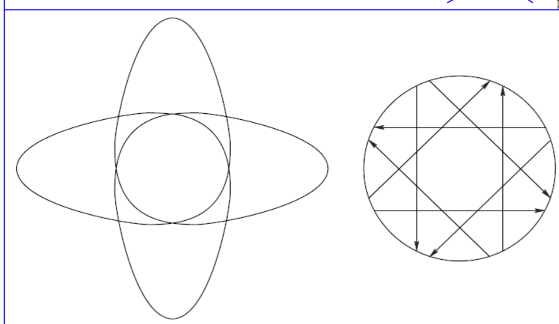


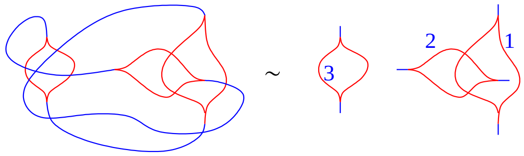
Figure 3. A non-trivial 1-doodle and its arrow diagram

From Merkov's [Me]

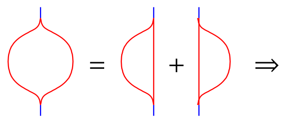
Chord Diagrams and an Upper Bound on $\mathcal{K}_n/\mathcal{K}_{n+1}$

The Rayman Principle. In $\mathcal{K}_n/\mathcal{K}_{n+1}$, "joins are irrelevant"

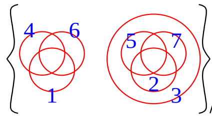

Rayman by Ubisoft



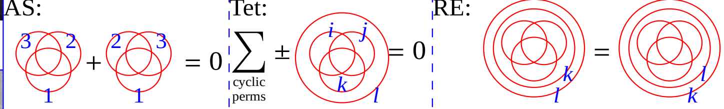
The Subdivision Relations. In $\mathcal{K}_n/\mathcal{K}_{n+1}$,



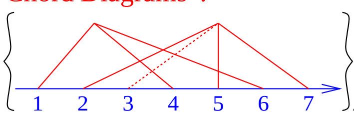
$\Rightarrow$ Rings can be subdivided until each one participates in just one "feature". $\Rightarrow$



/ Anti-Symmetry (AS), Tetrahedron (Tet), Ring Exchange (RE) $\longrightarrow\!\!\!\!\twoheadrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

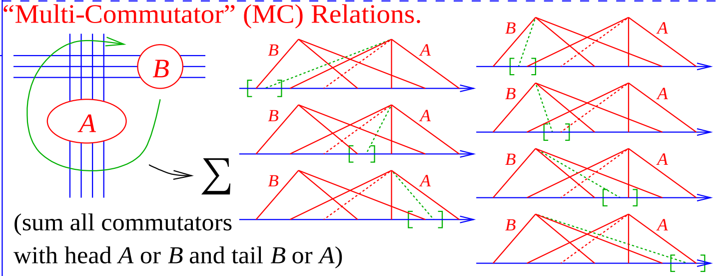


"Chord Diagrams".

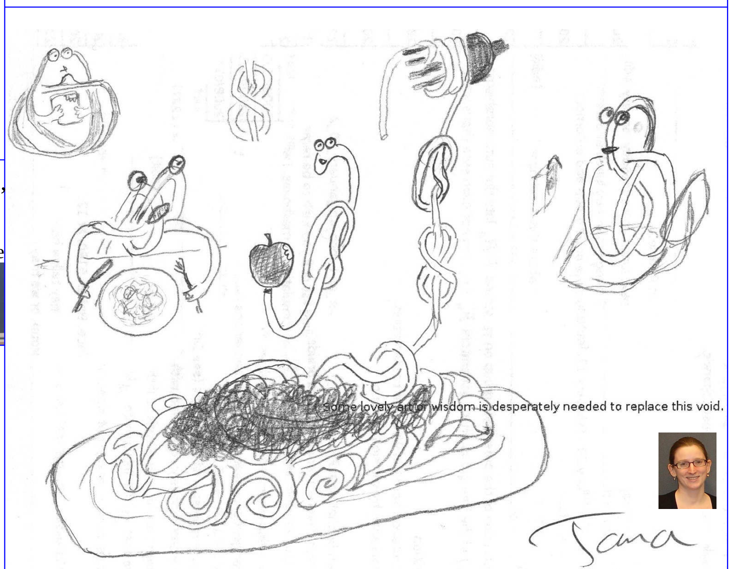


/ AS, Tet, RE, MC $\xrightarrow{\pi}\!\!\!\!\twoheadrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

"Multi-Commutator" (MC) Relations.



(sum all commutators with head A or B and tail B or A)



Some lovely artful wisdom is desperately needed to replace this void.

Doodles by my former student Jana Archibald

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org