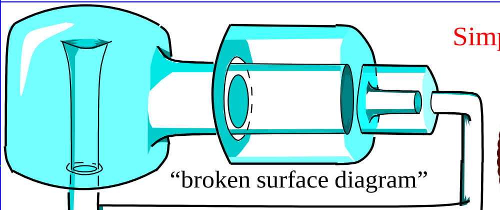
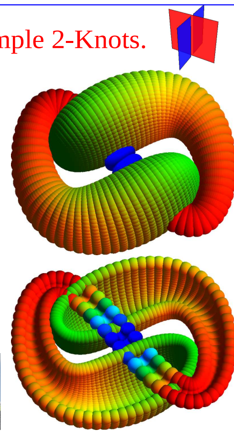
Simple 2-Knots.



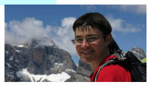
A 4D knot by Carter and Saito [CS]



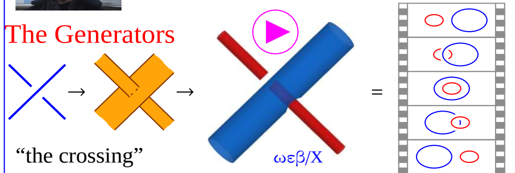
ωεβ/F

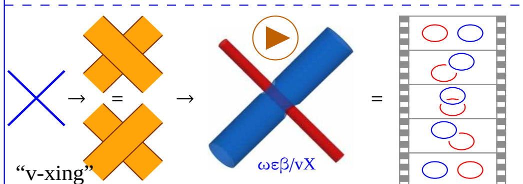
Satoh

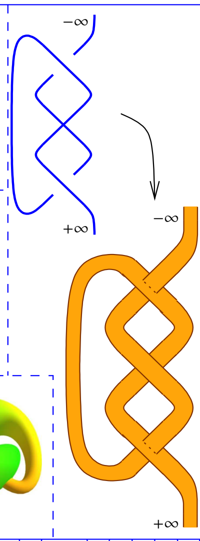
Dalvit
ωεβ/Dal

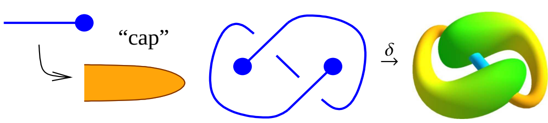


The Generators





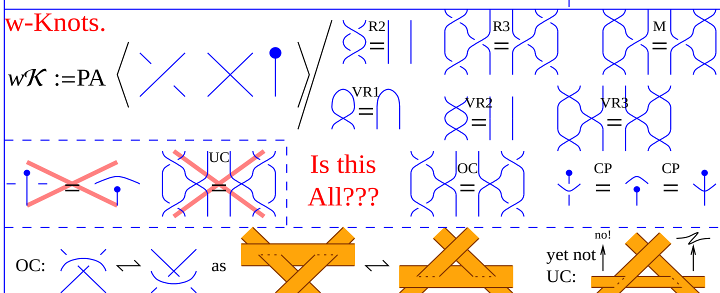




The Double Inflation Procedure δ.

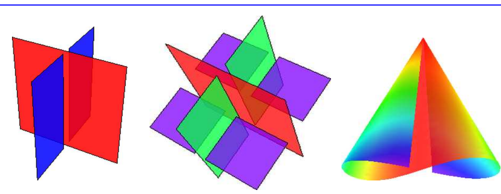
w-Knots.

$w\mathcal{K}$:=PA

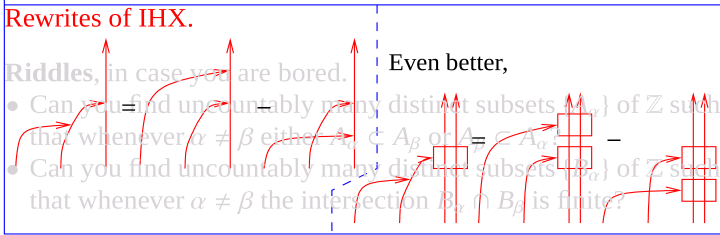


A Big Open Problem. δ maps w-knots onto simple 2-knots. To what extent is it a bijection? What other relations are required? In other words, **find a simple description of simple 2-knots**.

The Full 2-Knot Story



Rewrites of IHX.



Even better,

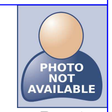
Riddles, in case you are bored.

- Can you find uncountably many distinct subsets $\{A_\alpha\}$ of $\mathbb{Z}$ such that whenever $\alpha\neq\beta$ either $A_\alpha\subset A_\beta$ or $A_\beta\subset A_\alpha$?
- Can you find uncountably many distinct subsets $\{B_\alpha\}$ of $\mathbb{Z}$ such that whenever $\alpha\neq\beta$ the intersection $B_\alpha\cap B_\beta$ is finite?

Question. Does it all extend to arbitrary 2-knots (not necessarily "simple")? To arbitrary codimension-2 knots?

BF Following [CR]. $A\in\Omega^1(M=\mathbb{R}^4,\mathfrak{g})$, $B\in\Omega^2(M,\mathfrak{g}^*)$,

$$S(A,B):=\int_M\langle B,F_A\rangle.$$



Rossi

With $\kappa\colon(S=\mathbb{R}^2)\to M$, $\beta\in\Omega^0(S,\mathfrak{g})$, $\alpha\in\Omega^1(S,\mathfrak{g}^*)$, set

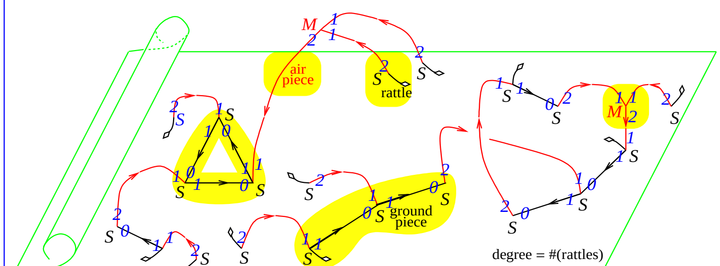
$$\mathcal{O}(A,B,\kappa):=\int\mathcal{D}\beta\mathcal{D}\alpha\exp\left(\frac{i}{\hbar}\int_S\langle\beta,d_{\kappa^*A}\alpha+\kappa^*B\rangle\right).$$

Cattaneo

The BF Feynman Rules. For an edge e, let Φ_e be its direction, in S^3 or S^1. Let ω_3 and ω_1 be volume forms on S^3 and S_1. Then

$$Z_{BF}=\sum_{\substack{\text{diagrams}\\ D}}\frac{[D]}{|\mathrm{Aut}(D)|}\underbrace{\int_{\mathbb{R}^2}\cdots\int_{\mathbb{R}^2}}_{S\text{-vertices}}\underbrace{\int_{\mathbb{R}^4}\cdots\int_{\mathbb{R}^4}}_{M\text{-vertices}}\prod_{\substack{\text{red}\\ e\in D}}\Phi_e^*\omega_3\prod_{\substack{\text{black}\\ e\in D}}\Phi_e^*\omega_1$$

(modulo some IHX-like relations). See also [Wa]



Issues. • Signs don't quite work out, and BF seems to reproduce only "half" of the wheels invariant on simple 2-knots.
• There are many more configuration space integrals than BF Feynman diagrams and than just trees and wheels.
• I don't know how to define / analyze "finite type" for general 2-knots.
• I don't know how to reduce Z_{BF} to combinatorics / algebra.

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"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org The Knot Atlas

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Hamilton-1412/