

Loading, initializing variables, setting default degree to 6.

Meaningless calculations.

(The Mathematica packages FreeLie' and AwCalculus' are at ⊞β/WKO4).

```
path = "C:/drorbn/AcademicPensieve/";
SetDirectory[path <> "2015-08/LesDiablerets-1508"];
Get[path <> "Projects/WKO4/FreeLie.m"];
Get[path <> "Projects/WKO4/AwCalculus.m"];
x = LW@"x"; y = LW@"y"; u = LW@"u";
$SeriesShowDegree = 6;
```

```
FreeLie' implements / extends
{*, +, **, $SeriesShowDegree, (<), ∫, ≡, ad, Ad, adSeries, AllCyclicWords,
AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence,
BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale,
DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA,
LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism,
New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t,
tb, TopBracketForm, tr, UndeterminedCoefficients, oMap, Γ, L, Δ, σ, h, r, r, r}.
```

FreeLie' is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

```
AwCalculus' implements / extends
{*, **, ≡, dA, dc, deg, dm, dS, dΔ, dh, dσ, E1, Es, hA, hm, hS, hΔ, hη,
hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.
```

AwCalculus' is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

BCH[x, y] (* Can raise degree to 22 *)

$$\text{LS} \left[\overline{x} + \overline{y}, \frac{\overline{xy}}{2}, \frac{1}{12} \overline{xx\overline{y}} + \frac{1}{12} \overline{x\overline{y}y}, \frac{1}{24} \overline{xx\overline{y}y}, \right. \\ \left. - \frac{1}{720} \overline{xxx\overline{xy}} + \frac{1}{180} \overline{xxx\overline{xy}y} + \frac{1}{180} \overline{xx\overline{y}yy} + \frac{1}{120} \overline{x\overline{y}xy} + \right. \\ \left. \frac{1}{360} \overline{xx\overline{y}xy} - \frac{1}{720} \overline{x\overline{y}yy} - \frac{1}{1440} \overline{xxx\overline{xy}y} + \frac{1}{360} \overline{xx\overline{y}yy} + \right. \\ \left. \frac{1}{240} \overline{x\overline{y}xy} + \frac{1}{720} \overline{xx\overline{y}xy} - \frac{1}{1440} \overline{xx\overline{y}yy}, \dots \right]$$

KV Direct.

{F = LS[{x, y}, Fs], G = LS[{x, y}, Gs]}; Fs["y"] = 1/2;

SeriesSolve[{F, G},

$$\hbar^{-1} (\text{LS}[x+y] - \text{BCH}[y, x] \equiv F - G - \text{Ad}[-x][F] + \text{Ad}[y][G]) \wedge \\ \text{div}_x[F] + \text{div}_y[G] \equiv \\ \frac{1}{2} \text{tr}_u \left[\text{adSeries} \left[\frac{\text{ad}}{\text{e}_{\text{ad}_1}}, x \right][u] + \text{adSeries} \left[\frac{\text{ad}}{\text{e}_{\text{ad}_1}}, y \right][u] - \right. \\ \left. \text{adSeries} \left[\frac{\text{ad}}{\text{e}_{\text{ad}_1}}, \text{BCH}[x, y] \right][u] \right];$$

{F, G} (* Can raise degree to 13 *)

$$\left\{ \text{LS} \left[\frac{\overline{y}}{2}, \frac{\overline{xy}}{6}, \frac{1}{24} \overline{x\overline{y}y}, -\frac{1}{180} \overline{xx\overline{xy}} + \frac{1}{80} \overline{x\overline{y}yy} + \frac{1}{360} \overline{x\overline{y}yy}, \right. \right. \\ \left. - \frac{1}{720} \overline{xxx\overline{xy}} + \frac{1}{240} \overline{xxx\overline{xy}y} + \frac{1}{240} \overline{x\overline{y}xy} + \frac{1}{720} \overline{xx\overline{y}xy} - \right. \\ \left. \frac{\overline{xy\overline{y}y}}{1440} + \frac{\overline{xxx\overline{xy}}}{5040} - \frac{\overline{xxx\overline{xy}y}}{1344} + \frac{13 \overline{xx\overline{y}yy}}{15120} + \frac{1}{840} \overline{x\overline{y}xy} + \right. \\ \left. \frac{\overline{xx\overline{y}xy}}{3360} + \frac{\overline{xx\overline{y}yy}}{6720} + \frac{\overline{xy\overline{y}y}}{1260} + \frac{\overline{xx\overline{y}xy}}{1680} - \frac{\overline{xy\overline{y}y}}{10080}, \dots \right], \\ \text{LS} \left[0, \frac{\overline{xy}}{12}, \frac{1}{24} \overline{x\overline{y}y}, -\frac{1}{360} \overline{xx\overline{xy}} + \frac{1}{120} \overline{x\overline{y}yy} + \frac{1}{180} \overline{x\overline{y}yy}, \right. \\ \left. - \frac{1}{720} \overline{xxx\overline{xy}} + \frac{1}{240} \overline{xxx\overline{xy}y} + \frac{1}{240} \overline{x\overline{y}xy} + \frac{1}{720} \overline{xx\overline{y}xy} - \right. \\ \left. \frac{\overline{xy\overline{y}y}}{1440} + \frac{\overline{xxx\overline{xy}}}{10080} - \frac{\overline{xxx\overline{xy}y}}{2016} + \frac{\overline{xx\overline{y}yy}}{1890} + \frac{\overline{xx\overline{y}xy}}{1120} + \frac{\overline{xx\overline{y}xy}}{5040} + \right. \\ \left. \frac{\overline{xx\overline{y}yy}}{2520} + \frac{1}{840} \overline{x\overline{y}xy} + \frac{\overline{xx\overline{y}xy}}{1260} - \frac{\overline{xy\overline{y}y}}{5040}, \dots \right] \}$$

{b[F, G], tr_x[F]}

$$\left\{ \text{LS} \left[0, 0, -\frac{1}{24} \overline{x\overline{y}y}, -\frac{1}{48} \overline{x\overline{y}yy}, \frac{1}{720} \overline{xx\overline{xy}} - \frac{1}{240} \overline{xx\overline{y}y} - \right. \right. \\ \left. \frac{\overline{xy\overline{y}y}}{1440} - \frac{1}{720} \overline{xx\overline{xy}} - \frac{1}{360} \overline{x\overline{y}yy}, \frac{\overline{xx\overline{y}y}}{1440} - \right. \\ \left. \frac{1}{480} \overline{xx\overline{y}yy} - \frac{1}{288} \overline{x\overline{y}xy} - \frac{7 \overline{xx\overline{y}xy}}{2880} + \frac{\overline{xy\overline{y}y}}{2880}, \dots \right], \\ \text{CWS} \left[-\frac{\overline{y}}{6}, \frac{\overline{xy}}{24}, \frac{\overline{xy}}{180} + \frac{\overline{xy}}{80} - \frac{\overline{xy}}{360}, -\frac{\overline{xy}}{180} + \frac{\overline{xy}}{240} - \frac{\overline{xy}}{240} - \frac{\overline{xy}}{1440}, \right. \\ \left. - \frac{\overline{xxxxy}}{5040} + \frac{\overline{xxxxy}}{6720} - \frac{\overline{xxxxy}}{1120} + \frac{2 \overline{xxxxy}}{945} - \frac{\overline{xxxxy}}{336} + \frac{\overline{xxxxy}}{6720} + \frac{\overline{xxxxy}}{10080}, \right. \\ \left. \frac{\overline{xxxxy}}{3360} - \frac{\overline{xxxxy}}{1344} - \frac{\overline{xxxxy}}{2240} + \frac{\overline{xxxxy}}{2016} + \frac{13 \overline{xxxxy}}{10080} + \frac{\overline{xxxxy}}{1680} - \right. \\ \left. \frac{\overline{xxxxy}}{3780} - \frac{\overline{xxxxy}}{840} + \frac{\overline{xxxxy}}{5040} + \frac{\overline{xxxxy}}{2240} + \frac{\overline{xxxxy}}{6720} + \frac{\overline{xxxxy}}{60480}, \dots \right] \}$$

(Also implemented: ∂_l and derivations in general, tb, e^{∂_l} and morphisms in general, div, j, Drinfel'd-Kohno, etc.)

The [BND] "vertex" equations.



```
α = LS[{x, y}, αs]; β = LS[{x, y}, βs];
γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn =
(V ** (V // dA) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
(Cap (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs["x"] = 1/2; βs["y"] = 0;
SeriesSolve[{α, β, γ, κ},
(ħ-1 R4Eqn) ∧ UnitarityEqn ∧ CapEqn];
{V, κ}
```

SeriesSolve:ArbitrarilySetting: In degree 1 arbitrarily setting {κs[x] → 0}.
SeriesSolve:ArbitrarilySetting: In degree 3 arbitrarily setting {αs[x, y] → 0}.
SeriesSolve:ArbitrarilySetting: In degree 5 arbitrarily setting {αs[x, x, y] → 0}.
General:stop:
Further output of SeriesSolve:ArbitrarilySetting will be suppressed during this calculation. >>

$$\left\{ \text{Es} \left[\overline{x} \rightarrow \text{LS} \left[0, -\frac{\overline{xy}}{24}, 0, \frac{7 \overline{xx\overline{xy}}}{5760} - \frac{7 \overline{xx\overline{y}y}}{5760} + \frac{\overline{xy\overline{y}y}}{1440}, 0, \right. \right. \right. \\ \left. - \frac{31 \overline{xxx\overline{xy}}}{967680} + \frac{31 \overline{xxx\overline{xy}y}}{483840} - \frac{83 \overline{xx\overline{y}yy}}{967680} - \frac{31 \overline{xx\overline{y}xy}}{725760} - \frac{31 \overline{xx\overline{y}xy}}{645120} + \right. \\ \left. \frac{13 \overline{xx\overline{y}yy}}{241920} + \frac{101 \overline{xy\overline{y}y}}{1451520} + \frac{527 \overline{xx\overline{y}xy}}{5806080} - \frac{\overline{xy\overline{y}y}}{60480}, \dots \right], \\ \overline{y} \rightarrow \text{LS} \left[\frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 0, \frac{\overline{xx\overline{xy}}}{5760} - \frac{1}{720} \overline{xx\overline{y}y} + \frac{1}{720} \overline{x\overline{y}yy}, -\frac{\overline{xx\overline{y}y}}{7680} + \right. \\ \left. \frac{\overline{xx\overline{y}y}}{3840} - \frac{\overline{xx\overline{y}xy}}{6912} - \frac{\overline{xxx\overline{xy}}}{645120} + \frac{23 \overline{xxx\overline{xy}}}{483840} - \frac{13 \overline{xx\overline{y}xy}}{161280} - \frac{\overline{xx\overline{y}xy}}{22680} - \right. \\ \left. \frac{41 \overline{xx\overline{y}xy}}{580608} + \frac{\overline{xx\overline{y}yy}}{15120} + \frac{\overline{xy\overline{y}y}}{12096} + \frac{71 \overline{xx\overline{y}xy}}{483840} - \frac{\overline{xy\overline{y}y}}{30240}, \dots \right], \\ \text{CWS} \left[0, -\frac{\overline{xy}}{48}, 0, \frac{\overline{xxxxy}}{2880} + \frac{\overline{xxxxy}}{2880} + \frac{\overline{xxxxy}}{5760} + \frac{\overline{xxxxy}}{2880}, 0, \right. \\ \left. - \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{120960} - \frac{241 \overline{xxxxxy}}{241920} - \frac{\overline{xxxxxy}}{120960} - \right. \\ \left. \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{362880} - \frac{\overline{xxxxxy}}{120960} - \frac{\overline{xxxxxy}}{241920} - \frac{\overline{xxxxxy}}{120960}, \dots \right], \\ \text{CWS} \left[0, -\frac{\overline{xx}}{96}, 0, \frac{\overline{xxxx}}{11520}, 0, -\frac{\overline{xxxx}}{725760}, \dots \right] \}$$