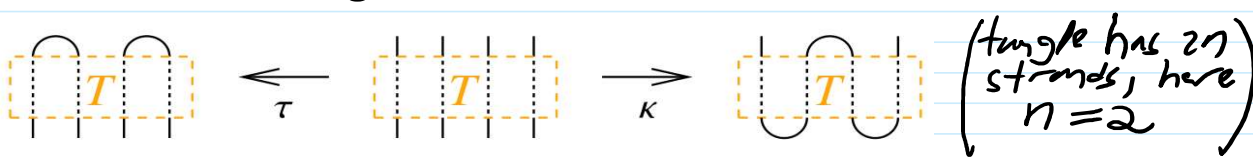
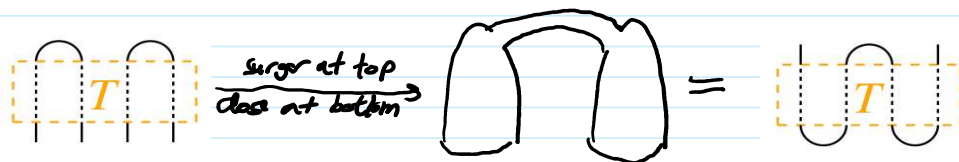


Proof of the Tangle Characterization of Ribbon Knots



Theorem. A knot K is ribbon iff there exists a tangle T whose τ closure is the untangle and whose κ closure is K .

Proof. The backward \Leftarrow implication is easy:

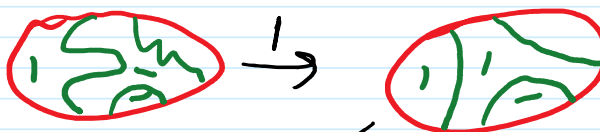


For the forward implication, follow the following 5 steps:



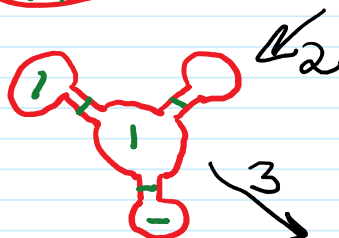
Step 1: In-situ cosmetics.

At end: D is a tree of chord-and-arc polygons.



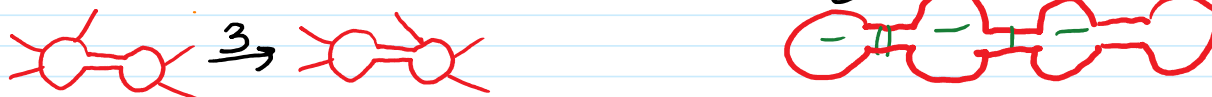
Step 2: Near-situ cosmetics.

At end: D is tree-band-sum of n unknotted disks.



Step 3: Slides.

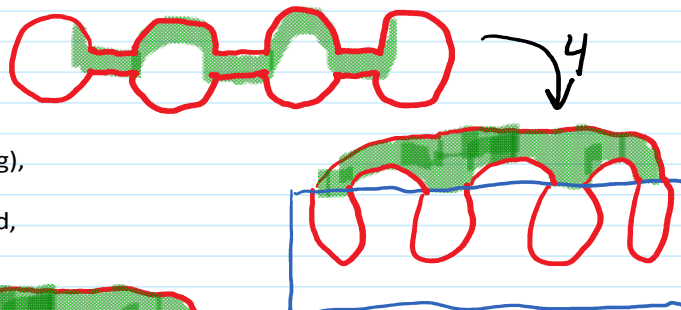
At end: D is a linear-band-sum of n unknotted disks.



Step 4: Exposure!

The green domain is contractible - so it can be shrank, moved at will (with the blue membrane following along), and expanded back again.

At end: D has $(n-1)$ exposed bridges which when turned, make D a union of n unknotted disks.



Step 5: Pulling bottom handles avoiding the obstacles.

At end: Theorem is proven.

