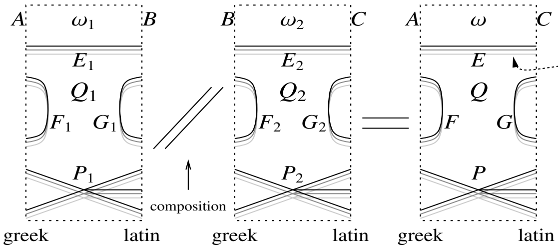
So What? If V is a representation, then $V^{\otimes n}$ explodes as a function of n, while in **DoPeGDO** up to a fixed power of ϵ, the ranks of mor$(A\to B)$ grow polynomially as a function of $|A|$ and $|B|$.

Compositions. In mor$(A\to B)$,

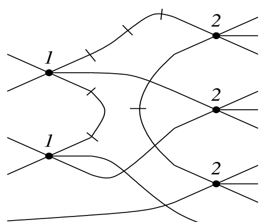
$$Q=\sum_{i\in A,j\in B}E_{ij}\zeta_i z_j+\frac12\sum_{i,j\in A}F_{ij}\zeta_i\zeta_j+\frac12\sum_{i,j\in B}G_{ij}z_iz_j,$$

and so (remember, $e^x=1+x+xx/2+xxx/6+\dots$)



$$E_1E_2+E_1F_2G_1E_2+E_1F_2G_1F_2G_1E_2+\dots=\sum_{r=0}^{\infty}E_1(F_2G_1)^rE_2$$

where • $E=E_1(I-F_2G_1)^{-1}E_2$.
• $F=F_1+E_1F_2(I-G_1F_2)^{-1}E_1^T$.
• $G=G_2+E_2^TG_1(I-F_2G_1)^{-1}E_2$.
• $\omega=\omega_1\omega_2\det(I-F_2G_1)^{-1}$.
• P is computed as the solution of a messy PDE or using "connected Feynman diagrams" (yet we're still in pure algebra!). Docility is preserved.



DoPeGDO Footnotes. Each variable has a "weight"$\in\{0,1,2\}$, and always wt z_i + wt $\zeta_i=2$.

†1. Really, "weight-graded finite sets" $A=A_0\sqcup A_1\sqcup A_2$.

†2. Really, a power series in the weight-0 variables†5.

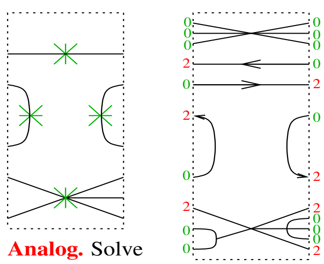
†3. The weight of Q must be 2, so it decomposes as $Q=Q_{20}+Q_{11}$. The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series†5.

†4. Setting wt $\epsilon=-2$, the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained)†5.

†5. In the knot-theoretic case, all weight-0 power series are rational functions of bounded degree in the exponentials of the weight-0 variables.

†6. There's also an obvious product mor$(A_1\to B_1)\times$mor$(A_2\to B_2)\to$ mor$(A_1\sqcup A_2\to B_1\sqcup B_2)$.

Full DoPeGDO. Compute compositions in two phases:
• A 1-1 phase over the ring of power series in the weight-0 variables, in which the weight-2 variables are spectators.
• A (slightly modified) 2-0 phase over $\mathbb{Q}$, in which the weight-1 variables are spectators.

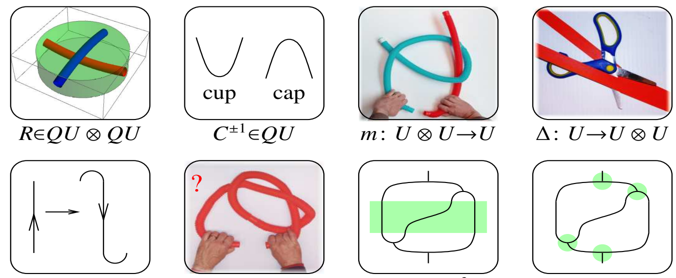


Analog. Solve $Ax=a,\ B(x)y=b$

Questions. • Are there QFT precedents for "two-step Gaussian integration"?
• In QFT, one saves even more by considering "one-particle-irreducible" diagrams and "effective actions". Does this mean anything here?
• Understanding Hom$(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B])$ seems like a good cause. Can you find other applications for the technology here?

($QU=\mathcal{U}_\hbar(sl^\epsilon_{2+})=A\langle y,b,a,x\rangle[\![\hbar]\!]$ with $[a,x]=x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-y$, $[b,x]=\epsilon x$, and $xy-qyx=(1-AB)/\hbar$, where $q=e^{\hbar\epsilon}$, $A=e^{-\hbar\epsilon a}$, and $B=e^{-\hbar b}$. Also $\Delta(y,b,a,x)=(y_1+B_1y_2,b_1+b_2,a_1+a_2,x_1+A_1x_2)$, $S(y,b,a,x)=(-B^{-1}y,-b,-a,-A^{-1}x)$, and $R=\sum\hbar^{j+k}y^kb^j\otimes a^jx^k/j![k]_q!$.)

Theorem. Everything of value regrading $U=CU$ and/or its quantization $U=QU$ is **DoPeGDO**:

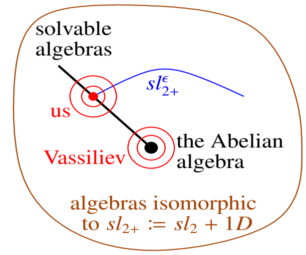


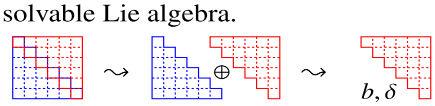
$R\in QU\otimes QU$ — $C^{\pm1}\in QU$ — $m\colon U\otimes U\to U$ — $\Delta\colon U\to U\otimes U$
$S:U\to U$ — tr: $U\to U/wx{=}xw$ — $\Phi\in CU^{\otimes 3}$ — $J\in CU\otimes CU$

also Cartan's θ, the $\mathbb{D}$equantizator, and more, and all of their compositions.

Solvable Approximation. In sl_n, half is enough! Indeed $sl_n\oplus\mathfrak{a}_{n-1}=\mathcal{D}(◹,b,\delta)$. Now define $sl^\epsilon_{n+}:=\mathcal{D}(◹,b,\epsilon\delta)$. Schematically, this is $[◹,◹]=◹$, $[◺,◺]=\epsilon◺$, and $[◹,◺]=◺+\epsilon◹$. The same process works for all semi-simple Lie algebras, and at $\epsilon^{k+1}=0$ always yields a solvable Lie algebra.

4D Metrized Lie Algebras





$b(◹)=b\colon ◹\otimes◹\to◹$
$b(◺)\rightsquigarrow\delta\colon ◹\to◹\otimes◹$

Conclusion. There are lots of poly-time-computable well-behaved near-Alexander knot invariants: • They extend to tangles with appropriate multiplicative behaviour. • They have cabling and strand reversal formulas. ωεβ/akt

The invariant for $sl^\epsilon_{2+}/(\epsilon^2=0)$ (prior art: ωεβ/Ov) attains 2,883 distinct values on the 2,978 prime knots with $\le$ 12 crossings. HOMFLY-PT and Khovanov homology together attain only 2,786 distinct values.

knot diag	n^t_k	Alexander's ω^+	genus / ribbon	$(\rho'_1)^+$	unknotting # / amphi?	$(\rho'_2)^+$
	0^a_1	1	0 / ✔	0	0 / ✔	0
	3^a_1	$T-1$	1 / ✘	T	1 / ✘	$3T^3-12T^2+26T-38$
	4^a_1	$3-T$	1 / ✘	0	1 / ✔	$T^4-3T^3-15T^2+74T-110$
	5^a_1	T^2-T+1	2 / ✘	$2T^3+3T$	2 / ✘	$5T^7-20T^6+55T^5-120T^4+217T^3-338T^2+450T-510$
	5^a_2	$2T-3$	1 / ✘	$5T-4$	1 / ✘	$-10T^4+120T^3-487T^2+1054T-1362$
	6^a_1	$5-2T$	1 / ✔	$T-4$	1 / ✘	$14T^4-16T^3-293T^2+1098T-1598$
	6^a_2	$-T^2+3T-3$	2 / ✘	T^3-4T^2+4T-4	1 / ✘	$3T^8-21T^7+49T^6+15T^5-433T^4+1543T^3-3431T^2+5482T-6410$
	6^a_3	T^2-3T+5	2 / ✘	0	1 / ✔	$4T^8-33T^7+121T^6-203T^5-111T^4+1499T^3-4210T^2+7186T-8510$
	7^a_1	T^3-T^2+T-1	3 / ✘	$3T^5+5T^3+6T$	3 / ✘	$7T^{11}-28T^{10}+77T^9-168T^8+322T^7-560T^6+891T^5-1310T^4+1777T^3-2238T^2+2604T-2772$

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Columbia-191125/