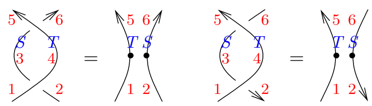
Reidemeister 2



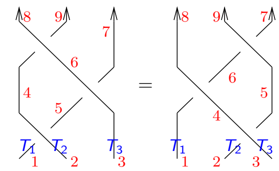
$\mathcal{A}@\{X_{2,4,3,1}[S,T], \overline{X}_{3,4,6,5}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{2,6}[S]\}$

True

$\mathcal{A}@\{\overline{X}_{3,1,2,4}[S,T], X_{6,5,3,4}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{6,2}[S]\}$

True

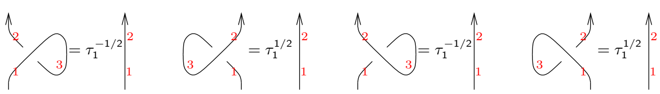
Reidemeister 3



$\mathcal{A}@\{X_{2,5,4,1}[T_2,T_1], X_{3,7,6,5}[T_3,T_1], X_{6,9,8,4}\} \equiv$
$\mathcal{A}@\{X_{3,5,4,2}[T_3,T_2], X_{4,6,8,1}[T_3,T_1], X_{5,7,9,6}\}$

True

Reidemeister 1

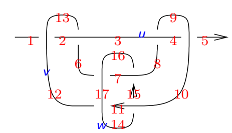


$\{\mathcal{A}@\{X_{3,3,2,1}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{X_{1,2,3,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\},$
$\mathcal{A}@\{\overline{X}_{1,3,3,2}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{\overline{X}_{3,1,2,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\}\}$

{True, True, True, True}

(So we have an invariant, up to rotation numbers).

The Relation with the Multivariable Alexander Polynomial



$\text{MVA} = u^{-1/2}\,v^{-1/2}\,w^{-1/2}\,(u-1)\,(v-1)\,(w-1);$

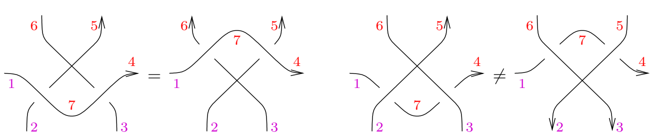
$A = \{\overline{X}_{1,12,2,13}[u,v], \overline{X}_{13,2,6,3}, X_{8,4,9,3}, X_{4,10,5,9}, X_{6,17,7,16}[v,w],$
$X_{15,8,16,7}, \overline{X}_{14,10,15,11}, \overline{X}_{11,17,12,14}\}$ // $\mathcal{A}$ // Last // Factor

$$\frac{(-1+u)^2\,(-1+v)\,(-1+w)\,(\text{Wedge}[\,] - x_5 \wedge \xi_1)}{u\,v}$$

$A == u^{-1/2}\,(u-1)\,u^{0}\,v^{-1/2}\,w^{1/2}\,\text{MVA}\,(\text{Wedge}[\wedge] - x_5 \wedge \xi_1)$

True

Overcrossings Commute but Undercrossings don't



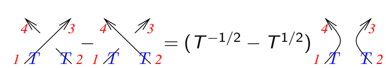
$\mathcal{A}@\{X_{2,7,5,1}, X_{3,4,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,1}, X_{2,4,5,7}\}$

True

$\mathcal{A}@\{\overline{X}_{1,2,7,5}, \overline{X}_{7,3,4,6}\} \equiv \mathcal{A}@\{\overline{X}_{1,3,7,6}, \overline{X}_{7,2,4,5}\}$

False

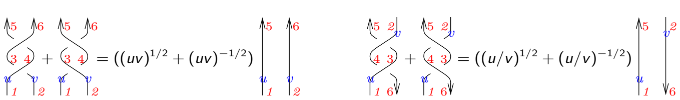
The Conway Relation (see [Co])



$\mathcal{A}@\{X_{2,3,4,1}[T,T]\} - \mathcal{A}@\{\overline{X}_{1,2,3,4}[T,T]\} \equiv (T^{-1/2} - T^{1/2})\,\mathcal{A}@\{P_{1,4}[T], P_{2,3}[T]\}$

True

Conway's Second Set of Identities (see [Co])



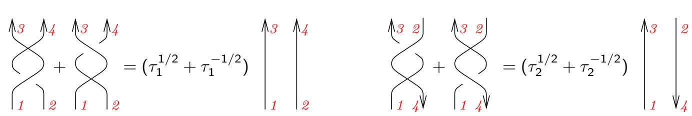
$\mathcal{A}@\{X_{2,4,3,1}[v,u], X_{4,6,5,3}\} + \mathcal{A}@\{\overline{X}_{1,2,4,3}[u,v], \overline{X}_{3,4,6,5}\} \equiv$
$(u^{1/2}\,v^{1/2} + u^{-1/2}\,v^{-1/2})\,\mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$

True

$\mathcal{A}@\{\overline{X}_{4,1,6,3}[v,u], \overline{X}_{3,2,5,4}\} + \mathcal{A}@\{X_{1,6,3,4}[u,v], X_{2,5,4,3}\} \equiv$
$(u^{1/2}\,v^{-1/2} + u^{-1/2}\,v^{1/2})\,\mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$

True

Virtual versions (Archibald, [Ar])



$\mathcal{A}@\{X_{2,3,4,1}\} + \mathcal{A}@\{\overline{X}_{2,1,4,3}\} \equiv (\tau_1^{1/2} + \tau_1^{-1/2})\,\mathcal{A}@\{P_{1,3}, P_{2,4}\}$

True

$\mathcal{A}@\{\overline{X}_{1,2,3,4}\} + \mathcal{A}@\{X_{1,4,3,2}\} \equiv (\tau_2^{1/2} + \tau_2^{-1/2})\,\mathcal{A}@\{P_{1,3}, P_{2,4}\}$

True

Video and more at http://www.math.toronto.edu/~drorbn/Talks/MoscowByWeb-2104/