Preliminaries

This is Rho.nb of http://drorbn.net/oa22/ap.

```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version
  of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/la22/ap
  to compute rotation numbers.
```

The Program

```
R1[s_, i_, j_] :=
  s (g_{j i} (g_{j+, j} + g_{j, j+} - g_{i j}) - g_{i i} (g_{j, j+} - 1) - 1/2);
Z[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :>
    (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1 ))];
                                   (  0     -1     )
  Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
  G = Inverse[A];
  ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (g_{k k} - 1/2);
  Factor@
   {Δ, Δ^2 ρ1 /. α_^+ :> α + 1 /. g_{α_, β_} :> G[[α, β]]}];
```

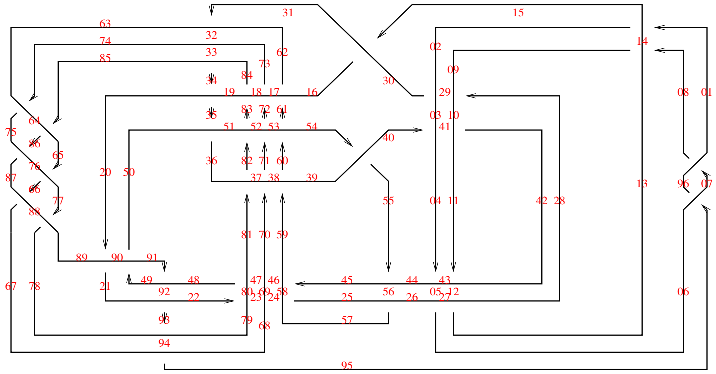
The First Few Knots

```
TableForm[Table[Join[{K[[1]]_{K[[2]]}}, Z[K]],
  {K, AllKnots[{3, 6}]}], TableAlignments → Center]
```

3_1	$\frac{1-T+T^2}{T}$	$\frac{(-1+T)^2(1+T^2)}{T^2}$
4_1	$-\frac{1-3T+T^2}{T}$	0
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$\frac{(-1+T)^2(1+T^2)(2+T^2+2T^4)}{T^4}$
5_2	$\frac{2-3T+2T^2}{T}$	$\frac{(-1+T)^2(5-4T+5T^2)}{T^2}$
6_1	$-\frac{(-2+T)(-1+2T)}{T}$	$\frac{(-1+T)^2(1-4T+T^2)}{T^2}$
6_2	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$\frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4}$
6_3	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	0

$p = 1 - T^s$

Fast!



```
Timing@
 Z[GST48 = EPD[X_{14,1}, X̄_{2,29}, X_{3,40}, X_{43,4}, X̄_{26,5}, X_{6,95},
    X_{96,7}, X_{13,8}, X̄_{9,28}, X_{10,41}, X_{42,11}, X̄_{27,12}, X_{30,15},
    X̄_{16,61}, X̄_{17,72}, X̄_{18,83}, X_{19,34}, X̄_{89,20}, X̄_{21,92},
    X̄_{79,22}, X̄_{68,23}, X̄_{57,24}, X̄_{25,56}, X_{62,31}, X̄_{73,32},
    X_{84,33}, X̄_{50,35}, X_{36,81}, X̄_{37,70}, X̄_{38,59}, X̄_{39,54}, X_{44,55},
    X_{58,45}, X_{69,46}, X_{80,47}, X_{48,91}, X_{90,49}, X_{51,82}, X_{52,71},
    X_{53,60}, X̄_{63,74}, X_{64,85}, X̄_{76,65}, X̄_{87,66}, X̄_{67,94},
    X̄_{75,86}, X̄_{88,77}, X̄_{78,93}]]
```

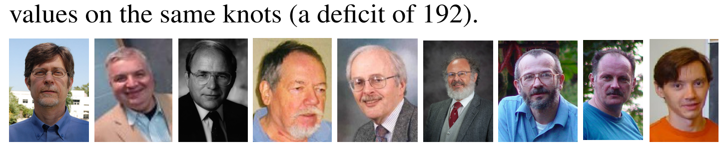
$$\Big\{170.313,\ \Big\{-\frac{1}{T^8}\left(-1+2T-T^2-T^3+2T^4-T^5+T^8\right)\left(-1+T^3-2T^4+T^5+T^6-2T^7+T^8\right),\ \frac{1}{T^{16}}(-1+T)^2\big(5-18T+33T^2-32T^3+2T^4+42T^5-62T^6-8T^7+166T^8-242T^9+108T^{10}+132T^{11}-226T^{12}+148T^{13}-11T^{14}-36T^{15}-11T^{16}+148T^{17}-226T^{18}+132T^{19}+108T^{20}-242T^{21}+166T^{22}-8T^{23}-62T^{24}+42T^{25}+2T^{26}-32T^{27}+33T^{28}-18T^{29}+5T^{30}\big)\Big\}\Big\}$$

Strong!

```
{NumberOfKnots[{3, 12}],
 Length@
  Union@Table[Z[K], {K, AllKnots[{3, 12}]}],
 Length@
  Union@Table[{HOMFLYPT[K], Kh[K]},
    {K, AllKnots[{3, 12}]}]}
```

```
{2977, 2882, 2785}
```

So the pair (Δ, ρ_1) attains 2,882 distinct values on the 2,977 prime knots with up to 12 crossings (a deficit of 95), whereas the pair (HOMFLYPT, Khovanov Homology) attains only 2,785 distinct values on the same knots (a deficit of 192).



Hoste Ocneanu Millett Freyd Lickorish Yetter Przytycki Traczyk Khovanov

Video: http://www.math.toronto.edu/~drorbn/Talks/Oaxaca-2210. Handout: http://www.math.toronto.edu/~drorbn/Talks/Nara-2308.