

Strand Operations. c for contract, mc for magnetic contract:

$$c_{i,j}@t : \Sigma_B[\{l_i, i, r_i\}, \{j, _ \}] [_] := t // GT_j, \text{First}[\{r_i, l_i\}] // \text{Cordon}_j$$

$$c_{i,j}@t : \Sigma_B[\{i, j, _ \}] [_] := \text{Cordon}_j @ t$$

$$c_{i,j}@t : \Sigma_B[\{j, _ \}, i, _ \}] [_] := \text{Cordon}_j @ t$$

$$c_{i,j}@t : \Sigma_B[\{i, j, _ \}] [_] := \text{Cordon}_i @ t$$

$$c_{i,j}@t : \Sigma_B[\{i, _ \}, j, _ \}] [_] := \text{Cordon}_i @ t$$

$$mc[\mathcal{E}] := \mathcal{E} //$$

$$t : \Sigma_B[\{i, _ \}, \{j, _ \}] [_] \mid \Sigma_B[\{i, j, _ \}] [_] \mid \Sigma_B[\{j, _ \}, i, _ \}] [_] / ; i + j = 0 \Rightarrow c_{i,j}@t$$

The Crossings (and empty strands).

$$\text{Kas}@P_{i,j} := \text{CF}@ \Sigma_B[\{i,j\}] [0, \text{PQ}[\{\}, \emptyset]] ;$$

$$\text{TL}@P_{i,j} := \text{CF}@ \Sigma_B[\{i,j\}] [0, \text{PQ}[\{\}, \emptyset]]$$

$$\text{Kas}[x : X[i, j, k, l]] :=$$

$$\text{Kas}@ \text{If}[\text{PositiveQ}[x], X_{-i,j,k,-l}, \bar{X}_{-j,k,l,-i}] ;$$

$$\text{Kas}[(x : X | \bar{X})_{fs}] := \text{Module}[\{v = 2u^2 - 1, p, \gamma s, m\},$$

$$\gamma s = \gamma_{\#} \& /@ \{fs\}; p = (x === X);$$

$$m = \text{If}[p, \begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}, -\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}];$$

$$\text{CF}@ \Sigma_B[\{fs\}] [\text{If}[p, -1, 1], \text{PQ}[\{\}, \gamma s^* \cdot m \cdot \gamma s]]]$$

$$\text{TL}[x : X[i, j, k, l]] :=$$

$$\text{TL}@ \text{If}[\text{PositiveQ}[x], X_{-i,j,k,-l}, \bar{X}_{-j,k,l,-i}] ;$$

$$\text{TL}[(x : X | \bar{X})_{fs}] := \text{Module}[\{t = 1 - \omega, r, \gamma s, m\},$$

$$r = t + t^*; \gamma s = \gamma_{\#} \& /@ \{fs\};$$

$$m = \text{If}[x === X,$$

$$\begin{pmatrix} -r & -t & 2t & t^* \\ -t^* & \theta & t^* & \theta \\ 2t^* & t & -r & -t^* \\ t & \theta & -t & \theta \end{pmatrix}, \begin{pmatrix} r & -t & -2t^* & t^* \\ -t^* & \theta & t^* & \theta \\ -2t & t & r & -t^* \\ t & \theta & -t & \theta \end{pmatrix}];$$

$$\text{CF}@ \Sigma_B[\{fs\}] [0, \text{PQ}[\{\}, \gamma s^* \cdot m \cdot \gamma s]]]$$

Evaluation on Tangles and Knots.

$$\text{Kas}[K] := \text{Fold}[\text{mc}[\#1 @ \#2] \&, \Sigma_B[0, \text{PQ}[\{\}, \emptyset]],$$

$$\text{List}@@(\text{Kas} /@ \text{PD}@K)];$$

$$\text{KasSig}[K] := \text{Expand}[\text{Kas}[K][1] / 2]$$

$$\text{TL}[K] :=$$

$$\text{Fold}[\text{mc}[\#1 @ \#2] \&, \Sigma_B[0, \text{PQ}[\{\}, \emptyset]],$$

$$\text{List}@@(\text{TL} /@ \text{PD}@K)] / .$$

$$\theta[c_+ + u] / ; \text{Abs}[c] \geq 1 \Rightarrow \theta[c];$$

$$\text{TL} \text{Sig}[K] := \text{TL}[K][1]$$

Reidemeister 3.

$$\text{R3L} = \text{PD}[X_{-2,5,4,-1}, X_{-3,7,6,-5},$$

$$X_{-6,9,8,-4}];$$

$$\text{R3R} = \text{PD}[X_{-3,5,4,-2}, X_{-4,6,8,-1},$$

$$X_{-5,7,9,-6}];$$

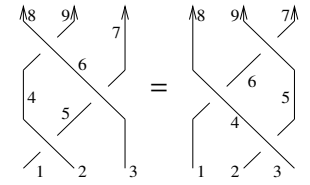
$$\{\text{TL}@R3L == \text{TL}@R3R, \text{Kas}@R3L == \text{Kas}@R3R\}$$

$$\{\text{True}, \text{True}\}$$

Kas@R3L

$$2\theta(u - \frac{1}{2}) - 2\theta(u + \frac{1}{2}) - 2$$

	γ_{-3}	γ_7	γ_9	γ_8	γ_{-1}	γ_{-2}
$\bar{\gamma}_{-3}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar{\gamma}_7$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar{\gamma}_9$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$
$\bar{\gamma}_8$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar{\gamma}_{-1}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar{\gamma}_{-2}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$



Reidemeister 2.

$$\text{TL}@ \text{PD}[X_{-2,4,3,-1}, \bar{X}_{-4,6,5,-3}]$$

$$\begin{matrix} & \theta & & & \\ & 1 & 0 & -1 & \theta \\ (\gamma_{-2} & \gamma_6 & \gamma_5 & \gamma_{-1}) & \\ \bar{\gamma}_{-2} & \theta & \theta & \theta & \theta \\ \bar{\gamma}_6 & \theta & \theta & \theta & \theta \\ \bar{\gamma}_5 & \theta & \theta & \theta & \theta \\ \bar{\gamma}_{-1} & \theta & \theta & \theta & \theta \end{matrix}$$

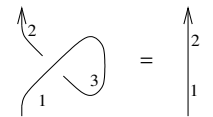
$$\{\text{TL}@ \text{PD}[X_{-2,4,3,-1}, \bar{X}_{-4,6,5,-3}] == \text{GT}_{5,-2} @ \text{TL}@ \text{PD}[P_{-1,5}, P_{-2,6}],$$

$$\{\text{True}, \text{True}\}$$

Reidemeister 1.

$$\{\text{TL}@ \text{PD}[X_{-3,3,2,-1}] == \text{TL}@ P_{-1,2},$$

$$\{\text{True}, \text{True}\}$$

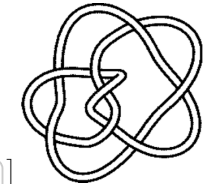


A Knot.

$$f = \text{TL} \text{Sig}[\text{Knot}[8, 5]]$$

$$2\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\theta\left[\frac{\sqrt{3}}{2} + u\right] -$$

$$2\theta\left[u - \sqrt{-0.630\dots}\right] + 2\theta\left[u - \sqrt{0.630\dots}\right]$$



$$\text{Plot}[f, \{u, -1, 1\}]$$

