

```
In[*]:= PF[ε_] := Collect[ε, τ | t | η | y | α | a | ξ | x, Simplify];
PF[ε_] := Collect[
  Total@Module[{vs = {τ, η, y, a, ξ, x}},
    vs = vs ∪ Flatten@Table[#1 & /@ vs, {i, 0, 2}];
    CoefficientRules[ε, vs] /. (p_ -> c_) => Simplify[c] Times @@ (vs^p)],
  ε]
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In[*]:= SΛ =

$$\text{Exp}\left[-\tau t + \frac{1}{\hbar T_1} \left(e^{\alpha \gamma \hbar} \eta \xi \hbar^2 - e^{\alpha \gamma \hbar} \eta \xi \hbar^2 T_1 - \alpha \hbar^2 a_1 T_1 - e^{\alpha \gamma \hbar} \xi \hbar^2 T_1 x_1 - e^{\alpha \gamma \hbar} \eta \hbar^2 y_1 \right) \right] \left(1 + \frac{1}{4 \hbar T_1^2} \right. \\ \left. \left(-3 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi^2 \hbar^4 - 4 e^{\alpha \gamma \hbar} \gamma \eta \xi \hbar^3 T_1 + 4 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi^2 \hbar^4 T_1 + 8 e^{\alpha \gamma \hbar} \eta \xi \hbar^3 a_1 T_1 + 4 e^{\alpha \gamma \hbar} \gamma \eta \xi \hbar^3 T_1^2 - e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi^2 \hbar^4 T_1^2 + 6 e^{2 \alpha \gamma \hbar} \gamma \eta \xi^2 \hbar^4 T_1 x_1 - 2 e^{2 \alpha \gamma \hbar} \gamma \eta \xi^2 \hbar^4 T_1^2 x_1 - 4 e^{\alpha \gamma \hbar} \xi \hbar^3 a_1 T_1^2 x_1 - 2 e^{2 \alpha \gamma \hbar} \gamma \xi^2 \hbar^4 T_1^2 x_1^2 + 6 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi \hbar^4 y_1 + 4 e^{\alpha \gamma \hbar} \gamma \eta \hbar^3 T_1 y_1 - 2 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi \hbar^4 T_1 y_1 - 4 e^{\alpha \gamma \hbar} \eta \hbar^3 a_1 T_1 y_1 - 4 e^{2 \alpha \gamma \hbar} \gamma \eta \xi \hbar^4 T_1 x_1 y_1 - 2 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi \hbar^4 y_1^2 \right) \right) \epsilon \Big) /. u_{-1} \Rightarrow u$$

$$\text{Out[*]} = e^{-\tau t + \frac{-a T \alpha \hbar^2 - e^{\alpha \gamma \hbar} \eta \hbar^2 - e^{\alpha \gamma \hbar} T x \xi \hbar^2 + e^{\alpha \gamma \hbar} \eta \xi \hbar^2 - e^{\alpha \gamma \hbar} T \eta \xi \hbar^2}{T \hbar}} \left(1 + \frac{1}{4 T^2 \hbar} \left(-4 a e^{\alpha \gamma \hbar} T \gamma \eta \hbar^3 + 4 e^{\alpha \gamma \hbar} T \gamma \gamma \eta \hbar^3 - 4 a e^{\alpha \gamma \hbar} T^2 x \xi \hbar^3 + 8 a e^{\alpha \gamma \hbar} T \eta \xi \hbar^3 - 4 e^{\alpha \gamma \hbar} T \gamma \eta \xi \hbar^3 + 4 e^{\alpha \gamma \hbar} T^2 \gamma \eta \xi \hbar^3 - 2 e^{2 \alpha \gamma \hbar} \gamma^2 \eta^2 \hbar^4 - 4 e^{2 \alpha \gamma \hbar} T x \gamma \eta \xi \hbar^4 + 6 e^{2 \alpha \gamma \hbar} \gamma \gamma \eta^2 \xi \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T \gamma \gamma \eta^2 \xi \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T^2 x^2 \gamma \xi^2 \hbar^4 + 6 e^{2 \alpha \gamma \hbar} T x \gamma \eta \xi^2 \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T^2 x \gamma \eta \xi^2 \hbar^4 - 3 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi^2 \hbar^4 + 4 e^{2 \alpha \gamma \hbar} T \gamma \eta^2 \xi^2 \hbar^4 - e^{2 \alpha \gamma \hbar} T^2 \gamma \eta^2 \xi^2 \hbar^4 \right) \right)$$

In[*]:= FullSimplify[Coefficient[SΛ, ε]]

$$\text{Out[*]} = \frac{1}{4 T^2} e^{-\tau t + \alpha (-a + \gamma) \hbar - \frac{e^{\alpha \gamma \hbar} (\gamma \eta + T x \xi + (-1 + T) \eta \xi) \hbar}{T}} \hbar^2 \left(4 T \gamma \eta (y + (-1 + T) \xi) - 4 a T (y \eta + T x \xi - 2 \eta \xi) - e^{\alpha \gamma \hbar} \gamma (2 y^2 \eta^2 + 2 y \eta (2 T x + (-3 + T) \eta) \xi + (2 T^2 x^2 + 2 (-3 + T) T x \eta + (-3 + T) (-1 + T) \eta^2) \xi^2) \hbar \right)$$

In[*]:= PF[-t τ + 1/(T ħ) (-a T α ħ² - e^{α γ ħ} y η ħ² - e^{α γ ħ} T x ξ ħ² + e^{α γ ħ} η ξ ħ² - e^{α γ ħ} T η ξ ħ²)] //

TraditionalForm

Out[*]//TraditionalForm=

$$-a \alpha \hbar - t \tau - \frac{\eta \xi (T - 1) \hbar e^{\alpha \gamma \hbar}}{T} - \frac{\eta y \hbar e^{\alpha \gamma \hbar}}{T} - \xi x \hbar e^{\alpha \gamma \hbar}$$

In[*]:= PF[-t τ + 1/(T ħ) (-a T α ħ² - e^{α γ ħ} y η ħ² - e^{α γ ħ} T x ξ ħ² + e^{α γ ħ} η ξ ħ² - e^{α γ ħ} T η ξ ħ²)] // TeXForm

$$\text{Out[*]//TeXForm} = -a \ \alpha \ \hbar - t \ \tau - \frac{\eta \ \xi \ (T - 1) \ \hbar \ e^{\{\alpha \ \gamma \ \hbar\}}}{T} - \frac{\eta \ y \ \hbar \ e^{\{\alpha \ \gamma \ \hbar\}}}{T} - \xi \ x \ \hbar \ e^{\{\alpha \ \gamma \ \hbar\}}$$

$$\text{In[*]:= PF} \left[\left(1 + \frac{1}{4 T^2 \hbar} \epsilon \left(-4 a e^{\alpha \gamma \hbar} T \gamma \eta \hbar^3 + 4 e^{\alpha \gamma \hbar} T \gamma \gamma \eta \hbar^3 - 4 a e^{\alpha \gamma \hbar} T^2 x \xi \hbar^3 + 8 a e^{\alpha \gamma \hbar} T \eta \xi \hbar^3 - 4 e^{\alpha \gamma \hbar} T \gamma \eta \xi \hbar^3 + 4 e^{\alpha \gamma \hbar} T^2 \gamma \eta \xi \hbar^3 - 2 e^{2 \alpha \gamma \hbar} y^2 \gamma \eta^2 \hbar^4 - 4 e^{2 \alpha \gamma \hbar} T x y \gamma \eta \xi \hbar^4 + 6 e^{2 \alpha \gamma \hbar} y \gamma \eta^2 \xi \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T y \gamma \eta^2 \xi \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T^2 x^2 \gamma \xi^2 \hbar^4 + 6 e^{2 \alpha \gamma \hbar} T x \gamma \eta \xi^2 \hbar^4 - 2 e^{2 \alpha \gamma \hbar} T^2 x \gamma \eta \xi^2 \hbar^4 - 3 e^{2 \alpha \gamma \hbar} \gamma \eta^2 \xi^2 \hbar^4 + 4 e^{2 \alpha \gamma \hbar} T \gamma \eta^2 \xi^2 \hbar^4 - e^{2 \alpha \gamma \hbar} T^2 \gamma \eta^2 \xi^2 \hbar^4 \right) \right] // \text{TeXForm}$$

$$\text{Out[*]//TeXForm} = \epsilon \left(\frac{2 a \eta x \hbar^2 e^{\alpha \gamma \hbar} T - \frac{a \eta x \hbar^2 e^{\alpha \gamma \hbar} T - \frac{\gamma \eta^2 x \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{4 T^2} - \frac{\gamma \eta^2 y \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{\gamma \eta^2} + \frac{\gamma \eta^2 x \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{\gamma \eta^2} - \frac{\gamma \eta^2 x \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{\gamma \eta^2} + \frac{\gamma \eta^2 x \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{\gamma \eta^2} + \frac{\gamma \eta^2 x \hbar^3 e^{2 \alpha \gamma \hbar} T^2}{\gamma \eta^2} \right) + 1$$

$$\text{In[*]:= PF} [\alpha \hbar a_1 + \alpha \hbar a_2 + \xi \hbar x_1 + \xi \hbar x_2 + \eta \hbar y_1 + \eta \hbar T_1 y_2]$$

$$\text{Out[*]:= } \alpha \hbar a_1 + \alpha \hbar a_2 + \xi \hbar x_1 + \xi \hbar x_2 + \eta \hbar y_1 + \eta \hbar T_1 y_2$$

$$\text{In[*]:= PF} \left[\frac{1}{2} \left(-2 \xi \hbar^2 a_1 x_2 + \gamma \xi^2 \hbar^3 x_1 x_2 - 2 \eta \hbar^2 a_1 T_1 y_2 + \gamma \eta^2 \hbar^3 T_1 y_1 y_2 \right) \right] // \text{TeXForm}$$

$$\text{Out[*]//TeXForm} = -a_1 \eta T_1 y_2 \hbar^2 - a_1 x_2 \hbar^2 + \frac{1}{2} \gamma \eta^2 T_1 y_1 y_2 \hbar^3$$

$$\text{In[*]:= } \mathbf{qo} = \mathbf{C}_{\mathbf{QU}}[\{\mathbf{y}_1, \mathbf{a}_1, \mathbf{x}_1\}_1, \{\mathbf{y}_2, \mathbf{a}_2, \mathbf{x}_2\}_2, \hbar (\eta_1 \mathbf{y}_1 + \alpha_1 \mathbf{a}_1 + \xi_1 \mathbf{x}_1 + \eta_2 \mathbf{y}_2 + \alpha_2 \mathbf{a}_2 + \xi_2 \mathbf{x}_2), \mathbf{1} + \mathbf{\theta}_1];$$

$$\text{Timing}@\{\mathbf{qo} // \mathbf{m}_{1,2 \rightarrow 0}, \mathbf{HL}@\mathbf{SimpT}[\mathbf{QU}[\mathbf{m}_{1,2 \rightarrow 0}[\mathbf{qo}]] - \mathbf{m}_{1,2 \rightarrow 0}[\mathbf{QU}[\mathbf{qo}]]]\}$$

$$\text{Out[*]:= } \{4.35938, \left\{ \mathbf{C}_{\mathbf{QU}}[\{\mathbf{y}_0, \mathbf{a}_0, \mathbf{x}_0\}_0, e^{-\gamma \hbar \alpha_1 - \gamma \hbar \alpha_2} (e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar a_0 \alpha_1 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar a_0 \alpha_2 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar y_0 \eta_1 + e^{\gamma \hbar \alpha_2} \hbar y_0 \eta_2 + e^{\gamma \hbar \alpha_1} \hbar x_0 \xi_1 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar \eta_2 \xi_1 - e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar T_0 \eta_2 \xi_1 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar x_0 \xi_2), \right. \\ \left. 1 + \frac{1}{4} e^{-\gamma \hbar \alpha_1 - \gamma \hbar \alpha_2} (8 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar^2 a_0 T_0 \eta_2 \xi_1 + 4 \gamma \hbar^3 x_0 y_0 \eta_2 \xi_1 + 2 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 y_0 \eta_2^2 \xi_1 - 6 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 T_0 y_0 \eta_2^2 \xi_1 + 2 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 x_0 \eta_2 \xi_1^2 - 6 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 T_0 x_0 \eta_2 \xi_1^2 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 \eta_2^2 \xi_1^2 - 4 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0 \eta_2^2 \xi_1^2 + 3 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0^2 \eta_2^2 \xi_1^2) \epsilon + \mathbf{O}[\epsilon]^2 \right\}, \mathbf{\theta} \}$$

$$\text{In[*]:= } \mathbf{qo} = \mathbf{C}_{\mathbf{QU}}[\{\mathbf{y}_1, \mathbf{a}_1, \mathbf{x}_1\}_1, \{\mathbf{y}_2, \mathbf{a}_2, \mathbf{x}_2\}_2, \eta_1 \mathbf{y}_1 + \alpha_1 \mathbf{a}_1 + \xi_1 \mathbf{x}_1 + \eta_2 \mathbf{y}_2 + \alpha_2 \mathbf{a}_2 + \xi_2 \mathbf{x}_2, \mathbf{1} + \mathbf{\theta}_1];$$

$$\text{Timing}@\{\mathbf{qo} // \mathbf{m}_{1,2 \rightarrow 0}\}$$

$$\text{Out[*]:= } \{2.125, \left\{ \mathbf{C}_{\mathbf{QU}}[\{\mathbf{y}_0, \mathbf{a}_0, \mathbf{x}_0\}_0, \frac{1}{\hbar} e^{-\gamma \alpha_1 - \gamma \alpha_2} (e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar a_0 \alpha_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar a_0 \alpha_2 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar y_0 \eta_1 + e^{\gamma \alpha_2} \hbar y_0 \eta_2 + e^{\gamma \alpha_1} \hbar x_0 \xi_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \eta_2 \xi_1 - e^{\gamma \alpha_1 + \gamma \alpha_2} T_0 \eta_2 \xi_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar x_0 \xi_2), \right. \\ \left. 1 + \frac{1}{4 \hbar} e^{-\gamma \alpha_1 - \gamma \alpha_2} (8 e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar a_0 T_0 \eta_2 \xi_1 + 4 \gamma \hbar^2 x_0 y_0 \eta_2 \xi_1 + 2 e^{\gamma \alpha_2} \gamma \hbar y_0 \eta_2^2 \xi_1 - 6 e^{\gamma \alpha_2} \gamma \hbar T_0 y_0 \eta_2^2 \xi_1 + 2 e^{\gamma \alpha_1} \gamma \hbar x_0 \eta_2 \xi_1^2 - 6 e^{\gamma \alpha_1} \gamma \hbar T_0 x_0 \eta_2 \xi_1^2 + e^{\gamma \alpha_1 + \gamma \alpha_2} \gamma \eta_2^2 \xi_1^2 - 4 e^{\gamma \alpha_1 + \gamma \alpha_2} \gamma T_0 \eta_2^2 \xi_1^2 + 3 e^{\gamma \alpha_1 + \gamma \alpha_2} \gamma T_0^2 \eta_2^2 \xi_1^2) \epsilon + \mathbf{O}[\epsilon]^2 \right\} \}$$

In[*]:= Simplify[$\frac{1}{\hbar} e^{-\gamma \alpha_1 - \gamma \alpha_2} (e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar a_0 \alpha_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar a_0 \alpha_2 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar y_0 \eta_1 + e^{\gamma \alpha_2} \hbar y_0 \eta_2 + e^{\gamma \alpha_1} \hbar x_0 \xi_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \eta_2 \xi_1 - e^{\gamma \alpha_1 + \gamma \alpha_2} T_0 \eta_2 \xi_1 + e^{\gamma \alpha_1 + \gamma \alpha_2} \hbar x_0 \xi_2)$ /. $u_{-0} := u$] // PF

Out[*]:= $a (\alpha_1 + \alpha_2) + y \eta_1 + e^{-\gamma \alpha_1} y \eta_2 + e^{-\gamma \alpha_2} x \xi_1 + \frac{(1 - T) \eta_2 \xi_1}{\hbar} + x \xi_2$

In[*]:= PF[$\frac{1}{4} e^{-\gamma \hbar \alpha_1 - \gamma \hbar \alpha_2} (8 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar^2 a_0 T_0 \eta_2 \xi_1 + 4 \gamma \hbar^3 x_0 y_0 \eta_2 \xi_1 + 2 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 y_0 \eta_2^2 \xi_1 - 6 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 T_0 y_0 \eta_2^2 \xi_1 + 2 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 x_0 \eta_2 \xi_1^2 - 6 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 T_0 x_0 \eta_2 \xi_1^2 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 \eta_2^2 \xi_1^2 - 4 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0 \eta_2^2 \xi_1^2 + 3 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0^2 \eta_2^2 \xi_1^2)$ /. $u_{-0} := u$]

Out[*]:= $2 a T \hbar^2 \eta_2 \xi_1 + e^{-\gamma \hbar (\alpha_1 + \alpha_2)} x y \gamma \hbar^3 \eta_2 \xi_1 - \frac{1}{2} e^{-\gamma \hbar \alpha_1} (-1 + 3 T) y \gamma \hbar^3 \eta_2^2 \xi_1 - \frac{1}{2} e^{-\gamma \hbar \alpha_2} (-1 + 3 T) x \gamma \hbar^3 \eta_2 \xi_1^2 + \frac{1}{4} (1 - 4 T + 3 T^2) \gamma \hbar^3 \eta_2^2 \xi_1^2$

In[*]:= PF[$\frac{1}{4} e^{-\gamma \hbar \alpha_1 - \gamma \hbar \alpha_2} (8 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \hbar^2 a_0 T_0 \eta_2 \xi_1 + 4 \gamma \hbar^3 x_0 y_0 \eta_2 \xi_1 + 2 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 y_0 \eta_2^2 \xi_1 - 6 e^{\gamma \hbar \alpha_2} \gamma \hbar^3 T_0 y_0 \eta_2^2 \xi_1 + 2 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 x_0 \eta_2 \xi_1^2 - 6 e^{\gamma \hbar \alpha_1} \gamma \hbar^3 T_0 x_0 \eta_2 \xi_1^2 + e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 \eta_2^2 \xi_1^2 - 4 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0 \eta_2^2 \xi_1^2 + 3 e^{\gamma \hbar \alpha_1 + \gamma \hbar \alpha_2} \gamma \hbar^3 T_0^2 \eta_2^2 \xi_1^2)$ /. $u_{-0} := u$] // TeXForm

Out[*]//TeXForm= $2 a \eta_2 \xi_1 T \hbar^2 + \frac{1}{4} \gamma \eta_2^2 \xi_1^2 \left(3 T^2 - 4 T + 1 \right) e^{-\gamma \hbar (\alpha_1 + \alpha_2)} x y \hbar^3 + \frac{1}{2} \gamma \eta_2 \xi_1^2 e^{-\gamma \hbar \alpha_2} (-1 + 3 T) x + \frac{1}{4} \gamma \eta_2^2 \xi_1^2 (1 - 4 T + 3 T^2) e^{-\gamma \hbar \alpha_1} (-1 + 3 T) y$