Pensieve header: A talk and a program about Archibald- ($\mathcal A$-) and Γ-calculus and the Halacheva map between them; the Γ part. Continues pensieve://2021-03/

Γ-Calculus

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\begin{frame}{\LARGE 6. An Implementation of Γ.}
If I didn't implement I wouldn't believe myself.
\vskip 2mm
Written in Mathematica~\cite{Wolfram:Mathematica}, available as the notebook {\tt Gamma.nb} at \url{http://drorbn.net/mo21/ap}. Code lines are highlighted in grey, demo lines are plain.
We start with canonical forms for quadratics with rational function coefficients:

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In[∘]:=
```
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_] := Module[{vs = Union@Cases[ℰ, (ξ | x)_, ∞]},
   Total[(CCF[#[[2]]] (Times @@ vs^#[[1]])) & /@ CoefficientRules[ℰ, vs]]];
```

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\end{frame}
\begin{frame}\null
Multiplying and comparing Γ objects:

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In[∘]:=
```
Γ /: Γ[is1_, os1_, cs1_, ω1_, λ1_] Γ[is2_, os2_, cs2_, ω2_, λ2_] :=
  Γ[is1 ⋃ is2, os1 ⋃ os2, Join[cs1, cs2], ω1 ω2, λ1 + λ2]
Γ /: Γ[is1_, os1_, _, ω1_, λ1_] ≡ Γ[is2_, os2_, _, ω2_, λ2_] := TrueQ[
   (Sort@is1 === Sort@is2) ∧ (Sort@os1 === Sort@os2) ∧ Simplify[ω1 == ω2] ∧ CF@λ1 == CF@λ2]
```

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No rules for linear operations!
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Contractions:

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In[∘]:=
```
c_{h_,t_}@Γ[is_, os_, cs_, ω_, λ_] := Module[{α, η, y, μ},
   α = ∂_{ξ_t,x_h} λ; μ = λ /. ξ_t | x_h → 0;
   η = ∂_{x_h} λ /. ξ_t → 0; y = ∂_{ξ_t} λ /. x_h → 0;
   Γ[
     DeleteCases[is, t], DeleteCases[os, h], KeyDrop[cs, {x_h, ξ_t}],
     CCF[(1 - α) ω], CF[μ + η y / (1 - α)]
    ] /. If[MatchQ[cs[ξ_t], τ_], cs[ξ_t] → cs[x_h], cs[x_h] → cs[ξ_t]]];
c@Γ[is_, os_, cs_, ω_, λ_] := Fold[c_{#2,#2}[#1] &, Γ[is, os, cs, ω, λ], is ⋂ os]
```

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The crossings and the point:

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In[•]:=
```
Γ[X_(i_,j_,k_,l_)[S_, T_]] := Γ[{l, i}, {j, k},
   ⟨|ξ_i → S, x_j → T, x_k → S, ξ_l → T|⟩, T^(-1/2), CF[{ξ_l, ξ_i}.( 1  1-T
                                                                      0   T  ).{x_j, x_k}]];
Γ[X̄_(i_,j_,k_,l_)[S_, T_]] := Γ[{i, j}, {k, l}, ⟨|ξ_i → S, ξ_j → T, x_k → S, x_l → T|⟩,
   T^(1/2), CF[{ξ_i, ξ_j}.(  T^-1   0
                            1-T^-1  1 ).{x_k, x_l}]];
Γ[X_(i_,j_,k_,l_)] := Γ[X_(i,j,k,l)[τ_i, τ_l]];
Γ[X̄_(i_,j_,k_,l_)] := Γ[X̄_(i,j,k,l)[τ_i, τ_j]];
```

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In[•]:=
```
Γ[P_(i_,j_)[T_]] := Γ[{i}, {j}, ⟨|ξ_i → T, x_j → T|⟩, 1, ξ_i x_j];
Γ[P_(i_,j_)] := Γ[P_(i,j)[τ_i]];
```

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\end{frame}
\begin{frame}\null
Automatic intelligent contractions:

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In[•]:=
```
Γ[{γ_Γ}] := c[γ];
Γ[{γ1_Γ, γs__Γ}] := Module[{γ2},
  γ2 = First@MaximalBy[{γs}, Length[γ1⟦1⟧ ⋂ #⟦2⟧] + Length[γ1⟦2⟧ ⋂ #⟦1⟧] &];
  Γ[Join[{c[γ1 γ2]}, DeleteCases[{γs}, γ2]]] ]
Γ[Γs_List] := Γ[Γ /@ Γs]
```

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\end{frame}
\begin{frame}\null
Conversions $\calA\leftrightarrow\Gamma$:

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In[•]:=
```
Γ@𝒜[is_, os_, cs_, w_] := Module[{i, j, ω = Coefficient[w, Wedge[]]},
   Γ[is, os, cs, ω, Sum[Cancel[-Coefficient[w, x_j ∧ ξ_i] ξ_i x_j / ω], {i, is}, {j, os}]]
  ];
𝒜@Γ[is_, os_, cs_, ω_, λ_] := 𝒜[is, os, cs, Expand[ω WExp[Expand[λ] /. ξ_a_ x_b_ :> ξ_a ∧ x_b]]];
```

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The conversions are inverses of each other:

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In[∘]:= $\Gamma[\{1,2,3\},\{1,2,3\},\{x_1\to\tau_1,\ x_2\to\tau_2,\ x_3\to\tau_3,\ \xi_1\to\tau_1,\ \xi_2\to\tau_2,\ \xi_3\to\tau_3\},\ \omega,$
$a_{11}\,x_1\,\xi_1+a_{12}\,x_2\,\xi_1+a_{13}\,x_3\,\xi_1+a_{21}\,x_1\,\xi_2+a_{22}\,x_2\,\xi_2+a_{23}\,x_3\,\xi_2+a_{31}\,x_1\,\xi_3+a_{32}\,x_2\,\xi_3+a_{33}\,x_3\,\xi_3];$
$\Gamma@$
$\mathcal{A}@$
$\gamma == \gamma$

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Out[∘]= True

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The conversions commute with contractions:

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In[∘]:= $\Gamma@c_{3,3}@\mathcal{A}@\gamma \equiv c_{3,3}@\gamma$

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Out[∘]= True

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\end{frame}

The Naik-Stanford Double Delta Move (again)

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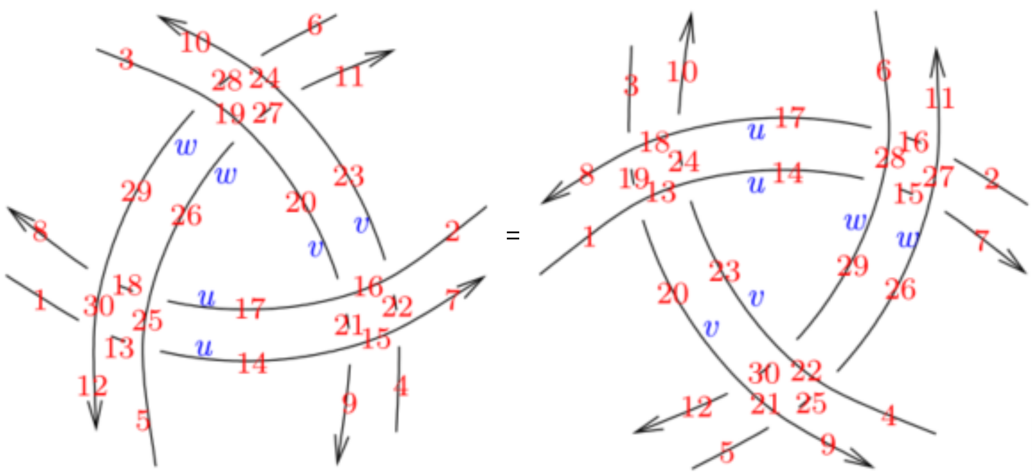
\begin{frame} {\large The Naik-Stanford Double Delta Move (again)}
\[\scalebox{0.8}{\input{figs/NaikStanford.pdf_t}} \]



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In[∘]:= $\text{Timing}\big[\Gamma@\{X_{6,10,28,24}[w,v],\ \overline{X}_{28,3,29,19}[w,v],\ X_{26,20,27,19}[w,v],\ \overline{X}_{27,23,11,24}[w,v],$
$X_{1,12,13,30}[u,w],\ \overline{X}_{13,5,14,25}[u,w],\ X_{17,26,18,25}[u,w],\ \overline{X}_{18,29,8,30}[u,w],$
$X_{4,7,22,15}[v,u],\ \overline{X}_{22,2,23,16}[v,u],\ X_{20,17,21,16}[v,u],\ \overline{X}_{21,14,9,15}[v,u]\} \equiv$
$\Gamma@\{X_{5,9,25,21}[w,v],\ \overline{X}_{25,4,26,22}[w,v],\ X_{29,23,30,22}[w,v],\ \overline{X}_{30,20,12,21}[w,v],$
$X_{2,11,16,27}[u,w],\ \overline{X}_{16,6,17,28}[u,w],\ X_{14,29,15,28}[u,w],\ \overline{X}_{15,26,7,27}[u,w],$
$X_{3,8,19,18}[v,u],\ \overline{X}_{19,1,20,13}[v,u],\ X_{23,14,24,13}[v,u],\ \overline{X}_{24,17,10,18}[v,u]\}\big]$

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Out[∘]= {1.28125, True}

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\end{frame}

Conway’s Third Identity (again)

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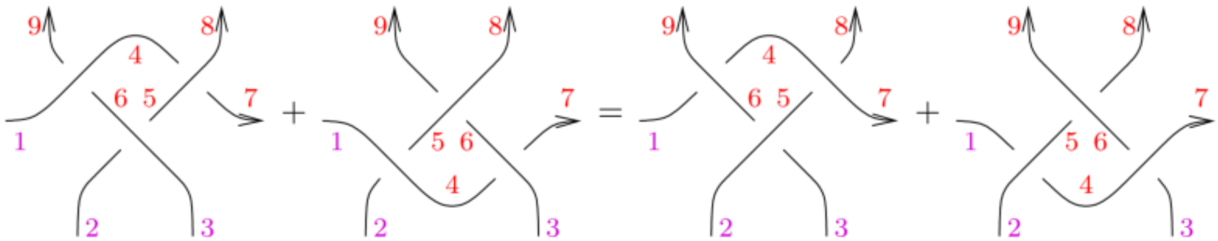
\begin{frame}{\large Conway’s Third Identity}
\[\input{figs/C3.pdf_t} \]



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Sorry, Γ has nothing to say about that...’
\end{frame}