Pensieve header: A talk and a program about Archibald- ($\mathcal A$-) and Γ-calculus and the Halacheva map between them; the $\mathcal A$ part. Continues pensieve://2021-03/

$\mathcal{A}$-Calculus

tex

\begin{frame}
{\LARGE 3. An Implementation of $\calA$}
If I didn't implement I wouldn't believe myself.
\vskip 2mm
Written in Mathematica~\cite{Wolfram:Mathematica}, available as the notebook {\tt Alpha.nb} at \url{http://drorbn.net/mo21/ap}. Code lines are highlighted in grey, demo lines are plain.
We start with an implementation of elements (``{\tt Wedge}'') of exterior algebras, and of the wedge product (``{\tt WP}''):
\vskip 2mm

pdf

In[ ]:=
```
WP[Wedge[u___], Wedge[v___]] := Signature[{u, v}] * Wedge @@ Sort[{u, v}];
WP[0, _] = WP[_, 0] = 0;
WP[A_, B_] :=
  Expand[Distribute[A ** B] /. (a_. * u_Wedge) ** (b_. * v_Wedge) :> a b WP[u, v]];
```

pdf

In[ ]:=
```
WP[Wedge[] + Wedge[a] - 2 b ∧ a, Wedge[] - 3 Wedge[b] + 7 c ∧ d]
```

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Out[ ]=
```
Wedge[] + Wedge[a] - 3 Wedge[b] - a ∧ b + 7 c ∧ d + 7 a ∧ c ∧ d + 14 a ∧ b ∧ c ∧ d
```

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\end{frame}
\begin{frame}\null
We then define the exponentiation map in exterior algebras (``{\tt WExp}'') by summing the series and stopping the sum once the current term (``{\tt t}'') vanishes:

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In[ ]:=
```
WExp[A_] := Module[{s = Wedge[], t = Wedge[], k = 0},
  While[t =!= 0, s += (t = Expand[WP[t, A] / (++k)])]; s]
```

pdf

In[ ]:=
```
WExp[a ∧ b + c ∧ d + e ∧ f]
```

pdf

Out[ ]=
```
Wedge[] + a ∧ b + c ∧ d + e ∧ f + a ∧ b ∧ c ∧ d + a ∧ b ∧ e ∧ f + c ∧ d ∧ e ∧ f + a ∧ b ∧ c ∧ d ∧ e ∧ f
```

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\end{frame}
\begin{frame}\null
Contractions!

pdf

In[∘]:=
```
c_{x_,y_}[w_Wedge] := Module[{i, j},
   {i} = FirstPosition[w, x, {0}]; {j} = FirstPosition[w, y, {0}];
   { w                                        (i == 0) ∧ (j == 0)
   { (-1)^(i+j+If[i>j,0,1]) Delete[w, {{i}, {j}}]   (i > 0) ∧ (j > 0)
  ];
c_{x_,y_}[ℰ_] := ℰ /. w_Wedge :> c_{x,y}[w]
```

pdf

In[∘]:=
```
WExp[a ∧ b + 2 c ∧ d]
c_{d,c}@WExp[a ∧ b + 2 c ∧ d]
```

pdf

Out[∘]= Wedge[] + a ∧ b + 2 c ∧ d + 2 a ∧ b ∧ c ∧ d

pdf

Out[∘]= −Wedge[] − a ∧ b

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\end{frame}

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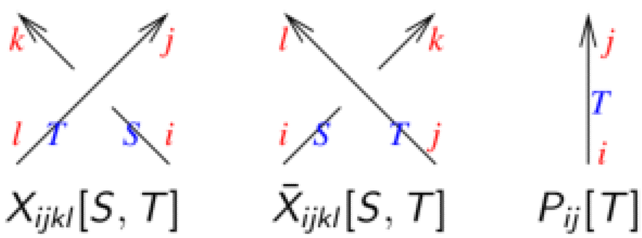
\begin{frame}

\parpic[r]{\def\X{$X_{ijkl}[S,T]$} \input{figs/Xp.pdf_t}}

$\calA${\tt [is,os,cs,w]} is also a container for the values of the $\calA$-invariant of a tangle. In it, {\tt is} are the labels of the input strands, {\tt os} are the labels of the output strands, {\tt cs} is an assignment of colours (namely, variables) to all the ends $\{\xi_i\}_{i\in\text{\tt is}}\sqcup\{x_j\}_{j\in\text{\tt os}}$, and {\tt w} is the ``payload'': an element of $\Lambda\left(\{\xi_i\}_{i\in\text{\tt is}}\sqcup\{x_j\}_{j\in\text{\tt os}}\right)$.



pdf

```
𝒜[X_{i_,j_,k_,l_}[S_, T_]] := 𝒜[{l, i}, {j, k}, ⟨|ξ_i → S, x_j → T, x_k → S, ξ_l → T|⟩,
   Expand[T^(-1/2) WExp[Expand[{ξ_l, ξ_i}.(1  1-T; 0  T).{x_j, x_k}] /. ξ_a_ x_b_ :> ξ_a ∧ x_b]]];
```

pdf

In[∘]:= 𝒜[X_{1,2,3,4}[u, v]]

pdf

Out[∘]= $\mathcal{A}\Big[\{4, 1\}, \{2, 3\}, \langle| \xi_1 \to u, x_2 \to v, x_3 \to u, \xi_4 \to v |\rangle,$

$\frac{\text{Wedge}[\,]}{\sqrt{v}} - \frac{x_2 \wedge \xi_4}{\sqrt{v}} - \sqrt{v}\, x_3 \wedge \xi_1 - \frac{x_3 \wedge \xi_4}{\sqrt{v}} + \sqrt{v}\, x_3 \wedge \xi_4 + \sqrt{v}\, x_2 \wedge x_3 \wedge \xi_1 \wedge \xi_4\Big]$

pdf

```
𝒜[X̄i_,j_,k_,l_] := 𝒜[X̄i,j,k,l[τi, τj]];
```

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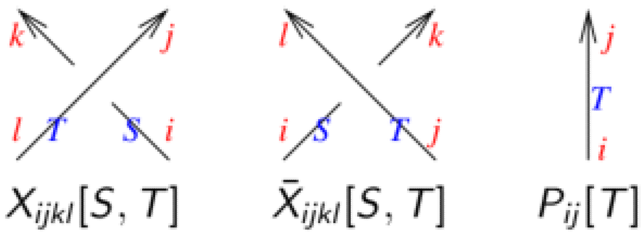
\end{frame}

tex

\begin{frame}\null
The negative crossing and the ``point'':
\[\def\Xbar{$\bar{X}_{ijkl}[S,T]$} \def\P{$P_{ij}[T]$}
 \input{figs/XmP.pdf_t}
\]



pdf

```
𝒜[X̄i_,j_,k_,l_[S_, T_]] := 𝒜[{i, j}, {k, l}, ⟨|ξi → S, ξj → T, xk → S, xl → T|⟩,
   Expand[T^(1/2) WExp[Expand[{ξi, ξj}.( T^-1    0 ).{xk, xl}] /. ξa_ xb_ :→ ξa ∧ xb]]];
                                       ( 1 - T^-1 1 )
𝒜[Xi_,j_,k_,l_] := 𝒜[Xi,j,k,l[τi, τl]];
```

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In[∘]:=

```
𝒜[Pi_,j_[T_]] := 𝒜[{i}, {j}, ⟨|ξi → T, xj → T|⟩, WExp[ξi ∧ xj]];
𝒜[Pi_,j_] := 𝒜[Pi,j[τi]];
```

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\end{frame}
\begin{frame}\null
The linear structure on $\calA$'s:

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```
𝒜 /: α_ × 𝒜[is_, os_, cs_, w_] := 𝒜[is, os, cs, Expand[α w]]
𝒜 /: 𝒜[is1_, os1_, cs1_, w1_] + 𝒜[is2_, os2_, cs2_, w2_] /;
   (Sort@is1 == Sort@is2) ∧ (Sort@os1 == Sort@os2) ∧
    (Sort@Normal@cs1 == Sort@Normal@cs2) := 𝒜[is1, os1, cs1, w1 + w2]
```

tex

Deciding if two $\calA$'s are equal:

pdf

```
𝒜 /: 𝒜[is1_, os1_, _, w1_] ≡ 𝒜[is2_, os2_, _, w2_] :=
 TrueQ[(Sort@is1 === Sort@is2) ∧ (Sort@os1 === Sort@os2) ∧ PowerExpand[w1 == w2]]
```

tex

\end{frame}
\begin{frame}\null

\vskip -12mm
\null\hfill\smash{\imagetop{\input{figs/R2Left.pdf_t}}}
\newline The union operation on $\calA$'s (implemented as ``multiplication''):

pdf

```
𝒜 /: 𝒜[is1_, os1_, cs1_, w1_] 𝒜[is2_, os2_, cs2_, w2_] :=
  𝒜[is1 ⋃ is2, os1 ⋃ os2, Join[cs1, cs2], WP[w1, w2]]
```

pdf

In[∘]:= `Short[𝒜[X2,4,3,1[S, T]] 𝒜[X̄3,4,6,5], 5]`

pdf

Out[∘]//Short=

$$\mathcal{A}\Big[\{1,2,3,4\},\{3,4,5,6\},\langle|\xi_2\to S,x_4\to T,x_3\to S,\xi_1\to T,\xi_3\to\tau_3,\xi_4\to\tau_4,x_6\to\tau_3,x_5\to\tau_4|\rangle,$$
$$\frac{\sqrt{\tau_4}\,\mathrm{Wedge}[\,]}{\sqrt{T}}-\frac{\sqrt{\tau_4}\,x_3\wedge\xi_1}{\sqrt{T}}+\sqrt{T}\sqrt{\tau_4}\,x_3\wedge\xi_1-\sqrt{T}\sqrt{\tau_4}\,x_3\wedge\xi_2-$$
$$\frac{\sqrt{\tau_4}\,x_4\wedge\xi_1}{\sqrt{T}}-\frac{\sqrt{\tau_4}\,x_5\wedge\xi_4}{\sqrt{T}}-\frac{x_6\wedge\xi_3}{\sqrt{T}\sqrt{\tau_4}}+\ll 40\gg+\frac{\sqrt{T}\,x_3\wedge x_5\wedge x_6\wedge\xi_1\wedge\xi_3\wedge\xi_4}{\sqrt{\tau_4}}-$$
$$\frac{\sqrt{T}\,x_3\wedge x_5\wedge x_6\wedge\xi_2\wedge\xi_3\wedge\xi_4}{\sqrt{\tau_4}}-\frac{x_4\wedge x_5\wedge x_6\wedge\xi_1\wedge\xi_3\wedge\xi_4}{\sqrt{T}\sqrt{\tau_4}}+\frac{\sqrt{T}\,x_3\wedge x_4\wedge x_5\wedge x_6\wedge\xi_1\wedge\xi_2\wedge\xi_3\wedge\xi_4}{\sqrt{\tau_4}}\Big]$$

tex

\end{frame}
\begin{frame}\null
Contractions of $\calA$-objects:

pdf

In[∘]:=

```
c_{h_,t_}@𝒜[is_, os_, cs_, w_] := 𝒜[
    DeleteCases[is, t], DeleteCases[os, h], KeyDrop[cs, {x_h, ξ_t}], c_{x_h,ξ_t}[w]
   ] /. If[MatchQ[cs[ξ_t], τ_], cs[ξ_t] → cs[x_h], cs[x_h] → cs[ξ_t]];
```

pdf

In[∘]:= `c4,4[𝒜[X2,4,3,1[S, T]] 𝒜[X̄3,4,6,5]]`

pdf

Out[∘]=

$$\mathcal{A}\Big[\{1,2,3\},\{3,5,6\},\langle|\xi_2\to S,x_3\to S,\xi_1\to T,\xi_3\to\tau_3,x_6\to\tau_3,x_5\to T|\rangle,$$
$$\mathrm{Wedge}[\,]-x_3\wedge\xi_1+T\,x_3\wedge\xi_1-T\,x_3\wedge\xi_2-x_5\wedge\xi_1-x_6\wedge\xi_1+\frac{x_6\wedge\xi_1}{T}-\frac{x_6\wedge\xi_3}{T}+$$
$$T\,x_3\wedge x_5\wedge\xi_1\wedge\xi_2-x_3\wedge x_6\wedge\xi_1\wedge\xi_2+T\,x_3\wedge x_6\wedge\xi_1\wedge\xi_2+x_3\wedge x_6\wedge\xi_1\wedge\xi_3-$$
$$\frac{x_3\wedge x_6\wedge\xi_1\wedge\xi_3}{T}-x_3\wedge x_6\wedge\xi_2\wedge\xi_3-\frac{x_5\wedge x_6\wedge\xi_1\wedge\xi_3}{T}-x_3\wedge x_5\wedge x_6\wedge\xi_1\wedge\xi_2\wedge\xi_3\Big]$$

tex

\end{frame}
\begin{frame}\null
Automatic and intelligent multiple contractions:

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In[ ]:=
```
c@𝒜[is_, os_, cs_, w_] := Fold[c#2,#2[#1] &, 𝒜[is, os, cs, w], is ⋂ os]
𝒜[{A_𝒜}] := c[A];
𝒜[{A1_𝒜, As__𝒜}] := Module[{A2},
  A2 = First@MaximalBy[{As}, Length[A1⟦1⟧ ⋂ #⟦2⟧] + Length[A1⟦2⟧ ⋂ #⟦1⟧] &];
  𝒜[Join[{c[A1 A2]}, DeleteCases[{As}, A2]]] ]
𝒜[𝒪s_List] := 𝒜[𝒜 /@ 𝒪s]
```

pdf

In[ ]:= c[𝒜[$X_{2,4,3,1}$[S, T]] 𝒜[$\overline{X}_{3,4,6,5}$]]

pdf

Out[ ]= $\mathcal{A}[\{1,2\},\{5,6\},\langle|\xi_2\to S,\xi_1\to T,x_6\to S,x_5\to T|\rangle,\text{Wedge}[\,]-x_5\wedge\xi_1-x_6\wedge\xi_2-x_5\wedge x_6\wedge\xi_1\wedge\xi_2]$

pdf

In[ ]:= 𝒜@{𝒜[$X_{2,4,3,1}$[S, T]], 𝒜[$\overline{X}_{3,4,6,5}$]}

pdf

Out[ ]= $\mathcal{A}[\{1,2\},\{5,6\},\langle|\xi_2\to S,\xi_1\to T,x_6\to S,x_5\to T|\rangle,\text{Wedge}[\,]-x_5\wedge\xi_1-x_6\wedge\xi_2-x_5\wedge x_6\wedge\xi_1\wedge\xi_2]$

tex

\end{frame}

Skein Relations

tex

\begin{frame}{\LARGE 4. Skein relations and evaluations for $\calA$}
\[\input{figs/SimpleTangle.pdf_t} \]

pdf

In[ ]:= 𝒜@{$\overline{X}_{4,1,6,3}$[v, u], $\overline{X}_{3,2,5,4}$}

pdf

Out[ ]= $\mathcal{A}\Big[\{1,2\},\{5,6\},\langle|\xi_2\to v,x_5\to u,\xi_1\to u,x_6\to v|\rangle,$

$$\sqrt{u}\,\sqrt{v}\,\text{Wedge}[\,]-\frac{\sqrt{u}\,x_5\wedge\xi_1}{\sqrt{v}}+\frac{\sqrt{u}\,x_5\wedge\xi_2}{\sqrt{v}}-\sqrt{u}\,\sqrt{v}\,x_5\wedge\xi_2+\frac{\sqrt{v}\,x_6\wedge\xi_1}{\sqrt{u}}-\sqrt{u}\,\sqrt{v}\,x_6\wedge\xi_1-$$
$$\frac{\sqrt{v}\,x_6\wedge\xi_2}{\sqrt{u}}-\frac{\sqrt{u}\,x_5\wedge x_6\wedge\xi_1\wedge\xi_2}{\sqrt{v}}-\frac{\sqrt{v}\,x_5\wedge x_6\wedge\xi_1\wedge\xi_2}{\sqrt{u}}+\sqrt{u}\,\sqrt{v}\,x_5\wedge x_6\wedge\xi_1\wedge\xi_2\Big]$$

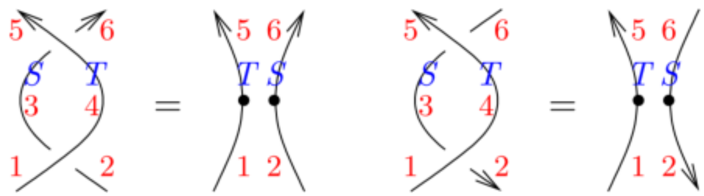
Reidemeister 2

tex

\end{frame}
\begin{frame}{\large Reidemeister 2}

tex

\[\input{figs/R2.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}$@$\{X_{2,4,3,1}[S, T], \overline{X}_{3,4,6,5}\}$ ≡ $\mathcal{A}$@$\{P_{1,5}[T], P_{2,6}[S]\}$

pdf

Out[∘]= True

pdf

In[∘]:= $\mathcal{A}$@$\{\overline{X}_{3,1,2,4}[S, T], X_{6,5,3,4}\}$ ≡ $\mathcal{A}$@$\{P_{1,5}[T], P_{6,2}[S]\}$

pdf

Out[∘]= True

Reidemeister 3

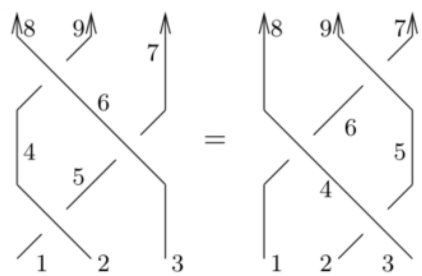
tex

\end{frame}

\begin{frame}{\large Reidemeister 3}

tex

\[\input{figs/R3.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}$@$\{X_{2,5,4,1}[T_2, T_1], X_{3,7,6,5}[T_3, T_1], X_{6,9,8,4}\}$ ≡ $\mathcal{A}$@$\{X_{3,5,4,2}[T_3, T_2], X_{4,6,8,1}[T_3, T_1], X_{5,7,9,6}\}$

pdf

Out[∘]= True

tex

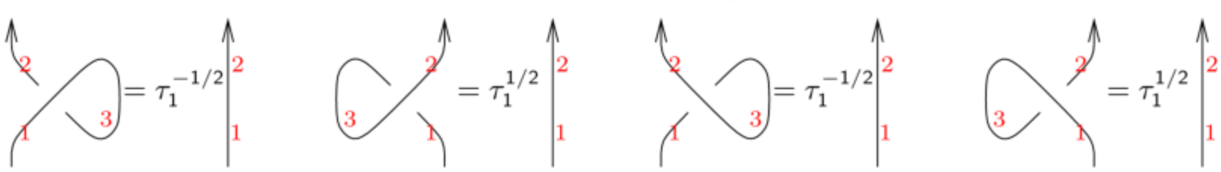
\end{frame}

Reidemeister 1

tex

\begin{frame}{\large Reidemeister 1}

\[\def\p{$=\tau_1^{1/2}$} \def\m{$=\tau_1^{-1/2}$} \input{figs/R1.pdf_t} \]



pdf

In[∘]:= $\{\mathcal{A}@\{X_{3,3,2,1}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{X_{1,2,3,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\},$
$\mathcal{A}@\{\overline{X}_{1,3,3,2}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{\overline{X}_{3,1,2,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\}\}$

pdf

Out[∘]= {True, True, True, True}

tex

(So we have an invariant, up to rotation numbers).
\end{frame}

The Relation with the Multivariable Alexander Polynomial

tex

\begin{frame}{\large The Relation with the Multivariable Alexander Polynomial}
\[\input{figs/Borromean.pdf_t} \]

pdf

$\text{MVA} = u^{-1/2}\,v^{-1/2}\,w^{-1/2}\,(u-1)\,(v-1)\,(w-1);$

pdf

In[∘]:= $A = \{\overline{X}_{1,12,2,13}[u, v],\ \overline{X}_{13,2,6,3},\ X_{8,4,9,3},\ X_{4,10,5,9},\ X_{6,17,7,16}[v, w],$
$X_{15,8,16,7},\ \overline{X}_{14,10,15,11},\ \overline{X}_{11,17,12,14}\}\ //\ \mathcal{A}\ //\ \text{Last}\ //\ \text{Factor}$

pdf

Out[∘]= $$\frac{(-1+u)^2\,(-1+v)\,(-1+w)\,(\text{Wedge}[\,]-x_5\wedge\xi_1)}{u\,v}$$

pdf

$A == u^{-1/2}\,(u-1)\,u^{0}\,v^{-1/2}\,w^{1/2}\,\text{MVA}\,(\text{Wedge}[\,]-x_5\wedge\xi_1)$

pdf

Out[∘]= True

tex

\end{frame}

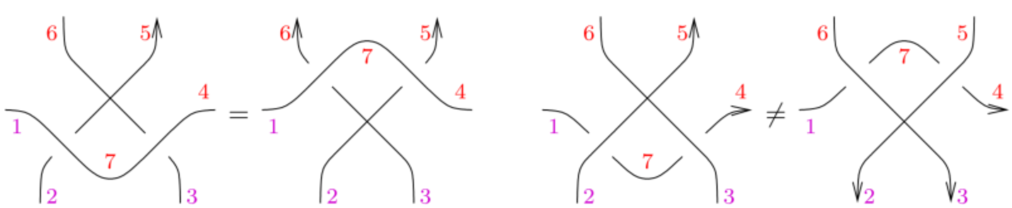
Overcrossings Commute but Undercrossings don’t

tex

\begin{frame}{\large Overcrossings Commute but Undercrossings don't}

tex

\[\input{figs/OUC.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}@\{X_{2,7,5,1},\ X_{3,4,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,1},\ X_{2,4,5,7}\}$

pdf

Out[∘]= True

pdf

In[ ]:= $\mathcal{A}@\{\overline{X}_{1,2,7,5}, \overline{X}_{7,3,4,6}\} \equiv \mathcal{A}@\{\overline{X}_{1,3,7,6}, \overline{X}_{7,2,4,5}\}$

pdf

Out[ ]= False

tex

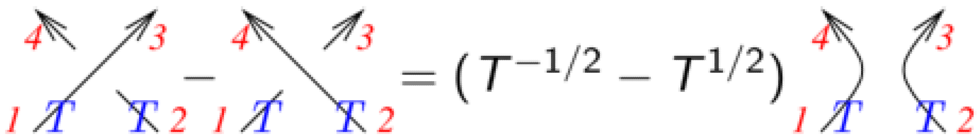
\end{frame}

The Conway Relation

tex

\begin{frame}{\large The Conway Relation\hfill (see~\cite{Conway:Enumeration})}

\[\input{figs/Conway.pdf_t} \]



pdf

In[ ]:= $\mathcal{A}@\{X_{2,3,4,1}[T, T]\} - \mathcal{A}@\{\overline{X}_{1,2,3,4}[T, T]\} \equiv \left(T^{-1/2} - T^{1/2}\right) \mathcal{A}@\{P_{1,4}[T], P_{2,3}[T]\}$

pdf

Out[ ]= True

tex

\[\includegraphics[height=36mm]{../../Projects/Gallery/Conway.png} \]

\end{frame}

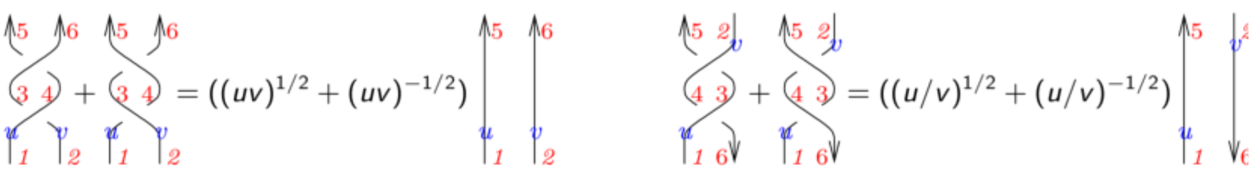
Conway's Second Set of Identities

tex

\begin{frame}{\large Conway's Second Set of Identities\hfill (see~\cite{Conway:Enumeration})}

\[\def\b{$=((uv)^{1/2}+(uv)^{-1/2})$} \def\c{$=((u/v)^{1/2}+(u/v)^{-1/2})$} \input{figs/Conway2nd.pdf_t}

\]



pdf

In[ ]:= $\mathcal{A}@\{X_{2,4,3,1}[v, u], X_{4,6,5,3}\} + \mathcal{A}@\{\overline{X}_{1,2,4,3}[u, v], \overline{X}_{3,4,6,5}\} \equiv$
$\left(u^{1/2} v^{1/2} + u^{-1/2} v^{-1/2}\right) \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$

pdf

Out[ ]= True

pdf

In[ ]:= $\mathcal{A}@\{\overline{X}_{4,1,6,3}[v, u], \overline{X}_{3,2,5,4}\} + \mathcal{A}@\{X_{1,6,3,4}[u, v], X_{2,5,4,3}\} \equiv$
$\left(u^{1/2} v^{-1/2} + u^{-1/2} v^{1/2}\right) \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}$

pdf

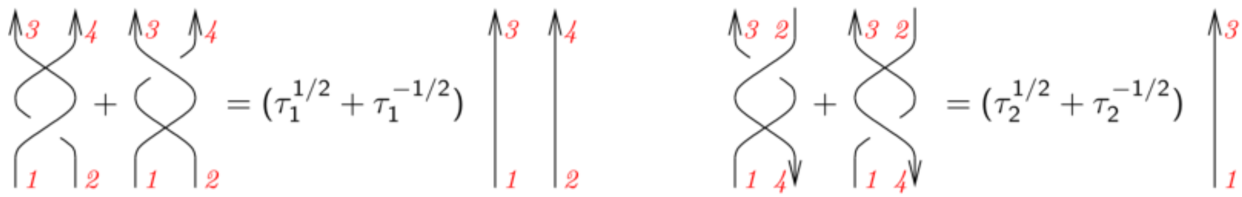
Out[ ]= True

tex

\end{frame}

tex

\begin{frame}\null
{\bf Virtual versions} (Archibald,~\cite{Archibald:Thesis})
\[\def\b{$=(\tau_1^{1/2}+\tau_1^{-1/2})$} \def\c{$=(\tau_2^{1/2}+\tau_2^{-1/2})$}
\input{figs/Conway2ndV.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}$@{$X_{2,3,4,1}$} + $\mathcal{A}$@{$\overline{X}_{2,1,4,3}$} ≡ $\left(\tau_1^{1/2}+\tau_1^{-1/2}\right)$ $\mathcal{A}$@{$P_{1,3}$, $P_{2,4}$}

pdf

Out[∘]= True

pdf

In[∘]:= $\mathcal{A}$@{$\overline{X}_{1,2,3,4}$} + $\mathcal{A}$@{$X_{1,4,3,2}$} ≡ $\left(\tau_2^{1/2}+\tau_2^{-1/2}\right)$ $\mathcal{A}$@{$P_{1,3}$, $P_{2,4}$}

pdf

Out[∘]= True

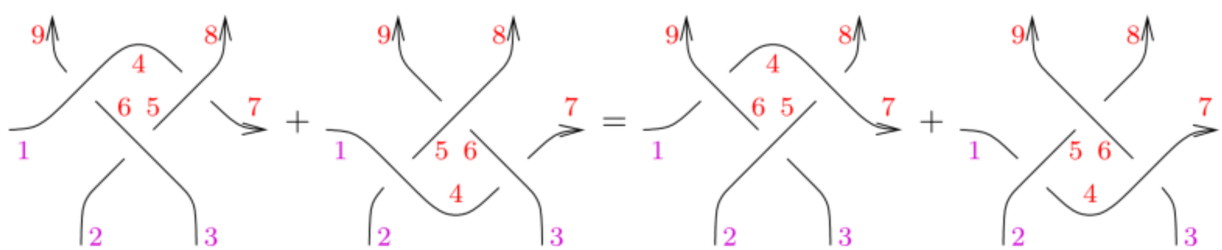
tex

\end{frame}

Conway’s Third Identity

tex

\begin{frame}{\large Conway’s Third Identity \hfill (see~\cite{Conway:Enumeration})}
\[\input{figs/C3.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}$@{$X_{6,4,9,1}$, $\overline{X}_{4,5,7,8}$, $\overline{X}_{2,3,5,6}$} + $\mathcal{A}$@{$X_{2,4,5,1}$, $\overline{X}_{4,3,7,6}$, $X_{6,8,9,5}$} ≡
$\mathcal{A}$@{$\overline{X}_{1,6,4,9}$, $X_{5,7,8,4}$, $X_{3,5,6,2}$} + $\mathcal{A}$@{$\overline{X}_{1,2,4,5}$, $X_{3,7,6,4}$, $\overline{X}_{5,6,8,9}$}

pdf

Out[∘]= True

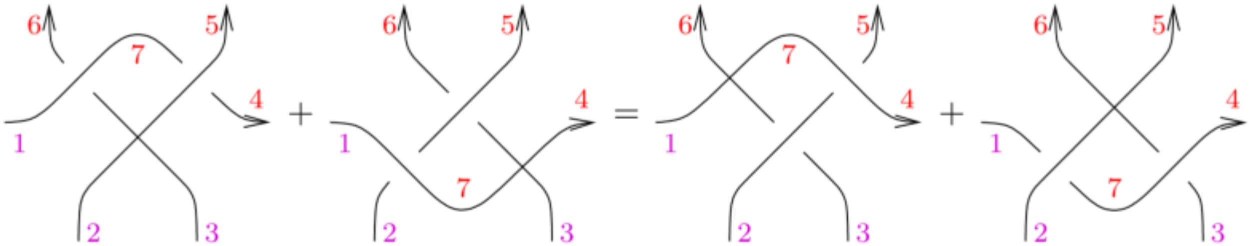
tex

\end{frame}

tex

\begin{frame}\null
{\bf Virtual version} (Archibald,~\cite{Archibald:Thesis})

\[\input{figs/C3V.pdf_t} \]



pdf

In[∘]:= $\mathcal{A}$@$\{X_{3,7,6,1}, \overline{X}_{7,2,4,5}\}$ + $\mathcal{A}$@$\{X_{2,4,7,1}, X_{3,5,6,7}\}$ ≡ $\mathcal{A}$@$\{X_{3,7,6,2}, X_{7,4,5,1}\}$ + $\mathcal{A}$@$\{\overline{X}_{1,2,7,5}, X_{3,4,6,7}\}$

pdf

Out[∘]= True

tex

\end{frame}

Jun Murakami's Fifth Axiom

tex

\begin{frame}{\large Jun Murakami's Fifth Axiom \hfill (see~\cite{MurakamiJ:StateModel})}
\[\def\prop{$=\frac{\sqrt{S}(1-T)}{\sqrt{T}}$} \input{figs/MJ5.pdf_t} \]

pdf

In[∘]:= $\mathcal{A}$@$\{X_{1,4,2,5}[T, S], X_{4,3,5,2}\}$ ≡ $\frac{\sqrt{S}\ (1-T)}{\sqrt{T}}$ $\mathcal{A}$@$\{P_{1,3}[T]\}$

pdf

Out[∘]= True

tex

\[\includegraphics[height=20mm]{../../Projects/Gallery/MurakamiJ.jpg} \]
\end{frame}

tex

\begin{frame}\null
{\bf Virtual versions} (Archibald,~\cite{Archibald:Thesis})
\[\def\prop{$=(T^{-1/2}-T^{1/2})$} \input{figs/MJ5V.pdf_t} \]

pdf

In[∘]:= $\mathcal{A}$@$\{X_{3,2,3,1}[S, T]\}$ ≡ $\left(T^{-1/2} - T^{1/2}\right)$ $\mathcal{A}$@$\{P_{1,2}[T]\}$

pdf

Out[∘]= True

pdf

In[∘]:= $\mathcal{A}$@$\{X_{1,3,2,3}\}$

pdf

Out[∘]= $\mathcal{A}[\{1\}, \{2\}, \langle| \xi_1 \to \tau_1, x_2 \to \tau_1 |\rangle, 0]$

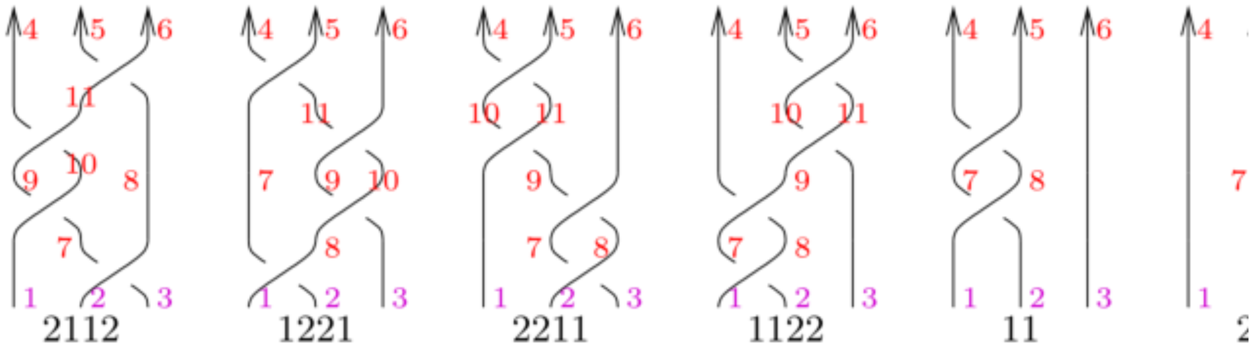
tex

\end{frame}

Jun Murakami's Third Axiom

tex

\begin{frame} {\large Jun Murakami’s Third Axiom\hfill(see~\cite{MurakamiJ:StateModel})}
\[\scalebox{0.64}{\input{figs/Murakami3.pdf_t}} \]



pdf

In[ ]:=
```
𝒜_{2112} = 𝒜@{X_{3,8,7,2}, X_{7,10,9,1}, X_{10,11,4,9}, X_{8,6,5,11}};
𝒜_{1221} = 𝒜@{X_{2,8,7,1}, X_{3,10,9,8}, X_{10,6,11,9}, X_{11,5,4,7}};
𝒜_{2211} = 𝒜@{X_{3,8,7,2}, X_{8,6,9,7}, X_{9,11,10,1}, X_{11,5,4,10}};
𝒜_{1122} = 𝒜@{X_{2,8,7,1}, X_{8,9,4,7}, X_{3,11,10,9}, X_{11,6,5,10}};
𝒜_{11} = 𝒜@{X_{2,8,7,1}, X_{8,5,4,7}, P_{3,6}};  𝒜_{22} = 𝒜@{X_{3,8,7,2}, X_{8,6,5,7}, P_{1,4}};
𝒜_{ϕ} = 𝒜@{P_{1,4}, P_{2,5}, P_{3,6}};
g_+[z_] := z^{1/2} + z^{-1/2}; g_-[z_] := z^{1/2} - z^{-1/2};
g_+[τ1] g_-[τ2] 𝒜_{2112} - g_-[τ2] g_+[τ3] 𝒜_{1221} - g_-[τ3 / τ1] (𝒜_{2211} + 𝒜_{1122}) +
   g_-[τ2 τ3 / τ1] g_+[τ3] 𝒜_{11} - g_+[τ1] g_-[τ1 τ2 / τ3] 𝒜_{22} ≡ g_-[τ3^2 / τ1^2] 𝒜_{ϕ}
```

pdf

Out[ ]= True

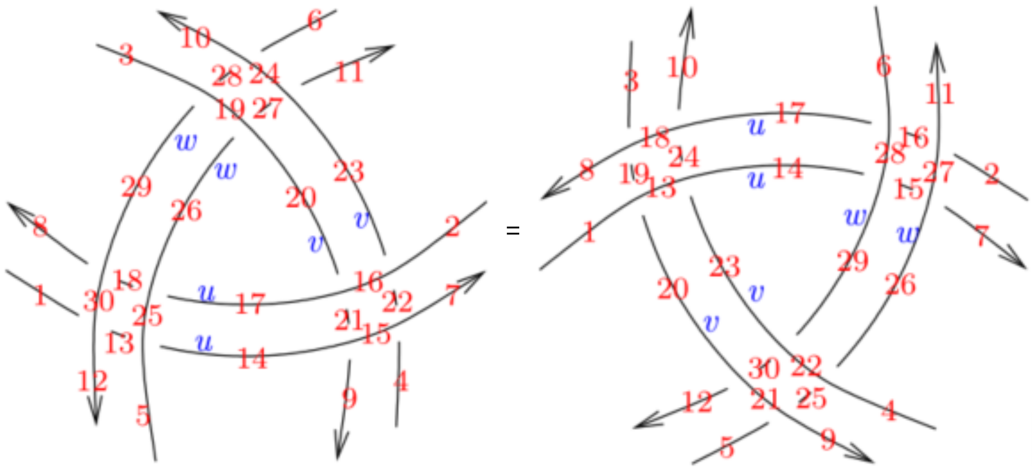
tex

\end{frame}

The Naik-Stanford Double Delta Move

tex

\begin{frame}{\large The Naik-Stanford Double Delta Move\hfill(see~\cite{NaikStanford:Move})}
\vskip -5mm
\[
 \includegraphics[height=20mm]{../../Projects/Gallery/Naik.jpg}
 \qquad\scalebox{0.8}{\input{figs/NaikStanford.pdf_t}}\qquad
 \includegraphics[height=20mm]{../../Projects/Gallery/Stanford.jpg}
\]



pdf

In[∘]:= $$\text{Timing}\big[\mathcal{A}@\{X_{6,10,28,24}[w,v], \overline{X}_{28,3,29,19}[w,v], X_{26,20,27,19}[w,v], \overline{X}_{27,23,11,24}[w,v], X_{1,12,13,30}[u,w], \overline{X}_{13,5,14,25}[u,w], X_{17,26,18,25}[u,w], \overline{X}_{18,29,8,30}[u,w], X_{4,7,22,15}[v,u], \overline{X}_{22,2,23,16}[v,u], X_{20,17,21,16}[v,u], \overline{X}_{21,14,9,15}[v,u]\} \equiv \mathcal{A}@\{X_{5,9,25,21}[w,v], \overline{X}_{25,4,26,22}[w,v], X_{29,23,30,22}[w,v], \overline{X}_{30,20,12,21}[w,v], X_{2,11,16,27}[u,w], \overline{X}_{16,6,17,28}[u,w], X_{14,29,15,28}[u,w], \overline{X}_{15,26,7,27}[u,w], X_{3,8,19,18}[v,u], \overline{X}_{19,1,20,13}[v,u], X_{23,14,24,13}[v,u], \overline{X}_{24,17,10,18}[v,u]\}\big]$$

pdf

Out[∘]= {251.156, True}

tex

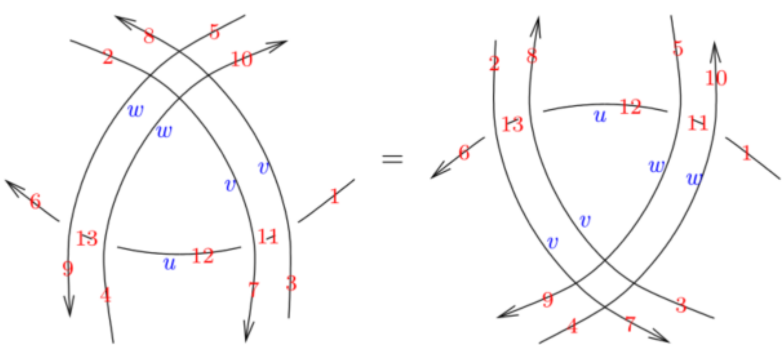
\end{frame}

tex

\begin{frame}\null
{\bf Virtual Version 1} (Archibald,~\cite{Archibald:Thesis})

tex

\[\input{figs/VNS1.pdf_t} \]



pdf

In[∘]:= $$\mathcal{A}@\{X_{1,8,11,3}[u,v], \overline{X}_{11,2,12,7}[u,v], X_{12,10,13,4}[u,w], \overline{X}_{13,5,6,9}[u,w]\} \equiv \mathcal{A}@\{X_{1,10,11,4}[u,w], \overline{X}_{11,5,12,9}[u,w], X_{12,8,13,3}[u,v], \overline{X}_{13,2,6,7}[u,v]\}$$

pdf

Out[∘]= True

tex

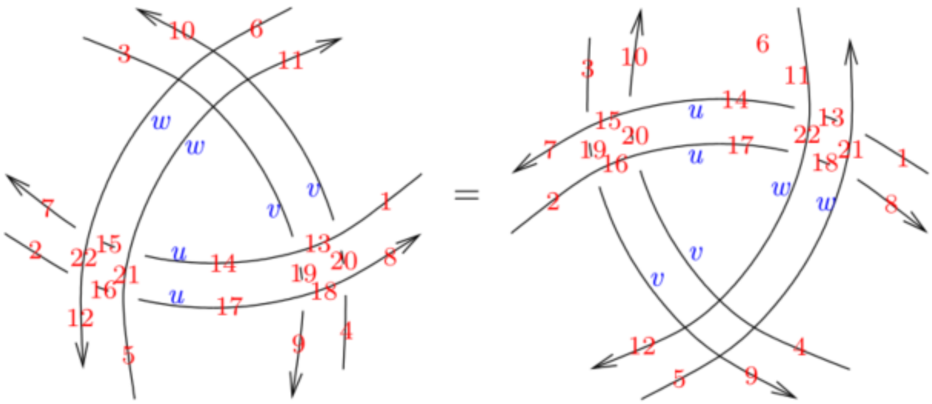
\end{frame}

tex

\begin{frame}\null
{\bf Virtual Version 2} (Archibald,~\cite{Archibald:Thesis})

tex

\[\input{figs/VNS2.pdf_t} \]



pdf

In[•]:= $\mathcal{A}$@$\{\overline{X}_{20,1,10,13}[v, u], X_{3,14,19,13}[v, u], X_{14,11,15,21}[u, w], \overline{X}_{15,6,7,22}[u, w], X_{2,12,16,22}[u, w], \overline{X}_{16,5,17,21}[u, w], \overline{X}_{19,17,9,18}[v, u], X_{4,8,20,18}[v, u]\} \equiv$
$\mathcal{A}$@$\{X_{1,11,13,21}[u, w], \overline{X}_{13,6,14,22}[u, w], \overline{X}_{20,14,10,15}[v, u], X_{3,7,19,15}[v, u], \overline{X}_{19,2,9,16}[v, u], X_{4,17,20,16}[v, u], X_{17,12,18,22}[u, w], \overline{X}_{18,5,8,21}[u, w]\}$

pdf

Out[•]= True

tex

\end{frame}