

In[ ]:=

```
SetDirectory@"C:\\drorbn\\AcademicPensieve\\Talks\\MoscowByWeb-2004"
```

Out[ ]:=

```
C:\\drorbn\\AcademicPensieve\\Talks\\MoscowByWeb-2004
```

```

In[ ]:= col = ImageCollage[
  Scaled[1] → ImagePad[#, 8, White] & /@
  ImageCrop /@ Import /@ FileNames["*.png", {"Clips"}],
  "Fit", 300 {8, 10.5},
  Method → "ClosestPacking", Background → White, Padding → Red, ImagePadding → 4
]

```

**OVER THEN UNDER TANGLES**

DROR BAR-NATAN, ZSUZSANNA DANCOS, AND ROLAND VAN DER VEEN

ABSTRACT. Brilliant wrong ideas should not be buried and forgotten. Instead, they should be mined for the gold that lies underneath the layer of wrong. In this paper we explain how "over then under tangles" lead to an easy classification of knots, and under the surface, also to some valid mathematics: a separation theorem for braids and virtual braids, a topological understanding of the Drinfel'd double construction of quantum group theory, and more.

**SetAttributes[VD, Orderless]**

```

Tidy[vd_VD] := Module[{ps = Union @@ (List @@@ vd)},
  Replace[vd, Thread[ps → Range[Length[ps]], {2}]]
R12Reduce1[vd_VD] := Tidy@Module[{R2s, R2}, Which[
  Length[R2s] = Cases[vd, X_>[i_>, j_>] > X_>[i+1, j+1]] ∩ (List @@@ vd) > 0,
  Complement[vd, vd[R2 = First@R2s, R2 /> X_>[i_>, j_>] > X_>[i-1, j-1]]],
  Length[R2s] = Cases[vd, X_>[i_>, j_>] > X_>[i+1, j-1]] ∩ (List @@@ vd) > 0,
  Complement[vd, vd[R2 = First@R2s, R2 /> X_>[i_>, j_>] > X_>[i-1, j+1]]],
  True, DeleteCases[vd, X_>[i_>, j_>] /> Abs[i-j] = 1]]
R12Reduce[vd_VD] := FixedPoint[R12Reduce1, vd]
γ[vd_VD] := Module[{js, s1, i1, j1, s2, i2, j2},
  js = Cases[vd, X_>[i_>, j_>] > j] ∩ Cases[vd, X_>[i_>, j_>] > i-1];
  If[Length[js] == 0, vd,
  j1 = RandomChoice[js]; i2 = j1+1;
  Cases[vd, X_>[i_>, j1] > s1 = s; i1 = i];
  Cases[vd, X_>[i2, j_>] > s2 = s; j2 = j];
  Tidy@Join[Complement[vd, vd[X_>[i1, j1], X_>[i2, j2]],
  vd[X_>[j1, j2], X_>[i1, i2], X_>[i1+s1/3, j2+s2/3],
  X_>[i1+s1/3, j2+s2/3]]
  ]
]
F[vd_VD] := FixedPoint[γ, vd, 2^8]
F[F_] /> Head[F] => VD := F[VD[F]]
VPB[n_, {cs_...}] := VPB[n, cs];
VD /> vd1_VD ** vd2_VD := Module[{es1, es2, m2},
  es1 = Cases[vd1, EOS[i_>] > i];
  m2 = Max[es2 = Cases[vd2, EOS[i_>] > i]];
  Tidy[
  vd1 ∪ Replace[DeleteCases[vd2, _EOS],
  i_> > i/m2-1+es1+Count[es2, e_> /> i > e]], {2}]]
]
VD[VPB[n_]] := VD @@@ (EOS /> Range[n]);
VD[VPB[n_, σ_>[i_>, j_>]]] := Tidy@Append[VD @@@ (EOS /> Range[n]), X_>[i-0.5, j-0.5]];
VD[VPB[n_, σ_>[i_>, j_>]]] := Tidy@Append[VD @@@ (EOS /> Range[n]), X_>[i-0.5, j-0.5]];
VD[VPB[n_, σ_>[i_>, j_>]]] := VD[VPB[n, σ]] ** VD[VPB[n, cs]]
VPBGenerators[n_] :=
  Flatten@Table[{σ_>[i_>, j_>], {i, n}, {j, DeleteCases[Range[n, 1]]}];
ProudFollowers[n_, σ_>[i_>, j_>]] := ProudFollowers[n, σ_>[i_>, j_>]] = Module[{p, q, s},
  Flatten@{σ_>[i_>, j_>], σ_>[i_>, j_>], {p, {i, j}}, {q, Complement[Range[n], {i, j}]},
  Table[{σ_>[i_>, j_>], {p, Complement[Range[i+1, n], {j}]}],
  {q, Complement[Range[n], {i, j, p}]}]
];
ProudFollowers[n_, σ_>[i_>, j_>]] :=
  ProudFollowers[n, σ_>[i_>, j_>]] = ProudFollowers[n, σ_>[i_>, j_>]] /. σ_>[i_>, j_>] > σ_>[i_>, j_>]
ProudVPBs[n_, 0] := {VPB[n]};
ProudVPBs[n_, 1] := VPB[n, #] & /> VPBGenerators[n];
ProudVPBs[n_, m_] /> m > 1 :=
  Flatten[ProudVPBs[n, m-1] /.
  VPB[n, cs_>, σ_>] > (VPB[n, cs, σ, #] & /> ProudFollowers[n, σ])]
CountOfForms[n_, m_] := Module[{k},
  Length@Union@Flatten@Table[F@vpb, {k, 0, m}, {vpb, ProudVPBs[n, k]}]]

```

**See <http://drorbn.net/mo20>**

**Credit to Manturov and Chterental!**

**• Subsets.**  
**• Supersets.**  
**• Subsets of supersets.**  
**• Completions.**  
**• Images.**  
**• Completions of sub-**  
**sets of supersets...**

**• Enriquez' universal quantization of Lie bi-algebra.**  
**• All else about quantization of Lie bi-algebra.**  
**• PBW / normal ordering.**  
**• Andoux-Meilhan "Characterization of the Reduced Peripheral System of Links"**  
**• B-N's "Balloons and Hoops" paper.**

```
In[ ]:= Export["OU.png", col]
```

```
Out[ ]:= OU.png
```