

Pensieve header: Fuller output of for V and Cap. Even fuller: <VCapSolution-to-12.m>.

```

SetDirectory["~drorbn/AcademicPensieve/Projects/WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn = (V ** (V // dA[x] // dA[y]) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
  (Cap * (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs[x] = 1/2; βs[y] = 0;
SeriesSolve[{α, β, γ, κ}, (ħ-1 R4Eqn) && UnitarityEqn && CapEqn];
{V, κ}

FreeLie` implements / extends
{*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, EulerE, Exp,
  Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ℓ, Λ, σ, ħ, ⌊, ⌋}.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg,
  dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.

Arbitrarily setting {κs[x] → 0}.
Arbitrarily setting {αs[x, y, y] → 0}.
{Es[⟨x → LS[0, - $\frac{\overline{xy}}{24}$ , 0, ...], y → LS[ $\frac{\overline{x}}{2}$ , - $\frac{\overline{xy}}{12}$ , 0, ...]⟩, CWS[0, - $\frac{\overline{xy}}{48}$ , 0, ...]],
  CWS[0, - $\frac{\overline{xx}}{96}$ , 0, ...]}

V@{8} // Timing
Arbitrarily setting {αs[x, x, x, y, y] → 0}.
Arbitrarily setting {αs[x, x, x, x, x, y, y] → 0}.
Arbitrarily setting {αs[x, x, x, x, y, x, y, y] → 0}.
{41.697661,
  Es[⟨x → LS[0, - $\frac{\overline{xy}}{24}$ , 0,  $\frac{7 \overline{xxxy}}{5760}$  -  $\frac{7 \overline{xyxy}}{5760}$  +  $\frac{\overline{xyyy}}{1440}$ , 0, - $\frac{31 \overline{xxxxxy}}{967680}$  +  $\frac{31 \overline{xxxxyy}}{483840}$  -

```

$$\begin{aligned}
& \frac{83 \times x \overline{x y} y}{967\,680} - \frac{31 \times \overline{x y} \overline{x y}}{725\,760} - \frac{31 \times x \overline{x y} \overline{x y}}{645\,120} + \frac{13 \times \overline{x y} y y}{241\,920} + \frac{101 \overline{x y} \overline{x y} y}{1\,451\,520} + \\
& \frac{527 \times \overline{x y} \overline{x y}}{5\,806\,080} - \frac{\overline{x y} y y y}{60\,480}, 0, \frac{127 \times x x x x \overline{x y}}{154\,828\,800} - \frac{127 \times x x x x \overline{x y}}{51\,609\,600} + \\
& \frac{2399 \times x x x \overline{x y} y}{464\,486\,400} + \frac{127 \times x x \overline{x y} \overline{x y}}{30\,965\,760} - \frac{2893 \times x x \overline{x y} y y}{464\,486\,400} - \frac{727 \times x \overline{x y} \overline{x y} y}{87\,091\,200} - \\
& \frac{1397 \times x x \overline{x y} \overline{x y}}{77\,414\,400} + \frac{271 \times x \overline{x y} y y y}{58\,060\,800} - \frac{15\,389 \times \overline{x y} \overline{x y} \overline{x y}}{1\,393\,459\,200} + \frac{12\,809 \times \overline{x y} \overline{x y} y y}{1\,393\,459\,200} - \\
& \frac{113 \times x \overline{x y} x \overline{x y}}{24\,883\,200} - \frac{127 \times x x \overline{x y} x \overline{x y}}{77\,414\,400} + \frac{13\,103 \times x \overline{x y} \overline{x y}}{1\,393\,459\,200} + \\
& \frac{12\,409 \times x \overline{x y} y y \overline{x y}}{696\,729\,600} + \frac{1207 \times \overline{x y} \overline{x y} y y}{278\,691\,840} + \frac{11 \times x \overline{x y} \overline{x y} \overline{x y}}{1\,382\,400} - \frac{19 \times \overline{x y} y y y y}{9\,676\,800} + \\
& \frac{67 \overline{x y} \overline{x y} \overline{x y} y}{7\,257\,600} - \frac{1327 \overline{x y} \overline{x y} y y y}{348\,364\,800} + \frac{x \overline{x y} x \overline{x y} y}{204\,800} + \frac{181 \times \overline{x y} \overline{x y} \overline{x y}}{38\,707\,200} + \\
& \frac{2021 \times x \overline{x y} x \overline{x y}}{696\,729\,600} - \frac{239 \times \overline{x y} \overline{x y} \overline{x y}}{199\,065\,600} - \frac{2171 \times \overline{x y} y \overline{x y}}{464\,486\,400} - \frac{631 \times \overline{x y} y y \overline{x y}}{92\,897\,280} - \\
& \frac{1751 \overline{x y} \overline{x y} y y}{696\,729\,600} - \frac{451 \overline{x y} \overline{x y} \overline{x y}}{154\,828\,800} - \frac{997 \times \overline{x y} \overline{x y} \overline{x y}}{77\,414\,400} + \frac{\overline{x y} y y y y}{241\,920}, \dots], \\
y \rightarrow & LS\left[\frac{\overline{x}}{2}, -\frac{\overline{x y}}{12}, 0, \frac{x \overline{x y}}{5760} - \frac{1}{720} x \overline{x y} + \frac{1}{720} \overline{x y} y, -\frac{x x \overline{x y}}{7680} + \frac{x x \overline{x y}}{3840} - \frac{\overline{x y} \overline{x y}}{6912}, \right. \\
& -\frac{x x x \overline{x y}}{645\,120} + \frac{23 \times x x \overline{x y}}{483\,840} - \frac{13 \times x \overline{x y} y}{161\,280} - \frac{x \overline{x y} \overline{x y}}{22\,680} - \\
& \frac{41 \times x \overline{x y} \overline{x y}}{580\,608} + \frac{x \overline{x y} y y}{15\,120} + \frac{\overline{x y} \overline{x y} y}{12\,096} + \frac{71 \times \overline{x y} \overline{x y}}{483\,840} - \frac{\overline{x y} y y y}{30\,240}, \\
& \frac{x x x x \overline{x y}}{258\,048} - \frac{5 \times x x x \overline{x y}}{387\,072} + \frac{x x x \overline{x y} y}{64\,512} + \frac{x x \overline{x y} \overline{x y}}{96\,768} + \frac{5 \times x x \overline{x y} \overline{x y}}{290\,304} - \frac{x x \overline{x y} y y}{96\,768} - \\
& \frac{17 \times \overline{x y} \overline{x y} y}{1\,451\,520} - \frac{x x \overline{x y} \overline{x y}}{60\,480} - \frac{\overline{x y} x \overline{x y}}{207\,360} - \frac{7 \times x \overline{x y} x \overline{x y}}{1\,658\,880} + \frac{x \overline{x y} y \overline{x y}}{207\,360}, \\
& \frac{x x x x x \overline{x y}}{77\,414\,400} - \frac{587 \times x x x x \overline{x y}}{464\,486\,400} + \frac{253 \times x x x \overline{x y} y}{66\,355\,200} + \frac{59 \times x x \overline{x y} \overline{x y}}{1\,393\,459\,200} +
\end{aligned}$$

$\kappa @ \{8\}$

The same thing, copy-paste ready and machine readable:

Sum[{V[[1]]_x[k], V[[1]]_y[k], V[[2]]_k}, {k, 8}] // InputForm

```
{-LW[x, y]/24 + (7*LW[x, x, x, y])/5760 - (7*LW[x, x, y, y])/5760 +
  LW[x, y, y, y]/1440 - (31*LW[x, x, x, x, x, y])/967680 +
  (31*LW[x, x, x, x, y, y])/483840 - (31*LW[x, x, x, y, x, y])/645120 -
  (83*LW[x, x, x, y, y, y])/967680 - (31*LW[x, x, y, x, y, y])/725760 +
  (527*LW[x, x, y, y, x, y])/5806080 + (13*LW[x, x, y, y, y, y])/241920 +
  (101*LW[x, y, x, y, y, y])/1451520 - LW[x, y, y, y, y, y]/60480 +
  (127*LW[x, x, x, x, x, x, x, y])/154828800 - (127*LW[x, x, x, x, x, x, y, y])/
  51609600 + (127*LW[x, x, x, x, x, y, x, y])/30965760 +
  (2399*LW[x, x, x, x, x, y, y, y])/464486400 - (127*LW[x, x, x, x, y, x, x, y])/
  77414400 - (1397*LW[x, x, x, x, y, y, x, y])/77414400 -
  (2893*LW[x, x, x, x, y, y, y, y])/464486400 - (113*LW[x, x, x, y, x, x, y, y])/
  24883200 + (11*LW[x, x, x, y, x, y, x, y])/1382400 -
  (727*LW[x, x, x, y, x, y, y, y])/87091200 + (2021*LW[x, x, x, y, y, x, x, y])/
  696729600 + (13103*LW[x, x, x, y, y, x, y, y])/1393459200 +
  (12409*LW[x, x, x, y, y, y, x, y])/696729600 + (271*LW[x, x, x, y, y, y, y, y])/
  58060800 + (181*LW[x, x, y, x, x, y, x, y])/38707200 +
  LW[x, x, y, x, x, y, y, y]/204800 - (15389*LW[x, x, y, x, y, x, y, y])/
  1393459200 - (239*LW[x, x, y, x, y, y, x, y])/199065600 +
  (12809*LW[x, x, y, x, y, y, y, y])/1393459200 - (997*LW[x, x, y, y, x, y, x, y])/
  77414400 + (1207*LW[x, x, y, y, x, y, y, y])/278691840 -
  (2171*LW[x, x, y, y, y, x, y, y])/464486400 - (631*LW[x, x, y, y, y, y, x, y])/
  92897280 - (19*LW[x, x, y, y, y, y, y, y])/9676800 +
  (67*LW[x, y, x, y, x, y, y, y])/7257600 - (451*LW[x, y, x, y, y, x, y, y])/
  154828800 - (1327*LW[x, y, x, y, y, y, y, y])/348364800 -
  (1751*LW[x, y, y, x, y, y, y, y])/696729600 + LW[x, y, y, y, y, y, y, y]/2419200,
  LW[x]/2 - LW[x, y]/12 + LW[x, x, x, y]/5760 - LW[x, x, y, y]/720 +
  LW[x, y, y, y]/720 - LW[x, x, x, x, y]/7680 + LW[x, x, x, y, y]/3840 -
  LW[x, x, y, x, y]/6912 - LW[x, x, x, x, x, y]/645120 +
  (23*LW[x, x, x, x, y, y])/483840 - (41*LW[x, x, x, y, x, y])/580608 -
  (13*LW[x, x, x, y, y, y])/161280 - LW[x, x, y, x, y, y]/22680 +
  (71*LW[x, x, y, y, x, y])/483840 + LW[x, x, y, y, y, y]/15120 +
  LW[x, y, x, y, y, y, y]/12096 - LW[x, y, y, y, y, y, y]/30240 +
  LW[x, x, x, x, x, x, y]/258048 - (5*LW[x, x, x, x, x, y, y])/387072 +
  (5*LW[x, x, x, x, y, x, y])/290304 + LW[x, x, x, x, y, y, y]/64512 -
  (7*LW[x, x, x, y, x, x, y])/1658880 + LW[x, x, x, y, x, y, y]/96768 -
  LW[x, x, x, y, y, x, y]/60480 - LW[x, x, x, y, y, y, y]/96768 -
  LW[x, x, y, x, x, y, y]/207360 - (17*LW[x, x, y, x, y, y, y])/1451520 +
  LW[x, x, y, y, y, x, y]/207360 + LW[x, x, x, x, x, x, x, y]/77414400 -
  (587*LW[x, x, x, x, x, x, y, y])/464486400 + (541*LW[x, x, x, x, x, y, x, y])/
  154828800 + (253*LW[x, x, x, x, x, y, y, y])/66355200 -
  (157*LW[x, x, x, x, y, x, x, y])/49766400 + (59*LW[x, x, x, x, y, x, y, y])/
  1393459200 - (24457*LW[x, x, x, x, y, y, x, y])/1393459200 -
  (43*LW[x, x, x, x, y, y, y, y])/7257600 - (199*LW[x, x, x, y, x, x, y, y])/
  29030400 + (271*LW[x, x, x, y, x, y, x, y])/30965760 -
  (12143*LW[x, x, x, y, x, y, y, y])/1393459200 + (2281*LW[x, x, x, y, y, x, x, y])/
  464486400 + (7367*LW[x, x, x, y, y, x, y, y])/696729600 +
  (5939*LW[x, x, x, y, y, y, x, y])/278691840 + (73*LW[x, x, x, y, y, y, y, y])/
  14515200 + (4057*LW[x, x, y, x, x, y, x, y])/696729600 +
  (47*LW[x, x, y, x, x, y, y, y])/6451200 - (1217*LW[x, x, y, x, y, x, y, y])/
```

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99532800 + (3*LW[x, x, y, x, y, y, x, y])/5734400 +
(1423*LW[x, x, y, x, y, y, y, y])/116121600 - (17513*LW[x, x, y, y, x, y, x, y])/
1393459200 + (3263*LW[x, x, y, y, x, y, y, y])/696729600 -
LW[x, x, y, y, y, x, y, y]/92160 - (151*LW[x, x, y, y, y, y, x, y])/14515200 -
LW[x, x, y, y, y, y, y, y]/403200 + (583*LW[x, y, x, y, x, y, y, y])/38707200 -
(4589*LW[x, y, x, y, y, x, y, y])/696729600 - LW[x, y, x, y, y, y, y, y]/172800 -
LW[x, y, y, x, y, y, y, y]/172800 + LW[x, y, y, y, y, y, y, y]/1209600,
-CW[x, y]/48 + CW[x, x, x, y]/2880 + CW[x, x, y, y]/2880 + CW[x, y, x, y]/5760 +
CW[x, y, y, y]/2880 - CW[x, x, x, x, x, y]/120960 - CW[x, x, x, x, y, y]/120960 -
CW[x, x, x, y, x, y]/120960 - CW[x, x, x, y, y, y]/120960 -
CW[x, x, y, x, x, y]/241920 - CW[x, x, y, x, y, y]/120960 -
CW[x, x, y, y, x, y]/120960 - CW[x, x, y, y, y, y]/120960 -
CW[x, y, x, y, x, y]/362880 - CW[x, y, x, y, y, y]/120960 -
CW[x, y, y, x, y, y]/241920 - CW[x, y, y, y, y, y]/120960 +
CW[x, x, x, x, x, x, x, y]/4838400 + CW[x, x, x, x, x, x, y, y]/4838400 +
CW[x, x, x, x, x, y, x, y]/4838400 + CW[x, x, x, x, x, y, y, y]/4838400 +
CW[x, x, x, x, y, x, x, y]/4838400 + CW[x, x, x, x, y, x, y, y]/4838400 +
CW[x, x, x, y, x, x, x, y]/9676800 + CW[x, x, x, y, x, x, y, y]/4838400 +
CW[x, x, x, y, x, y, x, y]/4838400 + CW[x, x, x, y, y, x, y, y]/4838400 +
CW[x, x, x, y, y, y, x, y]/4838400 + CW[x, x, x, y, y, y, y, y]/4838400 +
CW[x, x, y, x, x, y, x, y]/4838400 + CW[x, x, y, x, x, y, y, y]/4838400 +
CW[x, x, y, x, y, x, y, y]/4838400 + CW[x, x, y, x, y, y, x, y]/4838400 +
CW[x, x, y, x, y, y, y, y]/4838400 + CW[x, x, y, y, x, x, y, y]/9676800 +
CW[x, x, y, y, x, y, x, y]/4838400 + CW[x, x, y, y, x, y, y, y]/4838400 +
CW[x, x, y, y, y, x, y, y]/4838400 + CW[x, x, y, y, y, y, x, y]/4838400 +
CW[x, x, y, y, y, y, y, y]/4838400 + CW[x, y, x, y, x, y, x, y]/19353600 +
CW[x, y, x, y, x, y, y, y]/4838400 + CW[x, y, x, y, y, x, y, y]/4838400 +
CW[x, y, x, y, y, y, y, y]/4838400 + CW[x, y, y, x, y, y, y, y]/4838400 +
CW[x, y, y, y, x, y, y, y]/9676800 + CW[x, y, y, y, y, y, y, y]/4838400}

```

Sum[x[k], {k, 8}] // InputForm

```

-CW[x, x]/96 + CW[x, x, x, x]/11520 - CW[x, x, x, x, x, x]/725760 +
CW[x, x, x, x, x, x, x, x]/38707200

```

The same thing, machine readable and to degree 11, is in the file quoted below, in the same folder:

```

Put[Sum[{V[1]_x[k], V[1]_y[k], V[2][k]}, {k, 11}], "VCapSolution-to-11.m"];
PutAppend[Sum[x[k], {k, 11}], "VCapSolution-to-11.m"]

```

Arbitrarily setting {as[x, x, x, x, x, x, x, y, y] → 0}.

Arbitrarily setting {as[x, x, x, x, x, x, y, x, y, y] → 0}.

Arbitrarily setting

```
{as[x, x, x, x, x, x, x, x, y, y] → 0, as[x, x, x, x, x, x, y, x, y, y, y] → 0}.
```

```
{TimeUsed[], MaxMemoryUsed[]}
```

```
{17 123.2, 10 071 646 624}
```

```

TwistEqn = (V ≡ Rs[x, y] ** (V // dσ[{x, y} → {y, x}]) **
  (El[{x → LS[-LW@y/2], y → LS[-LW@x/2]}, CWS[0]] // r));
TwistEqn@
{6}
BS[7 True, ...]

TrueQ[TwistEqn]@{6}
BS[7 True, ...]

TrueQ[TwistEqn]@{11}
BS[12 True, ...]

{TimeUsed[], MaxMemoryUsed[]}
{35 991., 15 840 067 368}

```

The same thing, machine readable and to degree 12, is in the file quoted below, in the same folder:

```

Put[Sum[{V[[1]]x[k], V[[1]]y[k], V[[2]]k}], {k, 12}], "VCapSolution-to-12.m"];
PutAppend[Sum[x[k], {k, 12}], "VCapSolution-to-12.m"]

Arbitrarily setting
{αs[x, x, x, x, x, x, x, x, y, x, y, y] → 0, αs[x, x, x, x, x, x, y, x, y, y, y, y] → 0}.

{TimeUsed[], MaxMemoryUsed[]}
{140 028., 48 977 267 312}

TrueQ[TwistEqn]@{12}
BS[13 True, ...]

```