Pensieve header: The Objects.

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Echo@"Warning: On Sep 4 2019 I swapped the operations
   ϵ and η. Some incompatibilities may arise in older notebooks."
```

Program

The Objects

Program

Symmetric Algebra Objects

Program

$$\mathrm{sm}_{i\_,j\_\to k\_} := \mathbb{E}_{\{i,j\}\to\{k\}}[b_k(\beta_i+\beta_j)+t_k(\tau_i+\tau_j)+a_k(\alpha_i+\alpha_j)+y_k(\eta_i+\eta_j)+x_k(\xi_i+\xi_j)];$$
$$\mathrm{s}\Delta_{i\_\to j\_,k\_} := \mathbb{E}_{\{i\}\to\{j,k\}}\left[\beta_i\left(b_j+b_k\right)+\tau_i\left(t_j+t_k\right)+\alpha_i(a_j+a_k)+\eta_i(y_j+y_k)+\xi_i(x_j+x_k)\right];$$
$$\mathrm{sS}_{i\_} := \mathbb{E}_{\{i\}\to\{i\}}[-\beta_i b_i-\tau_i t_i-\alpha_i a_i-\eta_i y_i-\xi_i x_i];$$
$$\mathrm{s}\eta_{i\_} := \mathbb{E}_{\{\}\to\{i\}}[0];$$
$$\mathrm{s}\epsilon_{i\_} := \mathbb{E}_{\{i\}\to\{\}}[0];$$

Program

$$\mathrm{s}\sigma_{i\_\to j\_} := \mathbb{E}_{\{i\}\to\{j\}}[\beta_i b_j+\tau_i t_j+\alpha_i a_j+\eta_i y_j+\xi_i x_j];$$
$$\mathrm{s}\Upsilon_{i\_\to j\_,k\_,l\_,m\_} := \mathbb{E}_{\{i\}\to\{j,k,l,m\}}[\beta_i b_k+\tau_i t_k+\alpha_i a_l+\eta_i y_j+\xi_i x_m];$$

Program

The CU Definitions

Program

$$\mathrm{c}\Lambda = \left(\eta_i+\frac{e^{-\gamma\alpha_i-\epsilon\beta_i}\eta_j}{1+\gamma\epsilon\eta_j\xi_i}\right)y_k+\left(\beta_i+\beta_j+\frac{\mathrm{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\epsilon}\right)b_k+$$
$$\left(\alpha_i+\alpha_j+\frac{\mathrm{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\gamma}\right)a_k+\left(\frac{e^{-\gamma\alpha_j-\epsilon\beta_j}\xi_i}{1+\gamma\epsilon\eta_j\xi_i}+\xi_j\right)x_k;$$
$$\mathrm{Define}[\mathrm{cm}_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}[\mathrm{c}\Lambda]]$$

Program

$$\mathrm{Define}[\mathrm{c}\sigma_{i\to j}=\mathrm{s}\sigma_{i,j}\ /.\ \tau_i\to 0,\ \mathrm{c}\epsilon_i=\mathrm{s}\epsilon_i,\ \mathrm{c}\eta_i=\mathrm{s}\eta_i,\ \mathrm{c}\Delta_{i\to j,k}=\mathrm{s}\Delta_{i\to j,k},$$
$$\mathrm{cS}_i=\mathrm{sS}_i\ //\ \mathrm{s}\Upsilon_{i\to 1,2,3,4}\ //\ \mathrm{cm}_{4,3\to i}\ //\ \mathrm{cm}_{i,2\to i}\ //\ \mathrm{cm}_{i,1\to i}];$$

Program

Booting Up QU

Program

$$\mathrm{Define}[\mathrm{a}\sigma_{i\to j}=\mathbb{E}_{\{i\}\to\{j\}}[a_j\alpha_i+x_j\xi_i],\ \mathrm{b}\sigma_{i\to j}=\mathbb{E}_{\{i\}\to\{j\}}[b_j\beta_i+y_j\eta_i]]$$

Program

$$\mathrm{Define}\left[\mathrm{am}_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}\left[(\alpha_i+\alpha_j)a_k+\left(\mathcal{A}_j^{-1}\xi_i+\xi_j\right)x_k\right],\right.$$
$$\left.\mathrm{bm}_{i,j\to k}=\mathbb{E}_{\{i,j\}\to\{k\}}\left[(\beta_i+\beta_j)b_k+\left(\eta_i+e^{-\epsilon\beta_i}\eta_j\right)y_k\right]\right]$$

Three types of inverses appear below!
$\overline{R}$ is the inverse of R in the algebra $\mathbb{B}\otimes\mathbb{A}$.
P is the inverse of R as a quadratic form, like how an element of $V^*\otimes V^*$ can be the inverse of an element of $V\otimes V$.
$\overline{\mathrm{aS}}$ is the inverse of aS as an operator form, like how an element of $V^*\otimes V$ can be the inverse of another element of $V^*\otimes V$.

Program

$$\begin{aligned}&\text{Define}\Big[R_{i,j}=\mathbb{E}_{\{\}\to\{i,j\}}\Big[\hbar\, a_j\, b_i+\sum_{k=1}^{\$k+1}\frac{\left(1-e^{\gamma\epsilon\hbar}\right)^k(\hbar\, y_i\, x_j)^k}{k\left(1-e^{k\gamma\epsilon\hbar}\right)}\Big],\\
&\quad \overline{R}_{i,j}=\text{CF}@\mathbb{E}_{\{\}\to\{i,j\}}\Big[-\hbar\, a_j\, b_i,\ -\hbar\, x_j\, y_i\,/\,B_i,\ 1+\text{If}\big[\$k==0,\ 0,\ (\overline{R}_{\{i,j\},\$k-1})_{\$k}[3]-\\
&\qquad\quad \big(\big((\overline{R}_{\{i,j\},0})_{\$k}\, R_{1,2}\,(\overline{R}_{\{3,4\},\$k-1})_{\$k}\big)\ //\ \big(bm_{i,1\to i}\, am_{j,2\to j}\big)\ //\ \big(bm_{i,3\to i}\, am_{j,4\to j}\big)\big)[3]\big]\Big],\\
&\quad P_{i,j}=\mathbb{E}_{\{i,j\}\to\{\}}\big[\beta_i\,\alpha_j\,/\,\hbar,\ \eta_i\,\xi_j\,/\,\hbar,\ 1+\text{If}[\$k==0,\ 0,\ (P_{\{i,j\},\$k-1})_{\$k}[3]-\\
&\qquad\quad (R_{1,2}\ //\ ((P_{\{1,j\},0})_{\$k}\,(P_{\{i,2\},\$k-1})_{\$k}))[3]]\big]\Big]\end{aligned}$$

Program

$$\begin{aligned}&\text{Define}\Big[aS_i=\big(a\sigma_{i\to 2}\,\overline{R}_{1,i}\big)\ //\ P_{1,2},\\
&\quad \overline{aS}_i=\mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i\,\alpha_i,\ -x_i\,\mathcal{A}_i\,\xi_i,\ 1+\text{If}\Big[\$k==0,\ 0,\ (\overline{aS}_{\{i\},\$k-1})_{\$k}[3]-\\
&\qquad\quad \Big((\overline{aS}_{\{i\},0})_{\$k}\ //\ aS_i\ //\ (\overline{aS}_{\{i\},\$k-1})_{\$k}\Big)[3]\Big]\Big]\Big]\end{aligned}$$

(was $\mathrm{aS}_j=\overline{R}_{i,j}\sim B_i\sim P_{i,j}$).

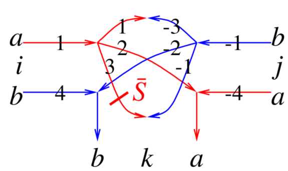
Program

$$\begin{aligned}&\text{Define}\big[bS_i=b\sigma_{i\to 1}\,R_{i,2}\ //\ aS_2\ //\ P_{1,2},\\
&\quad \overline{bS}_i=b\sigma_{i\to 1}\,R_{i,2}\ //\ \overline{aS}_2\ //\ P_{1,2},\\
&\quad a\Delta_{i\to j,k}=(R_{1,j}\,R_{2,k})\ //\ bm_{1,2\to 3}\ //\ P_{3,i},\\
&\quad b\Delta_{i\to j,k}=(R_{j,1}\,R_{k,2})\ //\ am_{1,2\to 3}\ //\ P_{i,3}\big]\end{aligned}$$

(was $\mathrm{bS}_i=R_{i,1}\sim B_1\sim \mathrm{aS}_1\sim B_1\sim P_{i,1}$, $\overline{\mathrm{bS}}_i=R_{i,1}\sim B_1\sim\overline{\mathrm{aS}}_1\sim B_1\sim P_{i,1}$).

Program

The Drinfel'd double:



Program

$$\begin{aligned}&\text{Define}\Big[\\
&\quad dm_{i,j\to k}=\Big(\big(s\Upsilon_{i\to 4,4,1,1}\ //\ a\Delta_{1\to 1,2}\ //\ a\Delta_{2\to 2,3}\ //\ \overline{aS}_3\big)\ \big(s\Upsilon_{j\to -1,-1,-4,-4}\ //\ b\Delta_{-1\to -1,-2}\ //\ b\Delta_{-2\to -2,-3}\big)\Big)\ //\\
&\qquad \big(P_{-1,3}\,P_{-3,1}\,am_{2,-4\to k}\,bm_{4,-2\to k}\big)\Big]\end{aligned}$$

Program

```
Define[dσ_{i→j} = aσ_{i→j} bσ_{i→j},
  dϵ_i = sϵ_i, dη_i = sη_i,
  dS_i = sϒ_{i→1,1,2,2} // (\overline{bS}_1 aS_2) // dm_{2,1→i},
  \overline{dS}_i = sϒ_{i→1,1,2,2} // (bS_1 \overline{aS}_2) // dm_{2,1→i},
  dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j})]
```

Program

In[∘]:=

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Define[C_i = 𝔼_{{}→{i}}[0, 0, B_i^{1/2} ⅇ^{-ℏ ϵ a_i/2}]_{$k},
  \overline{C}_i = 𝔼_{{}→{i}}[0, 0, B_i^{-1/2} ⅇ^{ℏ ϵ a_i/2}]_{$k},
  Kink_i = (R_{1,3} \overline{C}_2) // dm_{1,2→1} // dm_{1,3→i},
  \overline{Kink}_i = (\overline{R}_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
```

Program

Note. $t == \epsilon \mathrm{a} - \gamma \mathrm{b}$ and $b == -t/\gamma + \epsilon \mathrm{a}/\gamma$.

Program

```
Define[b2t_i = 𝔼_{{i}→{i}}[α_i a_i + β_i (ϵ a_i - t_i) / γ + ξ_i x_i + η_i y_i],
  t2b_i = 𝔼_{{i}→{i}}[α_i a_i + τ_i (ϵ a_i - γ b_i) + ξ_i x_i + η_i y_i]]
```

Program

The Knot Tensors

Program

```
Define[kR_{i,j} = (R_{i,j} // (b2t_i b2t_j)) /. t_{i|j} → t,
  \overline{kR}_{i,j} = (\overline{R}_{i,j} // (b2t_i b2t_j)) /. {t_{i|j} → t, T_{i|j} → T},
  km_{i,j→k} = ((t2b_i t2b_j) // dm_{i,j→k} // b2t_k ) /. {t_k → t, T_k → T, τ_{i|j} → 0},
  kC_i = (C_i // b2t_i) /. T_i → T,
  \overline{kC}_i = (\overline{C}_i // b2t_i) /. T_i → T,
  kKink_i = (Kink_i // b2t_i) /. {t_i → t, T_i → T},
  \overline{kKink}_i = (\overline{Kink}_i // b2t_i) /. {t_i → t, T_i → T}]
```