

Pensieve header: Profile with encapsulation of Zip3-Inner. Time to K31@\$k=3: 5816.8.

## Startup

```
In[ ]:= Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"..\\Profile\\Profile.m"];
$k = 1;
<< Objects.m
<< KT.m
```

```
Out[ ]:= {2021, 1, 3, 8, 46, 22.7844035}
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

## Engine

Canonical Forms:

```
In[ ]:= CCF[ $\mathcal{E}$ _] := PP_CCF@ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ]; (*Coefficient Canonical Form *)
CF[ $\mathcal{E}$ _] := PP_CF@Module[
  {vs = Cases[ $\mathcal{E}$ , (y | a | x |  $\eta$  |  $\beta$  |  $\tau$  |  $\xi$ )_,  $\infty$ ]  $\cup$  {y, a, x,  $\eta$ ,  $\beta$ ,  $\tau$ ,  $\xi$ }},
  Total[(CCF[#[[2]]] (Times@@vs^#[[1]])) & /@ CoefficientRules[ $\mathcal{E}$ , vs]]
];
CF[ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[E_sp___[ $\mathcal{E}$ s___]] := CF /@ E_sp[ $\mathcal{E}$ s];
```

Variables and their duals:

```
In[ ]:= {t*, b*, y*, a*, x*, z*,  $\tau$ *,  $\beta$ *,  $\eta$ *,  $\alpha$ *,  $\xi$ *,  $\zeta$ *} = { $\tau$ ,  $\beta$ ,  $\eta$ ,  $\alpha$ ,  $\xi$ ,  $\zeta$ , t, b, y, a, x, z};
(vs_List)* := (v  $\mapsto$  v*) /@ vs;
(u_i_)* := (u*)_i;
```

Weights:

```
In[ ]:= Clear[Wt];
Evaluate[Wt /@ {y, b, t, a, x,  $\eta$ ,  $\beta$ ,  $\tau$ ,  $\alpha$ ,  $\xi$ }] = {1, 0, 0, 2, 1, 1, 2, 2, 0, 1};
Wt[u_i_] := Wt[u];
```

The maximal weight \$n\$, i.e. the  $n$  of  $gl(n)$ . Initially and for a long while this will not be tested beyond  $n == 2$ .

```
In[ ]:= $n = 2;
```

Upper to lower and lower to Upper:

```

In[ ]:=
U21[ $\mathcal{E}_-$ ] :=  $\mathcal{E} / . \{ \mathbf{B}_{i-}^{p-} \mapsto \mathbf{e}^{-p \hbar \mathbf{b}_i}, \mathbf{B}_{i-}^{p-} \mapsto \mathbf{e}^{-p \hbar \mathbf{b}}, \mathbf{T}_{i-}^{p-} \mapsto \mathbf{e}^{p \hbar \mathbf{t}_i}, \mathbf{T}_{i-}^{p-} \mapsto \mathbf{e}^{p \hbar \mathbf{t}}, \mathcal{A}_{i-}^{p-} \mapsto \mathbf{e}^{p \alpha_i}, \mathcal{A}_{i-}^{p-} \mapsto \mathbf{e}^{p \alpha} \};$ 
12U[ $\mathcal{E}_-$ ] :=  $\mathcal{E} / . \{ \mathbf{e}^{c- \cdot \mathbf{b}_i + d- \cdot} \mapsto \mathbf{B}_i^{-c/\hbar} \mathbf{e}^d, \mathbf{e}^{c- \cdot \mathbf{b} + d- \cdot} \mapsto \mathbf{B}^{-c/\hbar} \mathbf{e}^d, \mathbf{e}^{c- \cdot \mathbf{t}_i + d- \cdot} \mapsto \mathbf{T}_i^{c/\hbar} \mathbf{e}^d, \mathbf{e}^{c- \cdot \mathbf{t} + d- \cdot} \mapsto \mathbf{T}^{c/\hbar} \mathbf{e}^d,$ 
 $\mathbf{e}^{c- \cdot \alpha_i + d- \cdot} \mapsto \mathcal{A}_i^c \mathbf{e}^d, \mathbf{e}^{c- \cdot \alpha + d- \cdot} \mapsto \mathcal{A}^c \mathbf{e}^d, \mathbf{e}^{\chi-} \mapsto \mathbf{e}^{\text{Expand@}\chi} \};$ 
12U[r_Rule] := Module[{U = r[[1]] /. {b → B, t → T, α → A}}, U → 12U[U21[U] /. r]];
AlsoUpper[rs_List] := rs ∪ (12U/@rs);

```

Derivatives in the presence of exponentiated variables:

```

In[ ]:=
D_b[f_] :=  $\partial_b f - \hbar \mathbf{B} \partial_B f$ ; D_b_i[f_] :=  $\partial_{b_i} f - \hbar \mathbf{B}_i \partial_{B_i} f$ ;
D_t[f_] :=  $\partial_t f + \hbar \mathbf{T} \partial_T f$ ; D_t_i[f_] :=  $\partial_{t_i} f + \hbar \mathbf{T}_i \partial_{T_i} f$ ;
D_alpha[f_] :=  $\partial_\alpha f + \mathcal{A} \partial_A f$ ; D_alpha_i[f_] :=  $\partial_{\alpha_i} f + \mathcal{A}_i \partial_{A_i} f$ ;
D_v[f_] :=  $\partial_v f$ ;

```

E operations:

```

In[ ]:=
 $\mathcal{E}_E[\$] := \text{Length}[\mathcal{E}] - 1$ ;  $\mathbf{E}_E[\mathcal{E}S\_][\$] := \mathbf{E}[\mathcal{E}S][\$]$ ;
 $\mathcal{E}_E[k\_Integer] := \mathcal{E}[[k+1]]$ ;  $\mathbf{E}_E[\mathcal{E}S\_][k\_Integer] := \{\mathcal{E}S\}[[k+1]]$ ;
 $\mathbf{E} /: \mathcal{E}1\_E \equiv \mathcal{E}2\_E := \text{Inner}[\text{CF}@\#1 == \text{CF}@\#2 \ \&, \mathcal{E}1, \mathcal{E}2, \text{And}]$ ;
 $\mathbf{E}_{d1 \rightarrow r1}[\mathcal{E}1S\_][\$] \equiv \mathbf{E}_{d2 \rightarrow r2}[\mathcal{E}2S\_][\$] \wedge := (\mathbf{d}1 == \mathbf{d}2) \wedge (\mathbf{r}1 == \mathbf{r}2) \wedge (\mathbf{E}[\mathcal{E}1S][\$] \equiv \mathbf{E}[\mathcal{E}2S][\$])$ ;
 $\mathbf{E} /: \mathcal{E}1\_E * \mathcal{E}2\_E := \mathbf{E} @@ \text{Table}[\text{CF}[\mathcal{E}1[\mathbf{k}k] + \mathcal{E}2[\mathbf{k}k]], \{\mathbf{k}k, 0, \text{Min}[\mathcal{E}1[\$], \mathcal{E}2[\$]]\}]$ ;
 $\mathbf{E}_{d1 \rightarrow r1}[\mathcal{E}1S\_][\$] \mathbf{E}_{d2 \rightarrow r2}[\mathcal{E}2S\_][\$] \wedge := \mathbf{E}_{(d1 \cup d2) \rightarrow (r1 \cup r2)} @ @ (\mathbf{E}[\mathcal{E}1S][\$] \times \mathbf{E}[\mathcal{E}2S][\$])$ ;

```

```

In[ ]:=
 $\mathbf{E}_{d1 \rightarrow r1}[\mathcal{E}1S\_][\$] // \mathbf{E}_{d2 \rightarrow r2}[\mathcal{E}2S\_][\$] := \text{Module}[\{\mathbf{is} = \mathbf{r}1 \cap \mathbf{d}2, \mathbf{lvs}\},$ 
 $\mathbf{lvs} = \text{Flatten@Table}[\{\mathbf{y}_{\$ei}, \mathbf{b}_{\$ei}, \mathbf{t}_{\$ei}, \mathbf{a}_{\$ei}, \mathbf{x}_{\$ei}\}, \{\mathbf{i}, \mathbf{is}\}]$ ;
 $\mathbf{E}_{(d1 \cup \text{Complement}[\mathbf{d}2, \mathbf{is}] \rightarrow (\mathbf{r}2 \cup \text{Complement}[\mathbf{r}1, \mathbf{is}])} @ @ (\text{Zip}_{\mathbf{lvs} \cup \mathbf{lvs}^*}[\{\mathbf{lvs}^*.\mathbf{lvs}, \text{Times}[\mathbf{E}[\mathcal{E}1S] /. \text{Table}[(\mathbf{v} : \mathbf{b} | \mathbf{B} | \mathbf{t} | \mathbf{T} | \mathbf{a} | \mathbf{x} | \mathbf{y})_i \rightarrow \mathbf{v}_{\$ei}, \{\mathbf{i}, \mathbf{is}\}],$ 
 $\mathbf{E}[\mathcal{E}2S] /. \text{Table}[(\mathbf{v} : \beta | \tau | \alpha | \mathcal{A} | \xi | \eta)_i \rightarrow \mathbf{v}_{\$ei}, \{\mathbf{i}, \mathbf{is}\}]\}$ 
 $\}])$ 
 $\]$ 

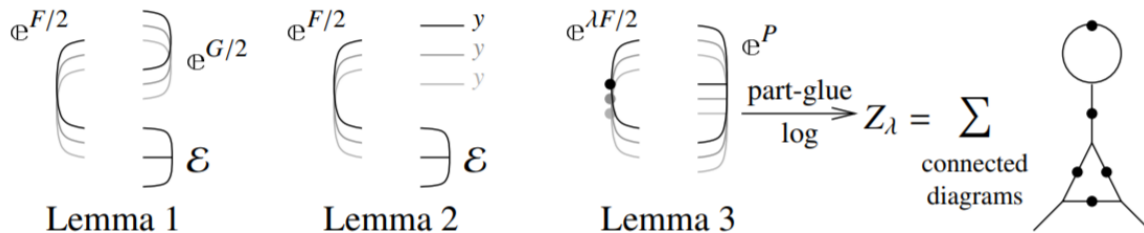
```

```

In[ ]:=
 $\Lambda 2 \mathbf{E}_{d \rightarrow r}[\mathcal{A}_-] := \text{Module}[\{\mathbf{k}\}, \mathbf{E}_{d \rightarrow r} @ @ 12U @ \text{Table}[\text{SeriesCoefficient}[\mathcal{A}, \{\mathbf{e}, 0, \mathbf{k}\}], \{\mathbf{k}, 0, \mathbf{\$k}\}]]$ ;

```

Ziping! Lemmas 2 and 3 are combined, yet they must be applied first to the middle weight variables and then to the heavy and light variables.



```

In[ ]:=
Zip_{vs\_}[\{\mathcal{F}_-, \mathcal{E}_-\}] := \{\mathcal{F}, \mathcal{E}\} // Zip1_{vs} // Zip2_{Select}[vs, (\theta < \text{Wt}[\#] < \$n) \&] // EZip3_{Select}[vs, (\theta < \text{Wt}[\#] < \$n) \&] //
Zip2_{Select}[vs, (\text{Wt}[\#] == \theta \text{Wt}[\#] == \$n) \&] // Zip3_{Select}[vs, (\text{Wt}[\#] == \theta \text{Wt}[\#] == \$n) \&] // Last;

```

Getting rid of the quadratic.

**Lemma 1.** With convergences left to the reader,

$$\left\langle F : \mathcal{E} \otimes^{\frac{1}{2}} \sum_{i,j \in B} G_{ij} z_i z_j \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```

In[ ]:=
Zip1_{ } = Identity;
Zip1_{vs_} @ {f_, E[Q_, P___]} := PPZip1@Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[If[Wt[u] + Wt[v] == $n, D[u*, v*, f, 0], {u, vs}, {v, vs}];
  G = Table[If[Wt[u] + Wt[v] == $n, D[u, v, Q, 0], {u, vs}, {v, vs}];
  {CF[vs*.(F.Inverse[I - G.F]).vs* / 2], E[CF[Q - Log[Det[I - G.F]] / 2 - vs.G.vs / 2], P]}
]

```

Getting rid of linear terms.

**Lemma 2.**  $\left\langle F: \mathcal{E}_{\oplus \sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F: \mathcal{E}_{|z_B \rightarrow z_B + F y_B} \right\rangle_B.$

```

In[ ]:=
Zip2_{ } = Identity;
Zip2_{vs_} @ {f_, E[Q_, P___]} := PPZip2@Module[{F, Y, u, v},
  F = Table[If[Wt[u] + Wt[v] == $n, D[u*, v*, f, 0], {u, vs}, {v, vs}];
  Y = Table[D_v, Q, {v, vs}] /. AlsoUpper@Table[v -> 0, {v, vs}];
  CF /@ ({f_, E[Q - Y.vs + Y.F.Y / 2, P]} /. AlsoUpper@Thread[vs -> vs + F.Y])
]

```

Dealing with Feynman diagrams.

**Lemma 3.** With an extra variable  $\lambda$ ,  $Z_\lambda := \log[\lambda F: \oplus^P]_B$  satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} \left( \partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda) \right).$$

Note that the power  $m$  of  $\lambda$  is at most  $k - 1 + \frac{2k+2}{2} = 2k$ . We write  $Z_\lambda = \sum Z[m] \lambda^m$ .

```

In[ ]:= Zip3vs@{ $\mathcal{F}$ _,  $\mathcal{E}$ _E} := PPZip3@Module[
  {F, u, v, Z, $k, kk, jj, $m = 0, m, n},
  $k = Length[ $\mathcal{E}$ ] - 1;
  Do[Z[0, kk] =  $\mathcal{E}$ [[kk + 1]], {kk, 0, $k}];
  F[u_, v_] := F[u, v] = CF@If[Wt[u] + Wt[v] == $n,  $\partial_{u^*, v^*} \mathcal{F}$ , 0];
  Z[m_, kk_, u_] := Z[m, kk, u] = Du[Z[m, kk]];
  Z[m_, kk_, u_, v_] := Z[m, kk, u, v] = Dv[Z[m, kk, u]];
  For[m = 0, m ≤ 2 $m, ++m, For[kk = 0, kk ≤ $k, ++kk,
    Z[m + 1, kk] = CF@Sum[
      If[F[u, v] === 0, 0,  $\frac{F[u, v]}{2 (m + 1)}$ 
        (Z[m, kk, u, v] + Sum[Z[n, jj, u] * Z[m - n, kk - jj, v], {n, 0, m}, {jj, 0, kk}])],
      {u, vs}, {v, vs}];
    If[Z[m + 1, kk] != 0, $m = m + 1]
  ]];
  CF /@ ({
     $\mathcal{F}$  - Sum[F[u, v] u* v* / 2, {u, vs}, {v, vs}],
    E@@Table[Sum[Z[m, kk], {m, 0, $m}], {kk, 0, $k}]
  } /. AlsoUpper@Table[v → 0, {v, vs}])
]

```

Encapsulation.

```

In[ ]:= EZip3vs@{ $\mathcal{F}$ _,  $\mathcal{E}$ _E} := PEZip3@Module[
  {n $\mathcal{E}$ , n $\mathcal{F}$ , rc, ps, rr = {(*release rules*)}},
  rc = 0; n $\mathcal{E}$  = Total[
    CoefficientRules[#, vs] /. (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{E}$ [++rc] → c]; c $\mathcal{E}$ [rc] (Times@@vsps))
  ] & /@  $\mathcal{E}$ ;
  rc = 0; n $\mathcal{F}$  = Total[CoefficientRules[ $\mathcal{F}$ , vs*] /.
    (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{F}$ [++rc] → c]; c $\mathcal{F}$ [rc] (Times@@(vs*)ps))];
  CF[Expand[{n $\mathcal{F}$ , n $\mathcal{E}$ } // Zip3vs] /. rr]
]

```

## Profiling

```

In[ ]:= BeginProfile[];

```

```

In[ ]:= Timing@Block[{ $k = 1}, Z[Knot[3, 1]]]

```

KnotTheory: Loading precomputed data in PD4Knots`.

$$\text{Out[ ]} = \left\{ 17.4844, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -4 \mathfrak{t} \hbar - \text{Log} \left[ \left( \frac{1}{\mathfrak{T}^3} - \frac{2}{\mathfrak{T}^2} + \frac{2}{\mathfrak{T}} \right)^2 \right] - \text{Log} \left[ \left( 1 + \frac{\mathfrak{T}}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} - \frac{\mathfrak{T}^2}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} \right)^2 \right] \right), \right. \\
\left. \frac{a \left( -2 \hbar + 2 \mathfrak{T}^2 \hbar \right)}{1 - \mathfrak{T} + \mathfrak{T}^2} + \frac{-2 \hbar + 3 \mathfrak{T} \hbar - 2 \mathfrak{T}^2 \hbar + \mathfrak{T}^3 \hbar}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{x y \left( -2 \hbar^2 - 2 \mathfrak{T} \hbar^2 \right)}{1 - \mathfrak{T} + \mathfrak{T}^2} \right] \right\}$$

```
In[ ]:= PrintProfile[]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 17.438
( 1) 0.093/ 17.440 above Z
( 1) 0/ 0 above RVK
CF: called 13065 times, time in 6.18/11.876
( 84) 0.139/ 0.391 under Z
( 76) 0.079/ 0.221 under Boot
( 135) 0.451/ 0.983 under EZip3
( 90) 0.094/ 0.299 under Zip1
( 270) 1.715/ 5.406 under Zip2
( 12410) 3.702/ 4.576 under Zip3
( 8889) 5.696/ 5.696 above CCF
CCF: called 8889 times, time in 5.696/5.696
( 8889) 5.696/ 5.696 under CF
Zip3: called 90 times, time in 2.486/7.062
( 22) 0.719/ 2.436 under Z
( 23) 1.000/ 2.611 under Boot
( 45) 0.767/ 2.015 under EZip3
( 12410) 3.702/ 4.576 above CF
Zip1: called 45 times, time in 1.105/1.404
( 22) 0.328/ 0.516 under Z
( 23) 0.777/ 0.888 under Boot
( 90) 0.094/ 0.299 above CF
EZip3: called 45 times, time in 0.969/3.967
( 22) 0.814/ 2.437 under Z
( 23) 0.155/ 1.530 under Boot
( 135) 0.451/ 0.983 above CF
( 45) 0.767/ 2.015 above Zip3
Zip2: called 90 times, time in 0.847/6.253
( 44) 0.313/ 5.220 under Z
( 46) 0.534/ 1.033 under Boot
( 270) 1.715/ 5.406 above CF
Z: called 1 times, time in 0.093/17.438
( 1) 0.093/ 17.440 under ProfileRoot
( 5) 0/ 6.345 above Boot
( 84) 0.139/ 0.391 above CF
( 22) 0.814/ 2.437 above EZip3
( 22) 0.328/ 0.516 above Zip1
( 44) 0.313/ 5.220 above Zip2
( 22) 0.719/ 2.436 above Zip3
Boot: called 23 times, time in 0.062/18.626
( 5) 0/ 6.345 under Z
( 18) 0.062/ 12.280 under Boot
( 18) 0.062/ 12.280 above Boot
( 76) 0.079/ 0.221 above CF
( 23) 0.155/ 1.530 above EZip3
( 23) 0.777/ 0.888 above Zip1
( 46) 0.534/ 1.033 above Zip2
( 23) 1.000/ 2.611 above Zip3
RVK: called 1 times, time in 0./0.
( 1) 0/ 0 under ProfileRoot
```

In[#:]= **Timing@Block**[{**\$k** = **1**}, **Z[Knot**[**8**, **17**]]]

$$\text{Out[#:]= } \left\{ 66.0469, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -2 \mathfrak{t} \hbar - \text{Log} \left[ \left( -1 - \frac{1}{\mathfrak{T}^4} + \frac{4}{\mathfrak{T}^3} - \frac{6}{\mathfrak{T}^2} + \frac{5}{\mathfrak{T}} \right)^2 \right] - \right. \right. \\ \left. \text{Log} \left[ \left( 1 + \frac{\mathfrak{T}}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{2 \mathfrak{T}^2}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{\mathfrak{T}^3}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} \right)^2 \right] - \right. \\ \left. \text{Log} \left[ \left( 1 - \frac{\mathfrak{T}}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{4 \mathfrak{T}^2}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{7 \mathfrak{T}^3}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \right. \right. \\ \left. \left. \frac{7 \mathfrak{T}^4}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{4 \mathfrak{T}^5}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{\mathfrak{T}^6}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} \right)^2 \right] \right] \right), \\ \frac{-3 \hbar + 8 \mathfrak{T} \hbar - 8 \mathfrak{T}^2 \hbar + 8 \mathfrak{T}^4 \hbar - 8 \mathfrak{T}^5 \hbar + 3 \mathfrak{T}^6 \hbar}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{\mathfrak{a} \left( -6 \hbar + 16 \mathfrak{T} \hbar - 16 \mathfrak{T}^2 \hbar + 16 \mathfrak{T}^4 \hbar - 16 \mathfrak{T}^5 \hbar + 6 \mathfrak{T}^6 \hbar \right)}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6} + \\ \frac{\mathfrak{x} \mathfrak{y} \left( -6 \hbar^2 + 10 \mathfrak{T} \hbar^2 - 6 \mathfrak{T}^2 \hbar^2 - 6 \mathfrak{T}^3 \hbar^2 + 10 \mathfrak{T}^4 \hbar^2 - 6 \mathfrak{T}^5 \hbar^2 \right)}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6} \left. \right] \right\}$$

In[ ]:= **PrintProfile**[]

```

Out[ ]:= ProfileRoot is root. Profiled time: 83.485
( 2) 0.309/ 83.485 above Z
( 2) 0/ 0 above RVK
CCF: called 26186 times, time in 34.842/34.842
( 26186) 34.842/ 34.842 under CF
CF: called 27296 times, time in 30.591/65.433
( 298) 1.007/ 2.304 under Z
( 88) 0.109/ 0.251 under Boot
( 318) 2.980/ 6.437 under EZip3
( 212) 0.313/ 0.785 under Zip1
( 636) 16.357/ 40.972 under Zip2
( 25744) 9.825/ 14.684 under Zip3
( 26186) 34.842/ 34.842 above CCF
EZip3: called 106 times, time in 8.204/18.654
( 79) 8.018/ 16.969 under Z
( 27) 0.186/ 1.685 under Boot
( 318) 2.980/ 6.437 above CF
( 106) 1.549/ 4.013 above Zip3
Zip3: called 212 times, time in 5.367/20.051
( 79) 2.725/ 13.224 under Z
( 27) 1.093/ 2.814 under Boot
( 106) 1.549/ 4.013 under EZip3
( 25744) 9.825/ 14.684 above CF
Zip2: called 212 times, time in 2.058/43.03
( 158) 1.428/ 41.870 under Z
( 54) 0.630/ 1.160 under Boot
( 636) 16.357/ 40.972 above CF
Zip1: called 106 times, time in 2.036/2.821
( 79) 1.211/ 1.855 under Z
( 27) 0.825/ 0.966 under Boot
( 212) 0.313/ 0.785 above CF
Z: called 2 times, time in 0.309/83.485
( 2) 0.309/ 83.485 under ProfileRoot
( 7) 0/ 6.954 above Boot
( 298) 1.007/ 2.304 above CF
( 79) 8.018/ 16.969 above EZip3
( 79) 1.211/ 1.855 above Zip1
( 158) 1.428/ 41.870 above Zip2
( 79) 2.725/ 13.224 above Zip3
Boot: called 27 times, time in 0.078/19.579
( 7) 0/ 6.954 under Z
( 20) 0.078/ 12.625 under Boot
( 20) 0.078/ 12.625 above Boot
( 88) 0.109/ 0.251 above CF
( 27) 0.186/ 1.685 above EZip3
( 27) 0.825/ 0.966 above Zip1
( 54) 0.630/ 1.160 above Zip2
( 27) 1.093/ 2.814 above Zip3
RVK: called 2 times, time in 0./0.
( 2) 0/ 0 under ProfileRoot

```

In[#:]= **Timing@Block**[{**\$k** = 2}, **Z[Knot**[3, 1]]]

$$\text{Out[#:]} = \left\{ 143.844, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -4 \mathfrak{t} \hbar - \text{Log} \left[ \left( \frac{1}{\mathfrak{T}^3} - \frac{2}{\mathfrak{T}^2} + \frac{2}{\mathfrak{T}} \right)^2 \right] - \text{Log} \left[ \left( 1 + \frac{\mathfrak{T}}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} - \frac{\mathfrak{T}^2}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} \right)^2 \right] \right), \right. \\ \frac{a \left( -2 \hbar + 2 \mathfrak{T}^2 \hbar \right)}{1 - \mathfrak{T} + \mathfrak{T}^2} + \frac{-2 \hbar + 3 \mathfrak{T} \hbar - 2 \mathfrak{T}^2 \hbar + \mathfrak{T}^3 \hbar}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{x y \left( -2 \hbar^2 - 2 \mathfrak{T} \hbar^2 \right)}{1 - \mathfrak{T} + \mathfrak{T}^2}, \frac{a^2 \left( 2 \mathfrak{T} \hbar^2 - 8 \mathfrak{T}^2 \hbar^2 + 2 \mathfrak{T}^3 \hbar^2 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \\ \frac{a \left( 2 \mathfrak{T} \hbar^2 - 14 \mathfrak{T}^2 \hbar^2 + 12 \mathfrak{T}^3 \hbar^2 - 6 \mathfrak{T}^4 \hbar^2 + 2 \mathfrak{T}^5 \hbar^2 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{\mathfrak{T} \hbar^2 - 11 \mathfrak{T}^2 \hbar^2 + 16 \mathfrak{T}^3 \hbar^2 - 12 \mathfrak{T}^4 \hbar^2 + 8 \mathfrak{T}^5 \hbar^2 - 3 \mathfrak{T}^6 \hbar^2 + \mathfrak{T}^7 \hbar^2}{2 - 8 \mathfrak{T} + 20 \mathfrak{T}^2 - 32 \mathfrak{T}^3 + 38 \mathfrak{T}^4 - 32 \mathfrak{T}^5 + 20 \mathfrak{T}^6 - 8 \mathfrak{T}^7 + 2 \mathfrak{T}^8} + \\ \left. \frac{a x y \left( 8 \mathfrak{T} \hbar^3 - 8 \mathfrak{T}^2 \hbar^3 - 4 \mathfrak{T}^3 \hbar^3 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{x y \left( -2 \hbar^3 - 2 \mathfrak{T}^2 \hbar^3 - 6 \mathfrak{T}^3 \hbar^3 + 2 \mathfrak{T}^5 \hbar^3 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{x^2 y^2 \left( \hbar^4 + 5 \mathfrak{T} \hbar^4 + \mathfrak{T}^2 \hbar^4 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} \right] \right\}$$



```
In[ ]:= PrintProfile[]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 227.329
( 3) 0.466/ 227.330 above Z
( 3) 0/ 0 above RVK
CCF: called 47610 times, time in 110.268/110.268
( 47610) 110.270/ 110.270 under CF
CF: called 41270 times, time in 82.741/193.009
( 424) 1.430/ 3.385 under Z
( 202) 0.202/ 0.485 under Boot
( 498) 17.104/ 36.795 under EZip3
( 302) 0.424/ 1.117 under Zip1
( 996) 31.594/ 92.989 under Zip2
( 38848) 31.987/ 58.238 under Zip3
( 47610) 110.270/ 110.270 above CCF
EZip3: called 151 times, time in 18.36/62.125
( 101) 18.001/ 58.814 under Z
( 50) 0.359/ 3.311 under Boot
( 498) 17.104/ 36.795 above CF
( 151) 2.742/ 6.970 above Zip3
Zip3: called 302 times, time in 9.438/67.676
( 101) 4.723/ 55.610 under Z
( 50) 1.973/ 5.096 under Boot
( 151) 2.742/ 6.970 under EZip3
( 38848) 31.987/ 58.238 above CF
Zip2: called 302 times, time in 2.929/95.918
( 202) 1.913/ 93.511 under Z
( 100) 1.016/ 2.407 under Boot
( 996) 31.594/ 92.989 above CF
Zip1: called 151 times, time in 2.923/4.04
( 101) 1.567/ 2.370 under Z
( 50) 1.356/ 1.670 under Boot
( 302) 0.424/ 1.117 above CF
Z: called 3 times, time in 0.466/227.329
( 3) 0.466/ 227.330 under ProfileRoot
( 12) 0.016/ 13.173 above Boot
( 424) 1.430/ 3.385 above CF
( 101) 18.001/ 58.814 above EZip3
( 101) 1.567/ 2.370 above Zip1
( 202) 1.913/ 93.511 above Zip2
( 101) 4.723/ 55.610 above Zip3
Boot: called 47 times, time in 0.204/35.61
( 12) 0.016/ 13.173 under Z
( 35) 0.188/ 22.437 under Boot
( 35) 0.188/ 22.437 above Boot
( 202) 0.202/ 0.485 above CF
( 50) 0.359/ 3.311 above EZip3
( 50) 1.356/ 1.670 above Zip1
( 100) 1.016/ 2.407 above Zip2
( 50) 1.973/ 5.096 above Zip3
RVK: called 3 times, time in 0./0.
( 3) 0/ 0 under ProfileRoot
```

In[ ]:= **Timing@Block**[{**\$k** = 2}, **Z[Knot**[8, 17]]]

$$\begin{aligned}
 \text{Out[ ]} = & \left\{ 1747.75, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -2 \mathfrak{t} \hbar - \text{Log} \left[ \left( -1 - \frac{1}{\mathfrak{T}^4} + \frac{4}{\mathfrak{T}^3} - \frac{6}{\mathfrak{T}^2} + \frac{5}{\mathfrak{T}} \right)^2 \right] - \right. \right. \\
 & \text{Log} \left[ \left( 1 + \frac{\mathfrak{T}}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{2 \mathfrak{T}^2}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{\mathfrak{T}^3}{1 - 4 \mathfrak{T} + 6 \mathfrak{T}^2 - 5 \mathfrak{T}^3 + \mathfrak{T}^4} \right)^2 \right] - \\
 & \text{Log} \left[ \left( 1 - \frac{\mathfrak{T}}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{4 \mathfrak{T}^2}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{7 \mathfrak{T}^3}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \right. \right. \\
 & \left. \left. \frac{7 \mathfrak{T}^4}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} - \frac{4 \mathfrak{T}^5}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{\mathfrak{T}^6}{1 - 3 \mathfrak{T} + 4 \mathfrak{T}^2 - 4 \mathfrak{T}^3 + \mathfrak{T}^4} \right)^2 \right] \right], \\
 & \frac{-3 \hbar + 8 \mathfrak{T} \hbar - 8 \mathfrak{T}^2 \hbar + 8 \mathfrak{T}^4 \hbar - 8 \mathfrak{T}^5 \hbar + 3 \mathfrak{T}^6 \hbar}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{a \left( -6 \hbar + 16 \mathfrak{T} \hbar - 16 \mathfrak{T}^2 \hbar + 16 \mathfrak{T}^4 \hbar - 16 \mathfrak{T}^5 \hbar + 6 \mathfrak{T}^6 \hbar \right)}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6} + \\
 & \frac{x y \left( -6 \hbar^2 + 10 \mathfrak{T} \hbar^2 - 6 \mathfrak{T}^2 \hbar^2 - 6 \mathfrak{T}^3 \hbar^2 + 10 \mathfrak{T}^4 \hbar^2 - 6 \mathfrak{T}^5 \hbar^2 \right)}{1 - 4 \mathfrak{T} + 8 \mathfrak{T}^2 - 11 \mathfrak{T}^3 + 8 \mathfrak{T}^4 - 4 \mathfrak{T}^5 + \mathfrak{T}^6}, \\
 & \left( a \left( 8 \mathfrak{T} \hbar^2 - 64 \mathfrak{T}^2 \hbar^2 + 262 \mathfrak{T}^3 \hbar^2 - 608 \mathfrak{T}^4 \hbar^2 + 952 \mathfrak{T}^5 \hbar^2 - 1096 \mathfrak{T}^6 \hbar^2 + 952 \mathfrak{T}^7 \hbar^2 - 608 \mathfrak{T}^8 \hbar^2 + 262 \mathfrak{T}^9 \hbar^2 - 64 \mathfrak{T}^{10} \hbar^2 + \right. \right. \\
 & \left. \left. 8 \mathfrak{T}^{11} \hbar^2 \right) \right) / \left( 1 - 8 \mathfrak{T} + 32 \mathfrak{T}^2 - 86 \mathfrak{T}^3 + 168 \mathfrak{T}^4 - 248 \mathfrak{T}^5 + 283 \mathfrak{T}^6 - 248 \mathfrak{T}^7 + 168 \mathfrak{T}^8 - 86 \mathfrak{T}^9 + 32 \mathfrak{T}^{10} - 8 \mathfrak{T}^{11} + \mathfrak{T}^{12} \right) + \\
 & \left( a^2 \left( 8 \mathfrak{T} \hbar^2 - 64 \mathfrak{T}^2 \hbar^2 + 262 \mathfrak{T}^3 \hbar^2 - 608 \mathfrak{T}^4 \hbar^2 + 952 \mathfrak{T}^5 \hbar^2 - 1096 \mathfrak{T}^6 \hbar^2 + 952 \mathfrak{T}^7 \hbar^2 - 608 \mathfrak{T}^8 \hbar^2 + 262 \mathfrak{T}^9 \hbar^2 - 64 \mathfrak{T}^{10} \hbar^2 + \right. \right. \\
 & \left. \left. 8 \mathfrak{T}^{11} \hbar^2 \right) \right) / \left( 1 - 8 \mathfrak{T} + 32 \mathfrak{T}^2 - 86 \mathfrak{T}^3 + 168 \mathfrak{T}^4 - 248 \mathfrak{T}^5 + 283 \mathfrak{T}^6 - 248 \mathfrak{T}^7 + 168 \mathfrak{T}^8 - 86 \mathfrak{T}^9 + 32 \mathfrak{T}^{10} - 8 \mathfrak{T}^{11} + \mathfrak{T}^{12} \right) + \\
 & \left( 4 \mathfrak{T} \hbar^2 - 50 \mathfrak{T}^2 \hbar^2 + 307 \mathfrak{T}^3 \hbar^2 - 1160 \mathfrak{T}^4 \hbar^2 + 3062 \mathfrak{T}^5 \hbar^2 - 6127 \mathfrak{T}^6 \hbar^2 + 9760 \mathfrak{T}^7 \hbar^2 - 12754 \mathfrak{T}^8 \hbar^2 + 13916 \mathfrak{T}^9 \hbar^2 - \right. \\
 & \left. 12754 \mathfrak{T}^{10} \hbar^2 + 9760 \mathfrak{T}^{11} \hbar^2 - 6127 \mathfrak{T}^{12} \hbar^2 + 3062 \mathfrak{T}^{13} \hbar^2 - 1160 \mathfrak{T}^{14} \hbar^2 + 307 \mathfrak{T}^{15} \hbar^2 - 50 \mathfrak{T}^{16} \hbar^2 + 4 \mathfrak{T}^{17} \hbar^2 \right) / \\
 & \left( 2 - 24 \mathfrak{T} + 144 \mathfrak{T}^2 - 578 \mathfrak{T}^3 + 1728 \mathfrak{T}^4 - 4056 \mathfrak{T}^5 + 7708 \mathfrak{T}^6 - 12072 \mathfrak{T}^7 + 15744 \mathfrak{T}^8 - 17194 \mathfrak{T}^9 + \right. \\
 & \left. 15744 \mathfrak{T}^{10} - 12072 \mathfrak{T}^{11} + 7708 \mathfrak{T}^{12} - 4056 \mathfrak{T}^{13} + 1728 \mathfrak{T}^{14} - 578 \mathfrak{T}^{15} + 144 \mathfrak{T}^{16} - 24 \mathfrak{T}^{17} + 2 \mathfrak{T}^{18} \right) + \\
 & \left( a x y \left( 28 \mathfrak{T} \hbar^3 - 168 \mathfrak{T}^2 \hbar^3 + 544 \mathfrak{T}^3 \hbar^3 - 1000 \mathfrak{T}^4 \hbar^3 + 1248 \mathfrak{T}^5 \hbar^3 - 1096 \mathfrak{T}^6 \hbar^3 + \right. \right. \\
 & \left. \left. 656 \mathfrak{T}^7 \hbar^3 - 216 \mathfrak{T}^8 \hbar^3 - 20 \mathfrak{T}^9 \hbar^3 + 40 \mathfrak{T}^{10} \hbar^3 - 12 \mathfrak{T}^{11} \hbar^3 \right) \right) / \\
 & \left( 1 - 8 \mathfrak{T} + 32 \mathfrak{T}^2 - 86 \mathfrak{T}^3 + 168 \mathfrak{T}^4 - 248 \mathfrak{T}^5 + 283 \mathfrak{T}^6 - 248 \mathfrak{T}^7 + 168 \mathfrak{T}^8 - 86 \mathfrak{T}^9 + 32 \mathfrak{T}^{10} - 8 \mathfrak{T}^{11} + \mathfrak{T}^{12} \right) + \\
 & \left( x y \left( -18 \hbar^3 + 78 \mathfrak{T} \hbar^3 - 146 \mathfrak{T}^2 \hbar^3 + 110 \mathfrak{T}^3 \hbar^3 + 78 \mathfrak{T}^4 \hbar^3 - 274 \mathfrak{T}^5 \hbar^3 + \right. \right. \\
 & \left. \left. 274 \mathfrak{T}^6 \hbar^3 - 78 \mathfrak{T}^7 \hbar^3 - 110 \mathfrak{T}^8 \hbar^3 + 146 \mathfrak{T}^9 \hbar^3 - 78 \mathfrak{T}^{10} \hbar^3 + 18 \mathfrak{T}^{11} \hbar^3 \right) \right) / \\
 & \left( 1 - 8 \mathfrak{T} + 32 \mathfrak{T}^2 - 86 \mathfrak{T}^3 + 168 \mathfrak{T}^4 - 248 \mathfrak{T}^5 + 283 \mathfrak{T}^6 - 248 \mathfrak{T}^7 + 168 \mathfrak{T}^8 - 86 \mathfrak{T}^9 + 32 \mathfrak{T}^{10} - 8 \mathfrak{T}^{11} + \mathfrak{T}^{12} \right) + \\
 & \left( x^2 y^2 \left( 3 \hbar^4 - 37 \mathfrak{T}^2 \hbar^4 + 153 \mathfrak{T}^3 \hbar^4 - 261 \mathfrak{T}^4 \hbar^4 + 325 \mathfrak{T}^5 \hbar^4 - 261 \mathfrak{T}^6 \hbar^4 + 153 \mathfrak{T}^7 \hbar^4 - 37 \mathfrak{T}^8 \hbar^4 + 3 \mathfrak{T}^{10} \hbar^4 \right) \right) / \\
 & \left. \left( 1 - 8 \mathfrak{T} + 32 \mathfrak{T}^2 - 86 \mathfrak{T}^3 + 168 \mathfrak{T}^4 - 248 \mathfrak{T}^5 + 283 \mathfrak{T}^6 - 248 \mathfrak{T}^7 + 168 \mathfrak{T}^8 - 86 \mathfrak{T}^9 + 32 \mathfrak{T}^{10} - 8 \mathfrak{T}^{11} + \mathfrak{T}^{12} \right) \right] \}
 \end{aligned}$$

```
In[ ]:= PrintProfile[]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 1975.08
(    4)    1.484/ 1975.080 above Z
(    4)      0/      0 above RVK
CCF: called 104354 times, time in 940.437/940.437
( 104354) 940.437/ 940.437 under CF
CF: called 56959 times, time in 838.722/1779.16
(   745)   7.192/ 16.274 under Z
(   220)   0.248/   0.547 under Boot
(   742) 244.828/ 463.079 under EZip3
(   424)   0.565/   1.568 under Zip1
(  1484) 307.379/ 756.116 under Zip2
( 53344) 278.510/ 541.575 under Zip3
( 104354) 940.437/ 940.437 above CCF
EZip3: called 212 times, time in 165.566/639.85
(   158) 165.177/ 636.352 under Z
(    54)   0.389/   3.498 under Boot
(   742) 244.828/ 463.079 above CF
(   212)   4.510/  11.205 above Zip3
Zip3: called 424 times, time in 20.098/561.673
(   158) 13.568/ 545.122 under Z
(    54)   2.020/   5.346 under Boot
(   212)   4.510/  11.205 under EZip3
( 53344) 278.510/ 541.575 above CF
Zip2: called 424 times, time in 4.72/760.836
(   316)   3.643/ 758.226 under Z
(   108)   1.077/   2.610 under Boot
(  1484) 307.379/ 756.116 above CF
Zip1: called 212 times, time in 3.801/5.369
(   158)   2.413/   3.667 under Z
(    54)   1.388/   1.702 under Boot
(   424)   0.565/   1.568 above CF
Z: called 4 times, time in 1.484/1975.08
(    4)   1.484/ 1975.080 under ProfileRoot
(   14)   0.016/  13.954 above Boot
(   745)   7.192/  16.274 above CF
(   158) 165.177/ 636.352 above EZip3
(   158)   2.413/   3.667 above Zip1
(   316)   3.643/ 758.226 above Zip2
(   158) 13.568/ 545.122 above Zip3
Boot: called 51 times, time in 0.251/36.86
(   14)   0.016/  13.954 under Z
(   37)   0.235/  22.906 under Boot
(   37)   0.235/  22.906 above Boot
(   220)   0.248/   0.547 above CF
(    54)   0.389/   3.498 above EZip3
(    54)   1.388/   1.702 above Zip1
(   108)   1.077/   2.610 above Zip2
(    54)   2.020/   5.346 above Zip3
RVK: called 4 times, time in 0./0.
(    4)      0/      0 under ProfileRoot
```

In[#:]= **Timing@Block**[{**\$k** = 3}, **Z[Knot**[3, 1]]]

$$\begin{aligned}
 \text{Out[#:]} = & \left\{ 3841.72, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -4 \mathfrak{t} \hbar - \text{Log} \left[ \left( \frac{1}{\mathfrak{T}^3} - \frac{2}{\mathfrak{T}^2} + \frac{2}{\mathfrak{T}} \right)^2 \right] - \text{Log} \left[ \left( 1 + \frac{\mathfrak{T}}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} - \frac{\mathfrak{T}^2}{1 - 2 \mathfrak{T} + 2 \mathfrak{T}^2} \right)^2 \right] \right), \right. \\
 & \frac{a \left( -2 \hbar + 2 \mathfrak{T}^2 \hbar \right)}{1 - \mathfrak{T} + \mathfrak{T}^2} + \frac{-2 \hbar + 3 \mathfrak{T} \hbar - 2 \mathfrak{T}^2 \hbar + \mathfrak{T}^3 \hbar}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{x y \left( -2 \hbar^2 - 2 \mathfrak{T} \hbar^2 \right)}{1 - \mathfrak{T} + \mathfrak{T}^2}, \frac{a^2 \left( 2 \mathfrak{T} \hbar^2 - 8 \mathfrak{T}^2 \hbar^2 + 2 \mathfrak{T}^3 \hbar^2 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \\
 & \frac{a \left( 2 \mathfrak{T} \hbar^2 - 14 \mathfrak{T}^2 \hbar^2 + 12 \mathfrak{T}^3 \hbar^2 - 6 \mathfrak{T}^4 \hbar^2 + 2 \mathfrak{T}^5 \hbar^2 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{\mathfrak{T} \hbar^2 - 11 \mathfrak{T}^2 \hbar^2 + 16 \mathfrak{T}^3 \hbar^2 - 12 \mathfrak{T}^4 \hbar^2 + 8 \mathfrak{T}^5 \hbar^2 - 3 \mathfrak{T}^6 \hbar^2 + \mathfrak{T}^7 \hbar^2}{2 - 8 \mathfrak{T} + 20 \mathfrak{T}^2 - 32 \mathfrak{T}^3 + 38 \mathfrak{T}^4 - 32 \mathfrak{T}^5 + 20 \mathfrak{T}^6 - 8 \mathfrak{T}^7 + 2 \mathfrak{T}^8} + \\
 & \frac{a x y \left( 8 \mathfrak{T} \hbar^3 - 8 \mathfrak{T}^2 \hbar^3 - 4 \mathfrak{T}^3 \hbar^3 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4} + \frac{x y \left( -2 \hbar^3 - 2 \mathfrak{T}^2 \hbar^3 - 6 \mathfrak{T}^3 \hbar^3 + 2 \mathfrak{T}^5 \hbar^3 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \frac{x^2 y^2 \left( \hbar^4 + 5 \mathfrak{T} \hbar^4 + \mathfrak{T}^2 \hbar^4 \right)}{1 - 2 \mathfrak{T} + 3 \mathfrak{T}^2 - 2 \mathfrak{T}^3 + \mathfrak{T}^4}, \\
 & \frac{a^3 \left( -4 \mathfrak{T} \hbar^3 + 28 \mathfrak{T}^2 \hbar^3 - 28 \mathfrak{T}^4 \hbar^3 + 4 \mathfrak{T}^5 \hbar^3 \right)}{3 - 9 \mathfrak{T} + 18 \mathfrak{T}^2 - 21 \mathfrak{T}^3 + 18 \mathfrak{T}^4 - 9 \mathfrak{T}^5 + 3 \mathfrak{T}^6} + \frac{a^2 \left( -2 \mathfrak{T} \hbar^3 + 24 \mathfrak{T}^2 \hbar^3 - 12 \mathfrak{T}^3 \hbar^3 - 32 \mathfrak{T}^4 \hbar^3 + 20 \mathfrak{T}^5 \hbar^3 - 8 \mathfrak{T}^6 \hbar^3 + 2 \mathfrak{T}^7 \hbar^3 \right)}{1 - 4 \mathfrak{T} + 10 \mathfrak{T}^2 - 16 \mathfrak{T}^3 + 19 \mathfrak{T}^4 - 16 \mathfrak{T}^5 + 10 \mathfrak{T}^6 - 4 \mathfrak{T}^7 + \mathfrak{T}^8} + \\
 & \left( a \left( -\mathfrak{T} \hbar^3 + 19 \mathfrak{T}^2 \hbar^3 - 19 \mathfrak{T}^3 \hbar^3 - 34 \mathfrak{T}^4 \hbar^3 + 40 \mathfrak{T}^5 \hbar^3 - 22 \mathfrak{T}^6 \hbar^3 + 11 \mathfrak{T}^7 \hbar^3 - 3 \mathfrak{T}^8 \hbar^3 + \mathfrak{T}^9 \hbar^3 \right) \right) / \\
 & \left( 1 - 5 \mathfrak{T} + 15 \mathfrak{T}^2 - 30 \mathfrak{T}^3 + 45 \mathfrak{T}^4 - 51 \mathfrak{T}^5 + 45 \mathfrak{T}^6 - 30 \mathfrak{T}^7 + 15 \mathfrak{T}^8 - 5 \mathfrak{T}^9 + \mathfrak{T}^{10} \right) + \\
 & \left( -\mathfrak{T} \hbar^3 + 29 \mathfrak{T}^2 \hbar^3 - 43 \mathfrak{T}^3 \hbar^3 - 71 \mathfrak{T}^4 \hbar^3 + 131 \mathfrak{T}^5 \hbar^3 - 84 \mathfrak{T}^6 \hbar^3 + 53 \mathfrak{T}^7 \hbar^3 - 23 \mathfrak{T}^8 \hbar^3 + 11 \mathfrak{T}^9 \hbar^3 - 3 \mathfrak{T}^{10} \hbar^3 + \mathfrak{T}^{11} \hbar^3 \right) / \\
 & \left( 6 - 36 \mathfrak{T} + 126 \mathfrak{T}^2 - 300 \mathfrak{T}^3 + 540 \mathfrak{T}^4 - 756 \mathfrak{T}^5 + 846 \mathfrak{T}^6 - 756 \mathfrak{T}^7 + 540 \mathfrak{T}^8 - 300 \mathfrak{T}^9 + 126 \mathfrak{T}^{10} - 36 \mathfrak{T}^{11} + 6 \mathfrak{T}^{12} \right) + \\
 & \frac{a^2 x y \left( -8 \mathfrak{T} \hbar^4 + 8 \mathfrak{T}^2 \hbar^4 + 36 \mathfrak{T}^3 \hbar^4 - 20 \mathfrak{T}^4 \hbar^4 - 4 \mathfrak{T}^5 \hbar^4 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \\
 & \frac{a x y \left( 12 \mathfrak{T} \hbar^4 - 16 \mathfrak{T}^2 \hbar^4 + 40 \mathfrak{T}^3 \hbar^4 - 16 \mathfrak{T}^4 \hbar^4 - 56 \mathfrak{T}^5 \hbar^4 + 8 \mathfrak{T}^6 \hbar^4 + 4 \mathfrak{T}^7 \hbar^4 \right)}{1 - 4 \mathfrak{T} + 10 \mathfrak{T}^2 - 16 \mathfrak{T}^3 + 19 \mathfrak{T}^4 - 16 \mathfrak{T}^5 + 10 \mathfrak{T}^6 - 4 \mathfrak{T}^7 + \mathfrak{T}^8} + \\
 & \left( x y \left( -4 \hbar^4 + 3 \mathfrak{T} \hbar^4 - 6 \mathfrak{T}^2 \hbar^4 - 9 \mathfrak{T}^3 \hbar^4 - 15 \mathfrak{T}^4 \hbar^4 - 63 \mathfrak{T}^5 \hbar^4 - 9 \mathfrak{T}^6 \hbar^4 + 42 \mathfrak{T}^7 \hbar^4 + 3 \mathfrak{T}^8 \hbar^4 - 4 \mathfrak{T}^9 \hbar^4 \right) \right) / \\
 & \left( 3 - 15 \mathfrak{T} + 45 \mathfrak{T}^2 - 90 \mathfrak{T}^3 + 135 \mathfrak{T}^4 - 153 \mathfrak{T}^5 + 135 \mathfrak{T}^6 - 90 \mathfrak{T}^7 + 45 \mathfrak{T}^8 - 15 \mathfrak{T}^9 + 3 \mathfrak{T}^{10} \right) + \\
 & \frac{a x^2 y^2 \left( -14 \mathfrak{T} \hbar^5 - 6 \mathfrak{T}^2 \hbar^5 + 30 \mathfrak{T}^3 \hbar^5 + 4 \mathfrak{T}^4 \hbar^5 \right)}{1 - 3 \mathfrak{T} + 6 \mathfrak{T}^2 - 7 \mathfrak{T}^3 + 6 \mathfrak{T}^4 - 3 \mathfrak{T}^5 + \mathfrak{T}^6} + \\
 & \frac{x^2 y^2 \left( 2 \hbar^5 + 23 \mathfrak{T} \hbar^5 - 10 \mathfrak{T}^2 \hbar^5 + 11 \mathfrak{T}^3 \hbar^5 + 42 \mathfrak{T}^4 \hbar^5 - 29 \mathfrak{T}^5 \hbar^5 - 8 \mathfrak{T}^6 \hbar^5 \right)}{1 - 4 \mathfrak{T} + 10 \mathfrak{T}^2 - 16 \mathfrak{T}^3 + 19 \mathfrak{T}^4 - 16 \mathfrak{T}^5 + 10 \mathfrak{T}^6 - 4 \mathfrak{T}^7 + \mathfrak{T}^8} + \\
 & \left. \frac{x^3 y^3 \left( -2 \hbar^6 - 24 \mathfrak{T} \hbar^6 - 24 \mathfrak{T}^2 \hbar^6 - 2 \mathfrak{T}^3 \hbar^6 \right)}{3 - 9 \mathfrak{T} + 18 \mathfrak{T}^2 - 21 \mathfrak{T}^3 + 18 \mathfrak{T}^4 - 9 \mathfrak{T}^5 + 3 \mathfrak{T}^6} \right\}
 \end{aligned}$$

In[ ]:= **PrintProfile**[ ]

Out[ ]:= ProfileRoot is root. Profiled time: 5816.8

```
( 5)      2.375/ 5816.798 above Z
( 5)      0/      0 above RVK
CCF: called 169628 times, time in 2757.93/2757.93
( 169628) 2757.925/ 2757.925 under CF
CF: called 71886 times, time in 2708.74/5466.67
( 913)    8.985/   20.634 under Z
( 372)    0.485/   1.144 under Boot
( 967)   1234.497/ 2279.516 under EZip3
( 514)    0.661/   1.821 under Zip1
( 1934)   470.278/ 1266.607 under Zip2
( 67186)  993.839/ 1896.948 under Zip3
( 169628) 2757.925/ 2757.925 above CCF
EZip3: called 257 times, time in 297.784/2604.69
( 180)   296.975/ 2598.603 under Z
( 77)    0.809/   6.089 under Boot
( 967)   1234.497/ 2279.516 above CF
( 257)    9.115/   27.392 above Zip3
Zip3: called 514 times, time in 38.768/1935.72
( 180)   26.381/ 1899.794 under Z
( 77)    3.272/   8.530 under Boot
( 257)    9.115/   27.392 under EZip3
( 67186)  993.839/ 1896.948 above CF
Zip2: called 514 times, time in 6.096/1272.7
( 360)    4.563/ 1267.181 under Z
( 154)    1.533/   5.522 under Boot
( 1934)   470.278/ 1266.607 above CF
Zip1: called 257 times, time in 4.697/6.518
( 180)    2.752/   4.148 under Z
( 77)     1.945/   2.370 under Boot
( 514)    0.661/   1.821 above CF
Z: called 5 times, time in 2.375/5816.8
( 5)      2.375/ 5816.798 under ProfileRoot
( 19)     0.031/   24.063 above Boot
( 913)    8.985/   20.634 above CF
( 180)   296.975/ 2598.603 above EZip3
( 180)    2.752/   4.148 above Zip1
( 360)    4.563/ 1267.181 above Zip2
( 180)   26.381/ 1899.794 above Zip3
Boot: called 71 times, time in 0.408/60.783
( 19)     0.031/   24.063 under Z
( 52)     0.377/   36.720 under Boot
( 52)     0.377/   36.720 above Boot
( 372)    0.485/   1.144 above CF
( 77)     0.809/   6.089 above EZip3
( 77)     1.945/   2.370 above Zip1
( 154)    1.533/   5.522 above Zip2
( 77)     3.272/   8.530 above Zip3
RVK: called 5 times, time in 0./0.
( 5)      0/      0 under ProfileRoot
```

In[ ]:= **Timing@Block**[{**\$k** = 3}, **Z**[**Knot**[8, 17]]]

Out[ ]:=  $\left\{ 79170.2, \mathbb{E}_{\{\cdot\} \rightarrow \{\emptyset\}} \left[ \frac{1}{2} \left( -2t\hbar - \text{Log} \left[ \left( -1 - \frac{1}{T^4} + \frac{4}{T^3} - \frac{6}{T^2} + \frac{5}{T} \right)^2 \right] \right) - \right. \right.$

$$\begin{aligned}
& \text{Log} \left[ \left( 1 + \frac{T}{1-4T+6T^2-5T^3+T^4} - \frac{2T^2}{1-4T+6T^2-5T^3+T^4} + \frac{T^3}{1-4T+6T^2-5T^3+T^4} \right)^2 \right] - \\
& \text{Log} \left[ \left( 1 - \frac{T}{1-3T+4T^2-4T^3+T^4} + \frac{4T^2}{1-3T+4T^2-4T^3+T^4} - \frac{7T^3}{1-3T+4T^2-4T^3+T^4} + \right. \right. \\
& \quad \left. \left. \frac{7T^4}{1-3T+4T^2-4T^3+T^4} - \frac{4T^5}{1-3T+4T^2-4T^3+T^4} + \frac{T^6}{1-3T+4T^2-4T^3+T^4} \right)^2 \right] , \\
& \frac{-3\hbar + 8T\hbar - 8T^2\hbar + 8T^4\hbar - 8T^5\hbar + 3T^6\hbar}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} + \frac{a(-6\hbar + 16T\hbar - 16T^2\hbar + 16T^4\hbar - 16T^5\hbar + 6T^6\hbar)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6} + \\
& \frac{xy(-6\hbar^2 + 10T\hbar^2 - 6T^2\hbar^2 - 6T^3\hbar^2 + 10T^4\hbar^2 - 6T^5\hbar^2)}{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}, \\
& (a(8T\hbar^2 - 64T^2\hbar^2 + 262T^3\hbar^2 - 608T^4\hbar^2 + 952T^5\hbar^2 - 1096T^6\hbar^2 + 952T^7\hbar^2 - 608T^8\hbar^2 + 262T^9\hbar^2 - 64T^{10}\hbar^2 + \\
& \quad 8T^{11}\hbar^2)) / (1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}) + \\
& (a^2(8T\hbar^2 - 64T^2\hbar^2 + 262T^3\hbar^2 - 608T^4\hbar^2 + 952T^5\hbar^2 - 1096T^6\hbar^2 + 952T^7\hbar^2 - 608T^8\hbar^2 + 262T^9\hbar^2 - 64T^{10}\hbar^2 + \\
& \quad 8T^{11}\hbar^2)) / (1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}) + \\
& (4T\hbar^2 - 50T^2\hbar^2 + 307T^3\hbar^2 - 1160T^4\hbar^2 + 3062T^5\hbar^2 - 6127T^6\hbar^2 + 9760T^7\hbar^2 - 12754T^8\hbar^2 + 13916T^9\hbar^2 - \\
& \quad 12754T^{10}\hbar^2 + 9760T^{11}\hbar^2 - 6127T^{12}\hbar^2 + 3062T^{13}\hbar^2 - 1160T^{14}\hbar^2 + 307T^{15}\hbar^2 - 50T^{16}\hbar^2 + 4T^{17}\hbar^2) / \\
& (2-24T+144T^2-578T^3+1728T^4-4056T^5+7708T^6-12072T^7+15744T^8-17194T^9+ \\
& \quad 15744T^{10}-12072T^{11}+7708T^{12}-4056T^{13}+1728T^{14}-578T^{15}+144T^{16}-24T^{17}+2T^{18}) + \\
& (axy(28T\hbar^3 - 168T^2\hbar^3 + 544T^3\hbar^3 - 1000T^4\hbar^3 + 1248T^5\hbar^3 - 1096T^6\hbar^3 + \\
& \quad 656T^7\hbar^3 - 216T^8\hbar^3 - 20T^9\hbar^3 + 40T^{10}\hbar^3 - 12T^{11}\hbar^3)) / \\
& (1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}) + \\
& (xy(-18\hbar^3 + 78T\hbar^3 - 146T^2\hbar^3 + 110T^3\hbar^3 + 78T^4\hbar^3 - 274T^5\hbar^3 + \\
& \quad 274T^6\hbar^3 - 78T^7\hbar^3 - 110T^8\hbar^3 + 146T^9\hbar^3 - 78T^{10}\hbar^3 + 18T^{11}\hbar^3)) / \\
& (1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}) + \\
& (x^2y^2(3\hbar^4 - 37T^2\hbar^4 + 153T^3\hbar^4 - 261T^4\hbar^4 + 325T^5\hbar^4 - 261T^6\hbar^4 + 153T^7\hbar^4 - 37T^8\hbar^4 + 3T^{10}\hbar^4)) / \\
& (1-8T+32T^2-86T^3+168T^4-248T^5+283T^6-248T^7+168T^8-86T^9+32T^{10}-8T^{11}+T^{12}), \\
& (a^2(-8T\hbar^3 + 96T^2\hbar^3 - 594T^3\hbar^3 + 2016T^4\hbar^3 - 4264T^5\hbar^3 + 5994T^6\hbar^3 - 5824T^7\hbar^3 + 3536T^8\hbar^3 - \\
& \quad 3536T^{10}\hbar^3 + 5824T^{11}\hbar^3 - 5994T^{12}\hbar^3 + 4264T^{13}\hbar^3 - 2016T^{14}\hbar^3 + 594T^{15}\hbar^3 - 96T^{16}\hbar^3 + 8T^{17}\hbar^3)) / \\
& (1-12T+72T^2-289T^3+864T^4-2028T^5+3854T^6-6036T^7+7872T^8-8597T^9+7872T^{10}- \\
& \quad 6036T^{11}+3854T^{12}-2028T^{13}+864T^{14}-289T^{15}+72T^{16}-12T^{17}+T^{18}) + \\
& (a^3(-16T\hbar^3 + 192T^2\hbar^3 - 1188T^3\hbar^3 + 4032T^4\hbar^3 - 8528T^5\hbar^3 + 11988T^6\hbar^3 - 11648T^7\hbar^3 + 7072T^8\hbar^3 - \\
& \quad 7072T^{10}\hbar^3 + 11648T^{11}\hbar^3 - 11988T^{12}\hbar^3 + 8528T^{13}\hbar^3 - 4032T^{14}\hbar^3 + 1188T^{15}\hbar^3 - 192T^{16}\hbar^3 + 16T^{17}\hbar^3)) / \\
& (3-36T+216T^2-867T^3+2592T^4-6084T^5+11562T^6-18108T^7+23616T^8-25791T^9+ \\
& \quad 23616T^{10}-18108T^{11}+11562T^{12}-6084T^{13}+2592T^{14}-867T^{15}+216T^{16}-36T^{17}+3T^{18}) + \\
& (-4T\hbar^3 + 76T^2\hbar^3 - 641T^3\hbar^3 + 2816T^4\hbar^3 - 6940T^5\hbar^3 + 8124T^6\hbar^3 + 4904T^7\hbar^3 - 39224T^8\hbar^3 + \\
& \quad 82152T^9\hbar^3 - 101684T^{10}\hbar^3 + 71608T^{11}\hbar^3 - 71608T^{13}\hbar^3 + 101684T^{14}\hbar^3 - 82152T^{15}\hbar^3 + \\
& \quad 39224T^{16}\hbar^3 - 4904T^{17}\hbar^3 - 8124T^{18}\hbar^3 + 6940T^{19}\hbar^3 - 2816T^{20}\hbar^3 + 641T^{21}\hbar^3 - 76T^{22}\hbar^3 + 4T^{23}\hbar^3) / \\
& (6-96T+768T^2-4104T^3+16416T^4-52128T^5+136092T^6-298752T^7+559776T^8-904416T^9+ \\
& \quad 1268640T^{10}-1551744T^{11}+1659090T^{12}-1551744T^{13}+1268640T^{14}-904416T^{15}+ \\
& \quad 559776T^{16}-298752T^{17}+136092T^{18}-52128T^{19}+16416T^{20}-4104T^{21}+768T^{22}-96T^{23}+6T^{24}) + \\
& (a(-4T\hbar^3 + 68T^2\hbar^3 - 561T^3\hbar^3 + 2688T^4\hbar^3 - 8380T^5\hbar^3 + 18212T^6\hbar^3 - 28776T^7\hbar^3 + 33688T^8\hbar^3 - \\
& \quad 29096T^9\hbar^3 + 18052T^{10}\hbar^3 - 7384T^{11}\hbar^3 + 7384T^{13}\hbar^3 - 18052T^{14}\hbar^3 + 29096T^{15}\hbar^3 - 33688T^{16}\hbar^3 + \\
& \quad 28776T^{17}\hbar^3 - 18212T^{18}\hbar^3 + 8380T^{19}\hbar^3 - 2688T^{20}\hbar^3 + 561T^{21}\hbar^3 - 68T^{22}\hbar^3 + 4T^{23}\hbar^3)) /
\end{aligned}$$

$$\begin{aligned}
& \left( 1 - 16 T + 128 T^2 - 684 T^3 + 2736 T^4 - 8688 T^5 + 22682 T^6 - 49792 T^7 + 93296 T^8 - 150736 T^9 + \right. \\
& \quad 211440 T^{10} - 258624 T^{11} + 276515 T^{12} - 258624 T^{13} + 211440 T^{14} - 150736 T^{15} + 93296 T^{16} - \\
& \quad 49792 T^{17} + 22682 T^{18} - 8688 T^{19} + 2736 T^{20} - 684 T^{21} + 128 T^{22} - 16 T^{23} + T^{24} \left. \right) + \left( a^2 x y \right. \\
& \quad \left( -28 T \hbar^4 + 224 T^2 \hbar^4 - 960 T^3 \hbar^4 + 1948 T^4 \hbar^4 - 928 T^5 \hbar^4 - 5472 T^6 \hbar^4 + 17332 T^7 \hbar^4 - 30256 T^8 \hbar^4 + 38100 T^9 \hbar^4 - \right. \\
& \quad \left. 37328 T^{10} \hbar^4 + 28980 T^{11} \hbar^4 - 17460 T^{12} \hbar^4 + 7600 T^{13} \hbar^4 - 2084 T^{14} \hbar^4 + 228 T^{15} \hbar^4 + 32 T^{16} \hbar^4 - 12 T^{17} \hbar^4 \right) \left. \right) / \\
& \left( 1 - 12 T + 72 T^2 - 289 T^3 + 864 T^4 - 2028 T^5 + 3854 T^6 - 6036 T^7 + 7872 T^8 - 8597 T^9 + 7872 T^{10} - \right. \\
& \quad 6036 T^{11} + 3854 T^{12} - 2028 T^{13} + 864 T^{14} - 289 T^{15} + 72 T^{16} - 12 T^{17} + T^{18} \left. \right) + \\
& \left( a x y \left( 132 T \hbar^4 - 1192 T^2 \hbar^4 + 5460 T^3 \hbar^4 - 15300 T^4 \hbar^4 + 28772 T^5 \hbar^4 - 37188 T^6 \hbar^4 + \right. \right. \\
& \quad 30672 T^7 \hbar^4 - 8188 T^8 \hbar^4 - 19080 T^9 \hbar^4 + 36036 T^{10} \hbar^4 - 35760 T^{11} \hbar^4 + \\
& \quad 23580 T^{12} \hbar^4 - 10236 T^{13} \hbar^4 + 2428 T^{14} \hbar^4 + 12 T^{15} \hbar^4 - 168 T^{16} \hbar^4 + 36 T^{17} \hbar^4 \left. \right) \left. \right) / \\
& \left( 1 - 12 T + 72 T^2 - 289 T^3 + 864 T^4 - 2028 T^5 + 3854 T^6 - 6036 T^7 + 7872 T^8 - 8597 T^9 + 7872 T^{10} - \right. \\
& \quad 6036 T^{11} + 3854 T^{12} - 2028 T^{13} + 864 T^{14} - 289 T^{15} + 72 T^{16} - 12 T^{17} + T^{18} \left. \right) + \\
& \left( x y \left( -108 \hbar^4 + 1184 T \hbar^4 - 6228 T^2 \hbar^4 + 20559 T^3 \hbar^4 - 46545 T^4 \hbar^4 + 72963 T^5 \hbar^4 - 70761 T^6 \hbar^4 + \right. \right. \\
& \quad 10415 T^7 \hbar^4 + 101655 T^8 \hbar^4 - 202605 T^9 \hbar^4 + 208095 T^{10} \hbar^4 - 88665 T^{11} \hbar^4 - 88665 T^{12} \hbar^4 + \\
& \quad 208095 T^{13} \hbar^4 - 202605 T^{14} \hbar^4 + 101655 T^{15} \hbar^4 + 10415 T^{16} \hbar^4 - 70761 T^{17} \hbar^4 + \\
& \quad 72963 T^{18} \hbar^4 - 46545 T^{19} \hbar^4 + 20559 T^{20} \hbar^4 - 6228 T^{21} \hbar^4 + 1184 T^{22} \hbar^4 - 108 T^{23} \hbar^4 \left. \right) \left. \right) / \\
& \left( 3 - 48 T + 384 T^2 - 2052 T^3 + 8208 T^4 - 26064 T^5 + 68046 T^6 - 149376 T^7 + 279888 T^8 - 452208 T^9 + \right. \\
& \quad 634320 T^{10} - 775872 T^{11} + 829545 T^{12} - 775872 T^{13} + 634320 T^{14} - 452208 T^{15} + 279888 T^{16} - \\
& \quad 149376 T^{17} + 68046 T^{18} - 26064 T^{19} + 8208 T^{20} - 2052 T^{21} + 384 T^{22} - 48 T^{23} + 3 T^{24} \left. \right) + \\
& \left( x^2 y^2 \left( 18 \hbar^5 + 60 T \hbar^5 - 988 T^2 \hbar^5 + 4723 T^3 \hbar^5 - 12050 T^4 \hbar^5 + 19335 T^5 \hbar^5 - 19017 T^6 \hbar^5 + 7023 T^7 \hbar^5 + 12209 T^8 \hbar^5 - \right. \right. \\
& \quad 27675 T^9 \hbar^5 + 31059 T^{10} \hbar^5 - 23091 T^{11} \hbar^5 + 11368 T^{12} \hbar^5 - 3167 T^{13} \hbar^5 + 74 T^{14} \hbar^5 + 228 T^{15} \hbar^5 - 60 T^{16} \hbar^5 \left. \right) \left. \right) / \\
& \left( 1 - 12 T + 72 T^2 - 289 T^3 + 864 T^4 - 2028 T^5 + 3854 T^6 - 6036 T^7 + 7872 T^8 - 8597 T^9 + 7872 T^{10} - \right. \\
& \quad 6036 T^{11} + 3854 T^{12} - 2028 T^{13} + 864 T^{14} - 289 T^{15} + 72 T^{16} - 12 T^{17} + T^{18} \left. \right) + \\
& \left( a x^2 y^2 \left( -48 T \hbar^5 + 340 T^2 \hbar^5 - 1314 T^3 \hbar^5 + 2512 T^4 \hbar^5 - 1962 T^5 \hbar^5 - 2646 T^6 \hbar^5 + 10398 T^7 \hbar^5 - 18098 T^8 \hbar^5 + \right. \right. \\
& \quad 21762 T^9 \hbar^5 - 19854 T^{10} \hbar^5 + 13914 T^{11} \hbar^5 - 7092 T^{12} \hbar^5 + 2386 T^{13} \hbar^5 - 392 T^{14} \hbar^5 + 12 T^{16} \hbar^5 \left. \right) \left. \right) / \\
& \left( 1 - 12 T + 72 T^2 - 289 T^3 + 864 T^4 - 2028 T^5 + 3854 T^6 - 6036 T^7 + 7872 T^8 - 8597 T^9 + 7872 T^{10} - \right. \\
& \quad 6036 T^{11} + 3854 T^{12} - 2028 T^{13} + 864 T^{14} - 289 T^{15} + 72 T^{16} - 12 T^{17} + T^{18} \left. \right) + \\
& \left( x^3 y^3 \left( -6 \hbar^6 - 30 T \hbar^6 + 336 T^2 \hbar^6 - 1514 T^3 \hbar^6 + 3288 T^4 \hbar^6 - 4650 T^5 \hbar^6 + 3954 T^6 \hbar^6 - 1728 T^7 \hbar^6 - \right. \right. \\
& \quad 1728 T^8 \hbar^6 + 3954 T^9 \hbar^6 - 4650 T^{10} \hbar^6 + 3288 T^{11} \hbar^6 - 1514 T^{12} \hbar^6 + 336 T^{13} \hbar^6 - 30 T^{14} \hbar^6 - 6 T^{15} \hbar^6 \left. \right) \left. \right) / \\
& \left( 3 - 36 T + 216 T^2 - 867 T^3 + 2592 T^4 - 6084 T^5 + 11562 T^6 - 18108 T^7 + 23616 T^8 - 25791 T^9 + \right. \\
& \quad 23616 T^{10} - 18108 T^{11} + 11562 T^{12} - 6084 T^{13} + 2592 T^{14} - 867 T^{15} + 216 T^{16} - 36 T^{17} + 3 T^{18} \left. \right) \left. \right\}
\end{aligned}$$

In[ ]:= [PrintProfile](#) [ ]

```

Out[ ]:= ProfileRoot is root. Profiled time: 84987.
(      6)      9.612/ 84986.954 above Z
(      6)      0/      0 above RVK
CF: called 89125 times, time in 48618.5/81272.3
(  1341)    44.693/   95.975 under Z
(   396)     0.563/    1.393 under Boot
(  1272)  34578.313/ 53311.779 under EZip3
(   636)     1.037/    2.495 under Zip1
(  2544)  4229.157/ 10143.332 under Zip2
( 82936)  9764.711/ 17717.324 under Zip3
(386706) 32653.824/ 32653.824 above CCF
CCF: called 386706 times, time in 32653.8/32653.8
( 386706) 32653.824/ 32653.824 under CF
EZip3: called 318 times, time in 3558.87/56927.8
(   237)  3557.966/ 56921.426 under Z
(    81)     0.902/    6.400 under Boot
(  1272)  34578.313/ 53311.779 above CF
(   318)    17.799/   57.179 above Zip3
Zip3: called 636 times, time in 126.055/17843.4
(   237)   104.814/ 17777.296 under Z
(    81)     3.442/    8.904 under Boot
(   318)    17.799/   57.179 under EZip3
( 82936)  9764.711/ 17717.324 above CF
Zip2: called 636 times, time in 13.689/10157.
(   474)    12.060/ 10150.950 under Z
(   162)     1.629/    6.071 under Boot
(  2544)  4229.157/ 10143.332 above CF
Z: called 6 times, time in 9.612/84987.
(      6)      9.612/ 84986.954 under ProfileRoot
(    21)     0.046/   25.766 above Boot
(  1341)    44.693/   95.975 above CF
(   237)  3557.966/ 56921.426 above EZip3
(   237)     3.907/    5.929 above Zip1
(   474)    12.060/ 10150.950 above Zip2
(   237)   104.814/ 17777.296 above Zip3
Zip1: called 318 times, time in 5.913/8.408
(   237)     3.907/    5.929 under Z
(    81)     2.006/    2.479 under Boot
(   636)     1.037/    2.495 above CF
Boot: called 75 times, time in 0.519/63.596
(    21)     0.046/   25.766 under Z
(    54)     0.473/   37.830 under Boot
(    54)     0.473/   37.830 above Boot
(   396)     0.563/    1.393 above CF
(    81)     0.902/    6.400 above EZip3
(    81)     2.006/    2.479 above Zip1
(   162)     1.629/    6.071 above Zip2
(    81)     3.442/    8.904 above Zip3
RVK: called 6 times, time in 0./0.
(      6)      0/      0 under ProfileRoot

```