Pensieve header: The Objects. Continues pensieve://Projects/SL2Portfoilo2/Objects.nb.

The Objects

“Define” Code

```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ; op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{{sis}}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

Symmetric Algebra Objects

$$sm_{i\_,j\_\to k\_} := \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\left[b_k\,(\beta_i+\beta_j)+t_k\,(\tau_i+\tau_j)+a_k\,(\alpha_i+\alpha_j)+y_k\,(\eta_i+\eta_j)+x_k\,(\xi_i+\xi_j)\right];$$

$$s\Delta_{i\_\to j\_,k\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{j,k\}}\left[\beta_i\,(b_j+b_k)+\tau_i\,(t_j+t_k)+\alpha_i\,(a_j+a_k)+\eta_i\,(y_j+y_k)+\xi_i\,(x_j+x_k)\right];$$

$$sS_{i\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{i\}}[-\beta_i\,b_i-\tau_i\,t_i-\alpha_i\,a_i-\eta_i\,y_i-\xi_i\,x_i];$$

$$s\eta_{i\_} := \Lambda 2\mathbb{E}_{\{\}\to\{i\}}[0];$$

$$s\epsilon_{i\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{\}}[0];$$

$$s\sigma_{i\_\to j\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{j\}}\left[\beta_i\,b_j+\tau_i\,t_j+\alpha_i\,a_j+\eta_i\,y_j+\xi_i\,x_j\right];$$

$$s\Upsilon_{i\_\to j\_,k\_,l\_,m\_} := \Lambda 2\mathbb{E}_{\{i\}\to\{j,k,l,m\}}\left[\beta_i\,b_k+\tau_i\,t_k+\alpha_i\,a_l+\eta_i\,y_j+\xi_i\,x_m\right];$$

The CU Definitions

$$c\Lambda = \left(\eta_i+\frac{e^{-\alpha_i-\epsilon\,\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right)y_k+\left(\beta_i+\beta_j+\frac{\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon}\right)b_k+$$
$$\left(\alpha_i+\alpha_j+\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]\right)a_k+\left(\frac{e^{-\alpha_j-\epsilon\,\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i}+\xi_j\right)x_k;$$

$$\mathrm{Define}\left[cm_{i,j\to k} = \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\left[\left(\eta_i+\frac{e^{-\alpha_i-\epsilon\,\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right)y_k+\left(\beta_i+\beta_j+\frac{\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon}\right)b_k+\right.\right.$$
$$\left.\left.\left(\alpha_i+\alpha_j+\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]\right)a_k+\left(\frac{e^{-\alpha_j-\epsilon\,\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i}+\xi_j\right)x_k\right]\right]$$

```
Define[cσ_{i→j} = sσ_{i,j} /. τ_i → 0, cϵ_i = sϵ_i, cη_i = sη_i, cΔ_{i→j,k} = sΔ_{i→j,k},
  cS_i = sS_i // sϒ_{i→1,2,3,4} // cm_{4,3→i} // cm_{i,2→i} // cm_{i,1→i}];
```

Booting Up QU

```
Define[aσ_{i→j} = Λ2𝔼_{{i}→{j}}[a_j α_i + x_j ξ_i], bσ_{i→j} = Λ2𝔼_{{i}→{j}}[b_j β_i + y_j η_i]]
```

```
Define[am_{i,j→k} = Λ2𝔼_{{i,j}→{k}}[(α_i + α_j) a_k + (𝒜_j^{-1} ξ_i + ξ_j) x_k],
 bm_{i,j→k} = Λ2𝔼_{{i,j}→{k}}[(β_i + β_j) b_k + (η_i + e^{-ϵ β_i} η_j) y_k]]
```

```
Define[R_{i,j} = Module[{k}, Λ2𝔼_{{}→{i,j}}[ℏ a_j b_i + Sum[(1 - e^{ϵ ℏ})^k (ℏ y_i x_j)^k / (k (1 - e^{k ϵ ℏ})), {k, 1, $k+1}]]]]
```

Three types of inverses appear below!

$\overline{R}$ is the inverse of R in the algebra $\mathbb{B}\otimes\mathbb{A}$.

P is the inverse of R as a quadratic form, like how an element of $V^*\otimes V^*$ can be the inverse of an element of $V\otimes V$.

$\overline{\mathrm{aS}}$ is the inverse of aS as an operator form, like how an element of $V^*\otimes V$ can be the inverse of another element of $V^*\otimes V$.

```
In[∘]:= Define[R̄_{i,j} = If[$k == 0, 𝔼_{{}→{i,j}}[-ℏ a_j b_i - ℏ x_j y_i / B_i],
    Append[R̄_{{i,j},$k-1}, -Last[PadRight[R̄_{{i,j},0}, $k + 1] R_{1,2} PadRight[R̄_{{3,4},$k-1}, $k + 1] //
        (bm_{i,1→i} am_{j,2→j}) // (bm_{i,3→i} am_{j,4→j})]]]
]
```

```
Define[P_{i,j} = If[$k == 0, 𝔼_{{i,j}→{}}[β_i α_j / ℏ + η_i ξ_j / ℏ], Append[P_{{i,j},$k-1},
   -Last[R_{1,2} // (PadRight[P_{{1,j},0}, $k + 1] * PadRight[P_{{i,2},$k-1}, $k + 1])]]]]
```

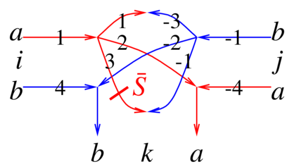
```
Define[aS_i = (aσ_{i→2} R̄_{1,i}) // P_{1,2}]
```

```
Define[aS̄_i = If[$k == 0, 𝔼_{{i}→{i}}[-a_i α_i - x_i 𝒜_i ξ_i],
    Append[aS̄_{{i},$k-1}, -Last[PadRight[aS̄_{{i},0}, $k + 1] // aS_i // PadRight[aS̄_{{i},$k-1}, $k + 1]]]
  ]]
```

Booting Up QU

$$\text{Define}\Big[bS_i = b\sigma_{i\to 1}\, R_{i,2} \,//\, aS_2 \,//\, P_{1,2},\quad \overline{bS}_i = b\sigma_{i\to 1}\, R_{i,2} \,//\, \overline{aS}_2 \,//\, P_{1,2},\quad a\Delta_{i\to j,k} = (R_{1,j}\, R_{2,k}) \,//\, bm_{1,2\to 3} \,//\, P_{3,i},\quad b\Delta_{i\to j,k} = (R_{j,1}\, R_{k,2}) \,//\, am_{1,2\to 3} \,//\, P_{i,3}\Big]$$

The Drinfel'd double:



$$\text{Define}\Big[dm_{i,j\to k} = \Big(\big(s\Upsilon_{i\to 4,4,1,1} \,//\, a\Delta_{1\to 1,2} \,//\, a\Delta_{2\to 2,3} \,//\, \overline{aS}_3\big)\,\big(s\Upsilon_{j\to -1,-1,-4,-4} \,//\, b\Delta_{-1\to -1,-2} \,//\, b\Delta_{-2\to -2,-3}\big)\Big) \,//\, (P_{-1,3}\, P_{-3,1}\, am_{2,-4\to k}\, bm_{4,-2\to k})\Big]$$

$$\text{Define}\Big[d\sigma_{i\to j} = a\sigma_{i\to j}\, b\sigma_{i\to j},\quad d\epsilon_i = s\epsilon_i,\ d\eta_i = s\eta_i,\quad dS_i = s\Upsilon_{i\to 1,1,2,2} \,//\, (\overline{bS}_1\, aS_2) \,//\, dm_{2,1\to i},\quad \overline{dS}_i = s\Upsilon_{i\to 1,1,2,2} \,//\, (bS_1\, \overline{aS}_2) \,//\, dm_{2,1\to i},\quad d\Delta_{i\to j,k} = (b\Delta_{i\to 3,1}\, a\Delta_{i\to 2,4}) \,//\, (dm_{3,4\to k}\, dm_{1,2\to j})\Big]$$

$$\text{Define}\Big[C_i = \Lambda 2\mathbb{E}_{\{\}\to\{i\}}\Big[-\frac{\hbar}{2}(b_i + \epsilon\, a_i)\Big],\quad \overline{C}_i = \Lambda 2\mathbb{E}_{\{\}\to\{i\}}\Big[\frac{\hbar}{2}(b_i + \epsilon\, a_i)\Big],\quad \text{Kink}_i = (R_{1,3}\, \overline{C}_2) \,//\, dm_{1,2\to 1} \,//\, dm_{1,3\to i},\quad \overline{\text{Kink}}_i = (\overline{R}_{1,3}\, C_2) \,//\, dm_{1,2\to 1} \,//\, dm_{1,3\to i}\Big]$$

Not yet verified

Note. $t == \epsilon\, a - b$ and $b == -t + \epsilon\, a$.

$$\text{Define}\big[b2t_i = \Lambda 2\mathbb{E}_{\{i\}\to\{i\}}[\alpha_i\, a_i + \beta_i\, (\epsilon\, a_i - t_i) + \xi_i\, x_i + \eta_i\, y_i],\quad t2b_i = \Lambda 2\mathbb{E}_{\{i\}\to\{i\}}[\alpha_i\, a_i + \tau_i\, (\epsilon\, a_i - b_i) + \xi_i\, x_i + \eta_i\, y_i]\big]$$

The Knot Tensors

$$
\begin{array}{l}
\text{Define}\big[\text{kR}_{i,j} = \big(\text{R}_{i,j} \mathbin{/\!/} (\text{b2t}_i\,\text{b2t}_j)\big) \text{ /. } \text{t}_{i|j} \to \text{t},\\
\quad \overline{\text{kR}}_{i,j} = \big(\overline{\text{R}}_{i,j} \mathbin{/\!/} (\text{b2t}_i\,\text{b2t}_j)\big) \text{ /. } \{\text{t}_{i|j} \to \text{t},\ \text{T}_{i|j} \to \text{T}\},\\
\quad \text{km}_{i,j\to k} = \big((\text{t2b}_i\,\text{t2b}_j) \mathbin{/\!/} \text{dm}_{i,j\to k} \mathbin{/\!/} \text{b2t}_k\big) \text{ /. } \{\text{t}_k \to \text{t},\ \text{T}_k \to \text{T},\ \tau_{i|j} \to 0\},\\
\quad \text{kC}_i = (\text{C}_i \mathbin{/\!/} \text{b2t}_i) \text{ /. } \text{t}_i \to \text{t},\\
\quad \overline{\text{kC}}_i = (\overline{\text{C}}_i \mathbin{/\!/} \text{b2t}_i) \text{ /. } \text{t}_i \to \text{t},\\
\quad \text{kKink}_i = (\text{Kink}_i \mathbin{/\!/} \text{b2t}_i) \text{ /. } \{\text{t}_i \to \text{t},\ \text{T}_i \to \text{T}\},\\
\quad \overline{\text{kKink}}_i = (\overline{\text{Kink}}_i \mathbin{/\!/} \text{b2t}_i) \text{ /. } \{\text{t}_i \to \text{t},\ \text{T}_i \to \text{T}\}\big]
\end{array}
$$