$G / m^{ij}_k = m^{ji}_k$

In $\mathcal{U}(\text{heis}(p,x)) / \text{commutators}$

$$0 = [p, F(p,x)] = \partial_x F$$

$$= pF - Fp \qquad F = \prod g_i$$

$$[p, F] = \sum \prod_{i<k} g_i \, [p, g_k] \prod_{i>k} g_i$$

$$\begin{pmatrix} \alpha & \beta & \phi \\ \gamma & \delta & \psi \\ \theta & \sigma & \end{pmatrix} \to \begin{pmatrix} \cdot & \cdot & \cdot \\ & \cdot & \end{pmatrix}$$

$$0 = [x, F(p,x)] = -\partial_p F$$

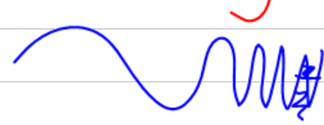
$$\begin{tikzcd} G_{2+s} \arrow[r, shift left, "m^{12}"] \arrow[r, shift right, "m^{21}"'] & G_{1+s} \arrow[r] \arrow[dr, "tr"] & G_{1+s}/m^{12}=m^{21} \\ & & G^{mod}_{0+s} \arrow[u] \arrow[r, hook] & G_{1+s} \end{tikzcd}$$

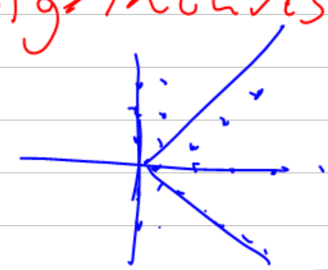
$\cap$ GDO represented by Gaussians in Greeks & Latins

$\not\ni$ GDO

Q: Can you get further "traces" using signatures?

$$\Lambda = (\lambda_{ab})$$





$$\int e^{-\frac{i}{2}\lambda_{ab}x^a x^b - \epsilon|x|^2} \xrightarrow[\epsilon \to 0]{} \frac{1}{(2\pi)^{n/2}\sqrt{\det(i\Lambda + \epsilon I)}}$$

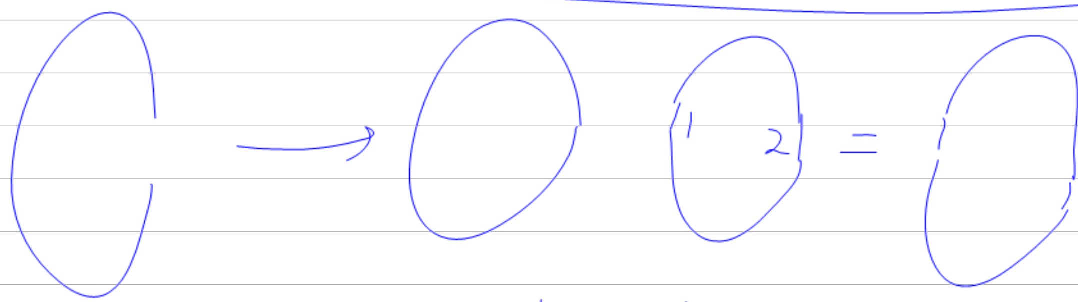
$$= \frac{1}{(2\pi)^{n/2}\sqrt{\det \Lambda}} \cdot e^{i\frac{\pi}{4}\operatorname{sign}(\Lambda)}$$

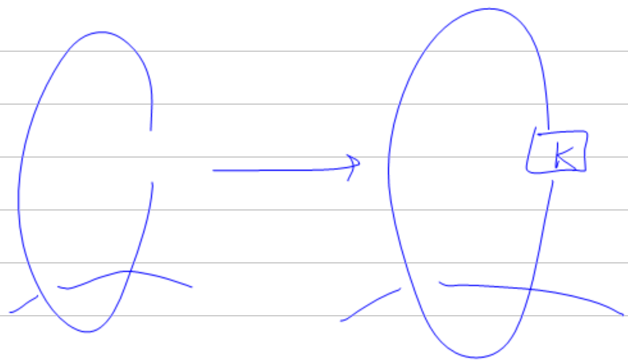
July 29, 2020

$$A^{\lambda} - 1 = e^{\alpha\lambda} - 1 = 1 + \alpha\lambda - 1 = \alpha\lambda$$

$$1 + \lambda\alpha - 1$$

Aug 6 2020

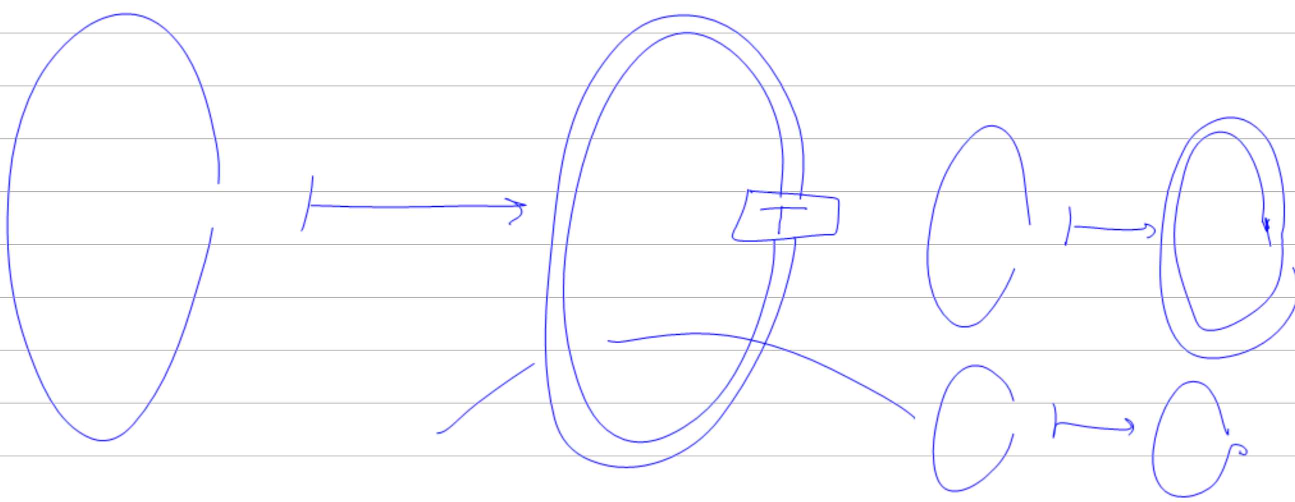


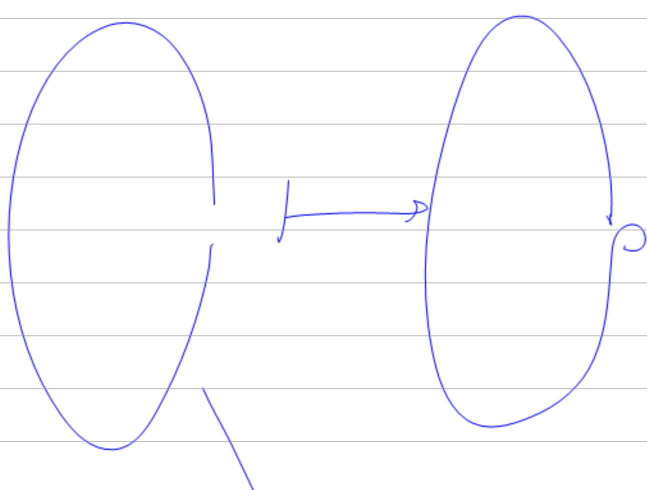


K is virtual

$K = \underbrace{K_1}_{\text{classical maximal}} \# \underbrace{K_2}_{\text{virtual}}$

Is K_1 unique?

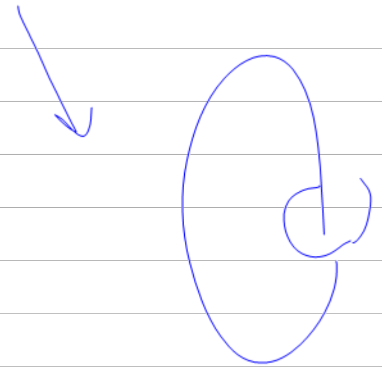


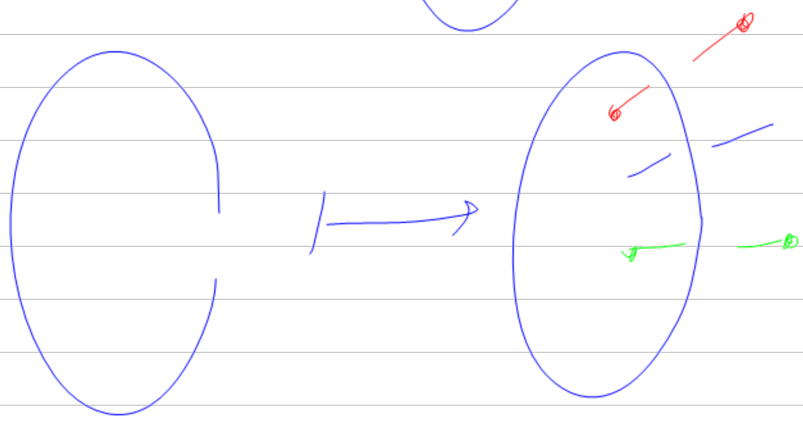


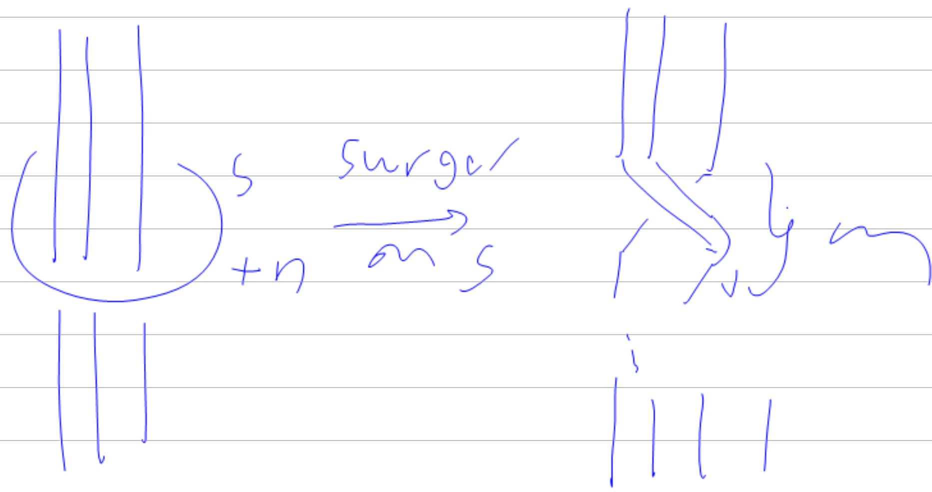
$$\begin{pmatrix} \alpha & \theta \\ \emptyset & \Xi \end{pmatrix} \xrightarrow{tr_H} (\ -\ )$$

$$\downarrow \Delta$$

$$\beta_m \xrightarrow{tr_H} (\ -\ )$$



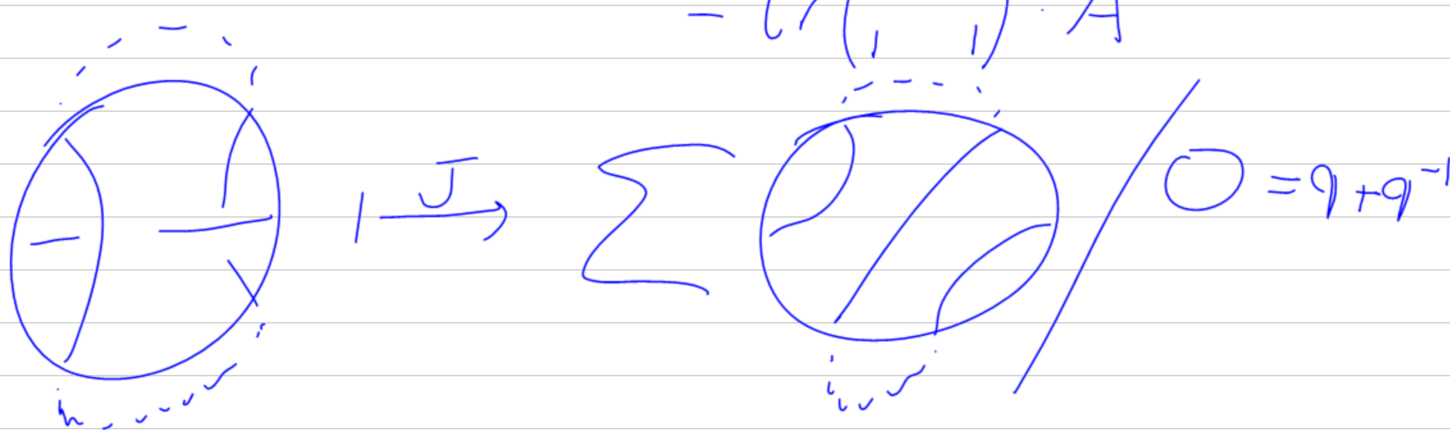




Aug 31, 2020

$(a_{ij}) = A \qquad \sum a_{ij} = (1 \cdots 1) A \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

$= tr \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \cdot A$



$$\circlearrowleft S^* \quad GDO \longleftrightarrow \Gamma \quad \circlearrowleft S^*$$

$$GDO \xleftrightarrow{S^*} GDO' \longleftrightarrow \Gamma$$

$$\begin{tikzcd} GDO \arrow[r, leftrightarrow, "\text{easy}"'] & \Gamma' \arrow[r, leftrightarrow] \arrow[loop below, "tr'_H"] & \Gamma \arrow[loop below, "tr_H"] \end{tikzcd}$$

Whirling (AP/2019-11)

$$W: \begin{array}{c} \begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \\ \begin{array}{cc} n-1 & 1 \end{array} \end{array} \longmapsto \frac{1}{\alpha} \begin{array}{c} \begin{pmatrix} 1 & -\theta \\ \phi & \alpha\Xi - \phi\theta \end{pmatrix} \\ \begin{array}{cc} 1 & n-1 \end{array} \end{array}$$

$W' \to$ cyclic

<u>Thm</u> $W^n(\underset{n\times n}{A}) = A^{-1}$

$$W': \begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \longmapsto \frac{1}{\alpha} \begin{pmatrix} \alpha\Xi - \phi\theta & \phi \\ -\theta & 1 \end{pmatrix}$$

$\{x^n n^k y^n\}$ $\quad \mathbb{Q}[x,y] \longrightarrow \mathbb{Q}[t]$

$$x^n y^m \longrightarrow \delta_{nm} t^n n!$$

$$e^{\xi x} e^{\eta y} \longrightarrow e^{\xi \eta t}$$

$$\quad\, \shortparallel \qquad\qquad\qquad\qquad \shortparallel$$

$$\sum \frac{\xi^n x^n}{m!} \frac{\eta^m y^m}{n!} \longmapsto \sum_n \frac{\xi^n \eta^n}{(n!)^2} n! t^n$$

$$\mathbb{E}\left(r\alpha_m\beta_m/k + s\cancel{\beta_m b_m} + t\alpha_m a_m \; \frac{A_m^{1-r}\left(A_m' - \cancel{B_m^s}\right)}{A_m - 1}\,\xi_m\eta_m \right)$$

$$Q[b,a] \xrightarrow{P} Q[b,a]$$

$$b^k a^n \longmapsto \begin{cases} b^k a^n & k \le n \\ 0 & k > n \end{cases}$$

$$g(P) = \sum_{k \le n} \frac{(\beta b)^k (\alpha a)^n}{\cancel{k!}\, n!}$$

$$= \sum_{k \le n} (\beta b)^k \frac{(\alpha a)^n}{n!}$$

$$= \sum_n \frac{(\beta b)^{n+1} - 1}{\beta b - 1} \cdot \frac{(\alpha a)^n}{n!}$$

$$= \frac{\beta b}{\beta b - 1} e^{\beta b \alpha a} - \frac{e^{\alpha a}}{\beta b - 1}$$

$$(x^k)' = k x^{k-1}$$

$$\frac{\Delta f}{\Delta x} = f(x+1) - f(x)$$

$$x^{(k)} = x(x-1)\cdots(x-k+1)$$

$$\frac{dx^{(k)}}{dx} = k\,x^{(k-1)}$$

$$L : x^k \longmapsto x^{(k)}$$

$$e^{x\log(1+\xi)}$$

$$g(L) = \sum \frac{\xi^k x^{(k)}}{k!} = (1+\xi)^x$$

$$= \sum \xi^k \binom{x}{k}$$

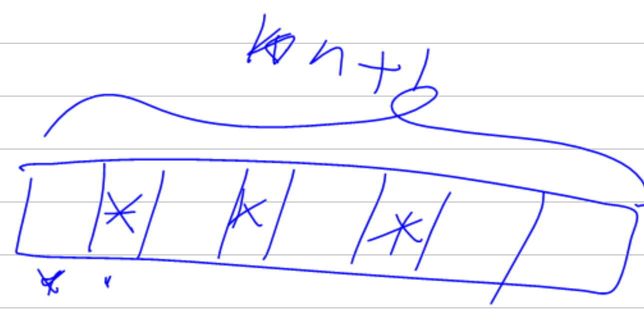
$$\frac{x^k}{k!} \qquad \binom{x}{k} \quad \binom{x}{k-1} \ \cdots$$

$$e^{\xi x} = \ \cdots$$

$$1+2+3+\cdots =$$

$$1^2+2^2+3^2+\cdots$$

$$1^k+\cdots+n^k = \cdots$$



$$\binom{1}{k}+\binom{2}{k}+\cdots+\binom{n}{k} = \binom{n+1}{k+1}$$

$$n^k \sim \binom{n}{k}$$

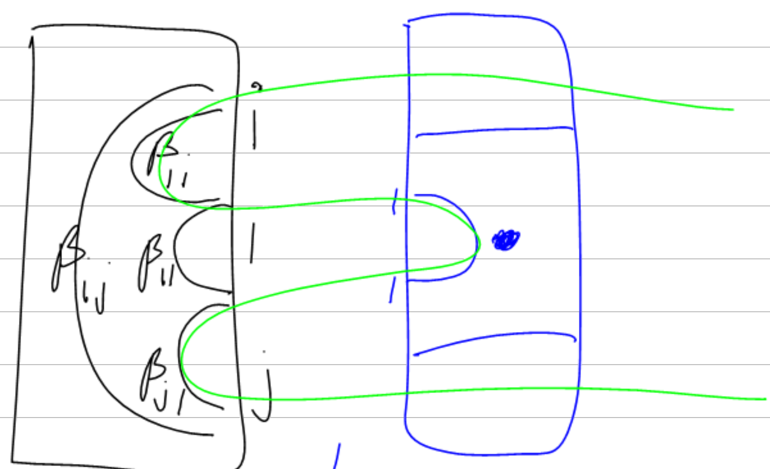
$$x^n \neq \sum_{k=0}^{n} \binom{n}{k} \binom{x}{k} k!$$

$$\cancel{x^2} = \underset{k=0}{1} + 2x + x(x-1) = x(x+1)$$

$$x^n = x^{(n)} + \underbrace{\binom{n}{2}} x^{(n-1)} + \cdots$$

$$+ \begin{pmatrix} \text{a poly} \\ \text{in } n \\ \text{of deg } 3 \end{pmatrix} x^{(n-2)}$$

$$+ \begin{pmatrix} \text{a poly in} \\ n \text{ of} \\ \text{deg } 4 \end{pmatrix} x^{(n-2)}$$



$$\frac{1}{1-\beta_{11}}$$

$$\bar{y} = A\bar{x}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x \end{pmatrix} \to \begin{pmatrix} x_1 \\ \vdots \\ x_{n-1} \\ y_n \end{pmatrix} \to \cdots \to \begin{pmatrix} x_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \to \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} |\sigma_1^{\pm 1}| & & 0 \\ & \ddots & \\ 0 & & |\sigma_n^{\pm 1}| \end{pmatrix} \pm A$$

$$\underline{H_n(x) e^{-x^2/2}} \qquad L_n(x) e^{-x}$$

$$\chi_{[-1,1]}$$

$$\langle f, g \rangle = \int e^{-x^2/2} f \bar{g} \, dx$$

$$y_1 = e_{11} x_1 + \dots + e_{1n} x$$

$$\vdots \qquad\qquad E = 1 + tF$$

$$y_n = e_{n1} x_1 + \dots + e_{nn} x_n$$

$$y' = A \underline{\underline{x'}} + B x'' \qquad \begin{pmatrix} x' \\ y'' \end{pmatrix} \longmapsto \begin{pmatrix} y' \\ x'' \end{pmatrix}$$

$$\underline{\underline{y''}} = C x' + D x''$$

$$x'' = D^{-1}(y'' - Cx')$$

$$\begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} - \begin{pmatrix} (1-\bar{B}_1)\beta_{11} & (1-\bar{B}_2)\beta_{12} \\ (1-\bar{B}_1)\beta_{2,1} & (1-\bar{B}_2)\beta_{2,2} \end{pmatrix}$$

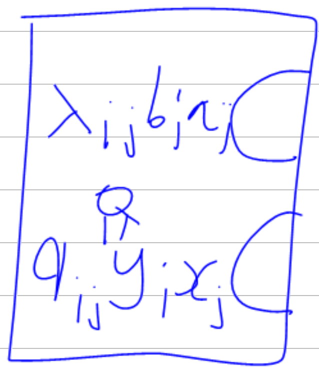
$$\begin{pmatrix} B_1^{-1} & 0 \\ 0 & B_2 \end{pmatrix} - \rho I \left(\begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} - \begin{pmatrix} \downarrow \end{pmatrix} \right)$$

$$\beta = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$$

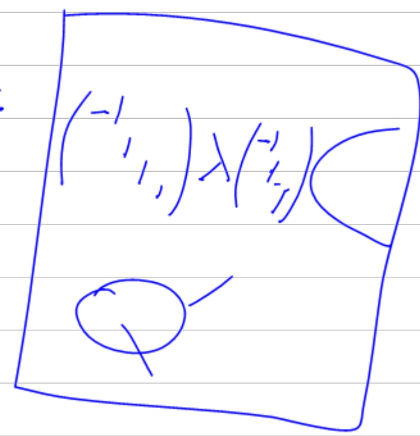
$$S_1\beta = \begin{pmatrix}\delta^{-1}(B_1^{-1}) & \\ & \delta(B_1)\end{pmatrix}\left[\begin{pmatrix}\gamma(B_1^{-1}) & \\ & \gamma(B_2)\end{pmatrix} P I_1 \left(\begin{pmatrix}\gamma(B_1) & 0 \\ 0 & \gamma(B_2)\end{pmatrix} - \begin{pmatrix}\delta(B_1) & \\ & \gamma(B_2)\end{pmatrix}\beta\begin{pmatrix}\epsilon(B_1) & \\ & \epsilon(B_2)\end{pmatrix}\right)\right]\begin{pmatrix}\epsilon^{-1}(\sigma_1^{-1}) & \\ & \epsilon^{-1}(B_2)\end{pmatrix}$$

$$ds_1 = e^{-\alpha_1 a_1 - b_1\beta_1 - \frac{\eta_1 A_1}{B_1} y_1 - \xi_1 A_1 x_1 + \frac{1-B_1}{B_1} A_1 \eta_1 \xi_1}$$

$$= e^{\alpha_1}$$



$// \; ds_1 =$



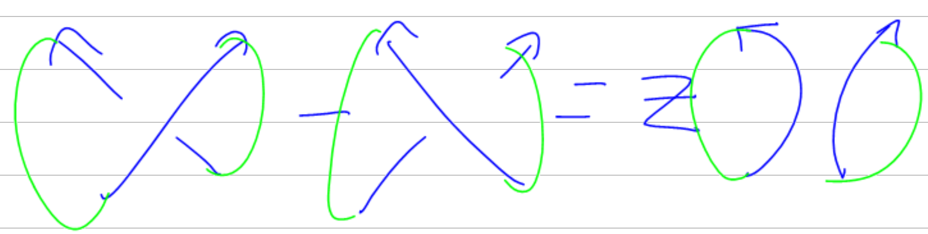
$$\tilde{Q} = (y_1 \ldots y_n)\begin{pmatrix}\tilde{\gamma} & \tilde{\theta} \\ \tilde{\phi} & \tilde{\Xi}\end{pmatrix}\begin{pmatrix}x_1 \\ \vdots \\ x_n\end{pmatrix}$$

$$\Xi' = \Xi + \phi\theta \cdot \frac{(1-B_1)}{B_1} \prod_{j=1}^{n} B_1^{-\lambda_{j1}}$$

$$\underbrace{I \in g^* \otimes g}$$ $\quad g = \langle a, x \rangle$

$$R_{ij} = e^{b_i a_j + x_i y_j}$$

$R_{ij}, \; m^{ij}_k,$



$S \; \Delta, \; t$

$$\mathbb{E}[\hbar l_{ij} b_i a_j, \hbar q_{ij} y_j x_j]$$
$$\hat{z} \qquad F(\hat{e}^{\hbar b_i})$$

$xw = \cancel{x}\, w x$

$$\to \sum \hbar^m \, CV^{\otimes n}_{\leq 2m}$$

$$Z = \sum \hbar^m Z_m \quad Z_m \in CU^{\otimes n}_{\leq 2m}$$

Proj into coinv $\to \overline{Z}_m \in CoCV^{\sim}_{\leq 2m}$

$$\text{Coinv} = \langle y^k a^n x^k \rangle$$
$$\qquad \qquad \quad \| $$
$$\qquad \qquad a^n t^k$$

$$CU_{\leq 2m} \xrightarrow{\pi} $$

$$g_{\leq 2m}(\pi) = \sum a^* t^* \underbrace{\eta_0 \beta_0 \alpha_0 \xi_0}_{\deg \leq 2m}$$

$$\Downarrow$$

$$tr^m = \mathbb{E}[0, 0, g_{\leq 2m}(\pi)]$$

$$cm_0^{ij} /\!/ tr_0^m - cm_0^{ji} /\!/ tr_0^m$$

should vanish to deg $2m$ in greeks.

$\mathbb{Z} \,//\, tr_0^m$ mod out by having many gens

is a link invt,

Either trivial or not.

$$x_i^n = x_i^n$$

{i,j,k,l} = Exponent[word, #] &

/@ {y, b, y, x}

$$F_k U \nearrow U$$

$$F_n(U^{\otimes 2}) = \sum_{j+k \leq n} F_j U \otimes F_k U \xrightarrow{m} F_n(U)$$

$$m_k^{ij}$$

$$cR_{ij} = e^{b_i a_j} + \underbrace{\frac{e^{b_i}-1}{b_i}}_{2} y_i x_j$$