

15-344 Combinatorics on Oct 8, hour 12: More graph colourings

Thursday, September 17, 2015 7:59 PM

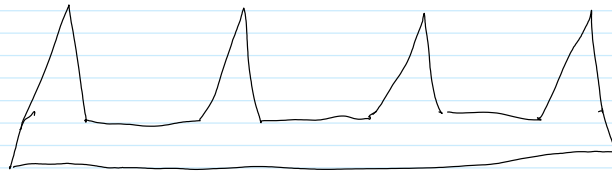
class photo on web. Add your name!

Read Along: sects 2.3-2.4.

HW3 on web.

Recall: $\chi(G) := \min \text{ number of colours needed to colour } V \text{ s.t. adjacent vertices have distinct colours.}$ Big big Thm: $G \text{ planar} \Rightarrow \chi(G) \leq 4$
(4CT, more later) $\chi(K_{m,n}) = 2$ $\chi(Q_n) = 2$ $\chi(G) = 2 \Leftrightarrow G \text{ is bipartite}$ $\chi(K_n) = n$ Thm If T is a triangulation, $\chi(T) = 3$ Thm $\lfloor \frac{n}{3} \rfloor$ guards are enough to watch an n -gon art gallery on boards

Example



Thm (The 5-colour thm) every planar graph can be 5-coloured.

Lemma A planar graph has at least one vertex of valency ≤ 5 .pf of lemma $e \leq 3V - 6$ so $2e \leq 6V - 12$ so $\sum \deg(v) \leq 6V - 12$ so $\sum (\deg(v) - 6) \leq -12$

so at least one of these numbers is negative.

pf of thm. (by induction...)

done line

warning To the rest of this class T may be full on line

pf of the 4CT,
cases:

