

## Loading Data Files

<http://drorbn.net/AcademicPensieve/2016-10/#MathematicaNotebooks>

```

184 c t31 - 3 t32 - 120 c t32 + 35 c t33 - 4 c t34 -  $\frac{138459 uw}{2}$  +  $\frac{16 uw}{t^{32}}$  -  $\frac{104 uw}{t^{31}}$  +
 $\frac{281 uw}{t^{30}}$  -  $\frac{395 uw}{t^{29}}$  +  $\frac{389 uw}{t^{28}}$  -  $\frac{809 uw}{t^{27}}$  +  $\frac{4175 uw}{2 t^{26}}$  -  $\frac{2697 uw}{t^{25}}$  +  $\frac{221 uw}{t^{24}}$  +  $\frac{3495 uw}{t^{23}}$  -
 $\frac{4933 uw}{2 t^{22}}$  -  $\frac{3641 uw}{2 t^{21}}$  -  $\frac{4435 uw}{t^{20}}$  +  $\frac{25096 uw}{t^{19}}$  -  $\frac{33239 uw}{t^{18}}$  +  $\frac{3260 uw}{t^{17}}$  +  $\frac{34472 uw}{t^{16}}$  -
 $\frac{21176 uw}{t^{15}}$  -  $\frac{52101 uw}{2 t^{14}}$  +  $\frac{7643 uw}{t^{13}}$  +  $\frac{99049 uw}{t^{12}}$  -  $\frac{302603 uw}{2 t^{11}}$  +  $\frac{29271 uw}{t^{10}}$  +  $\frac{144364 uw}{t^9}$  -
 $\frac{286931 uw}{2 t^8}$  -  $\frac{15940 uw}{t^7}$  +  $\frac{158735 uw}{2 t^6}$  +  $\frac{49059 uw}{t^5}$  -  $\frac{296115 uw}{2 t^4}$  +  $\frac{86001 uw}{2 t^3}$  +
 $\frac{203097 uw}{2 t^2}$  -  $\frac{121203 uw}{2 t}$  + 40278 t uw +  $\frac{230547}{2} t^2 uw$  -  $\frac{276873}{2} t^3 uw$  -  $\frac{22989}{2} t^4 uw$  +
 $\frac{171075}{2} t^5 uw$  +  $\frac{68763}{2} t^6 uw$  - 138055 t7 uw +  $\frac{65845}{2} t^8 uw$  + 149872 t9 uw -
 $\frac{336737}{2} t^{10} uw$  + 26206 t11 uw + 73031 t12 uw -  $\frac{75537}{2} t^{13} uw$  - 26567 t14 uw +
17967 t15 uw +  $\frac{60193}{2} t^{16} uw$  - 40798 t17 uw + 10900 t18 uw + 11876 t19 uw - 8409 t20 uw -
1500 t21 uw + 1677 t22 uw +  $\frac{5857}{2} t^{23} uw$  -  $\frac{7395}{2} t^{24} uw$  + 1171 t25 uw + 443 t26 uw -
305 t27 uw -  $\frac{163}{2} t^{28} uw$  + 39 t29 uw + 95 t30 uw - 89 t31 uw + 31 t32 uw - 4 t33 uw] } }
(* Computed in http://drorbn.net/AcademicPensieve/2016-09/GST48.nb *)
];
tab /. (K_ -> {_, z_}) -> (z1[K] = z);
AllKs = First /@ tab;
Ribbons = {Knot[0, 1], Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27],
  Knot[9, 41], Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42],
  Knot[10, 48], Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123],
  Knot[10, 129], Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]};
(* From Kawauchi via Andrey Khesin *)

```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

**z1[Knot[3, 1]]**

$$\begin{aligned}
& \mathbb{E} \left[ -1 + \frac{1}{t} + t, 0, 0, \right. \\
& 16 + \frac{9c}{2} + \frac{2}{t^4} - \frac{2c}{t^4} - \frac{7}{t^3} + \frac{11c}{2t^3} + \frac{14}{t^2} - \frac{8c}{t^2} - \frac{18}{t} + \frac{4c}{t} - 10t - 10ct + 4t^2 + 8ct^2 - t^3 - \frac{3ct^3}{2} - 2ct^4 + \\
& \left. 2ct^5 - \frac{ct^6}{2} - 4uw + \frac{2uw}{t^4} - \frac{7uw}{2t^3} + \frac{9uw}{2t^2} + \frac{uw}{2t} + 6t uw - 2t^2 uw - \frac{1}{2} t^3 uw + \frac{3}{2} t^4 uw - \frac{1}{2} t^5 uw \right]
\end{aligned}$$

## The Essence

```

al[K_] := z1[K][[1]];
ap[K_] := al[K] /. tn -> ; n < 0 -> 0;

```

```

e[K_] := e[K] = Expand[Together[
$$\frac{(t \, z1[K] \, \mathbb{I}4) /. c | u | w \rightarrow 0 + a1[K]^3 t^2 D[a1[K], t]}{(t - 1)^2 a1[K]^2}$$
]];
ep[K_] := e[K] /. tn -> 0; n < 0 -> 0;

```

## Work

```

Column@Sort@DeleteDuplicatesBy[
  Table[{UnknottingNumber[K], Exponent[ap[K], t], Exponent[ep[K], t], K},
    {K, AllKnots[{0, 11}]}],
  Most
]

```

```

{0, 0, -∞, Knot[0, 1]}
{1, 0, 2, Knot[11, NonAlternating, 34]}
{1, 1, 1, Knot[3, 1]}
{1, 1, -∞, Knot[4, 1]}
{1, 2, 1, Knot[7, 7]}
{1, 2, 2, Knot[9, 44]}
{1, 2, 3, Knot[6, 2]}
{1, 2, -∞, Knot[6, 3]}
{1, 3, 2, Knot[9, 24]}
{1, 3, 3, Knot[9, 27]}
{1, 3, 4, Knot[11, NonAlternating, 21]}
{1, 3, 5, Knot[8, 7]}
{1, 3, -∞, Knot[8, 9]}
{1, 4, 4, Knot[11, Alternating, 35]}
{1, 4, 5, Knot[10, 91]}
{1, 4, 7, Knot[10, 9]}
{1, 4, -∞, Knot[10, 17]}
{2, 1, 1, Knot[7, 4]}
{2, 1, -∞, Knot[8, 3]}
{2, 2, 1, Knot[10, 140]}
{2, 2, 3, Knot[5, 1]}
{2, 2, -∞, Knot[8, 12]}
{2, 3, 2, Knot[10, 158]}
{2, 3, 3, Knot[10, 75]}
{2, 3, 5, Knot[8, 2]}
{2, 3, -∞, Knot[8, 18]}
{2, 4, 5, Knot[10, 48]}
{2, 4, 7, Knot[10, 5]}
{2, 4, -∞, Knot[10, 99]}
{3, 2, 3, Knot[9, 49]}
{3, 3, 5, Knot[7, 1]}
{3, 4, 7, Knot[10, 2]}
{4, 4, 7, Knot[9, 1]}
{5, 5, 9, Knot[11, Alternating, 367]}
{{1, 2}, 1, 1, Knot[11, NonAlternating, 139]}
{{1, 2}, 1, 2, Knot[11, NonAlternating, 67]}
{{1, 2}, 2, 2, Knot[11, Alternating, 201]}
{{1, 2}, 2, 3, Knot[11, Alternating, 21]}
{{1, 2}, 3, 3, Knot[11, Alternating, 5]}
{{1, 2}, 3, 4, Knot[11, NonAlternating, 6]}
{{1, 2}, 3, 5, Knot[10, 163]}
{{1, 2}, 4, 4, Knot[11, Alternating, 28]}
{{1, 2}, 4, 5, Knot[11, Alternating, 24]}
{{1, 2}, 4, 7, Knot[11, Alternating, 3]}
{{2, 3}, 1, 1, Knot[9, 35]}
{{2, 3}, 2, 3, Knot[9, 10]}
{{2, 3}, 3, 5, Knot[10, 51]}
{{2, 3}, 4, 7, Knot[10, 47]}
{{2, 3}, 4, -∞, Knot[10, 79]}
{{3, 4}, 3, 5, Knot[11, Alternating, 291]}
{{1, 2, 3}, 1, 1, Knot[11, Alternating, 362]}
{{1, 2, 3}, 2, 3, Knot[11, Alternating, 45]}
{{1, 2, 3}, 3, 5, Knot[11, Alternating, 18]}
{{1, 2, 3}, 4, 5, Knot[11, Alternating, 14]}
{{1, 2, 3}, 4, 7, Knot[11, NonAlternating, 148]}
{{2, 3, 4}, 2, 3, Knot[11, Alternating, 354]}

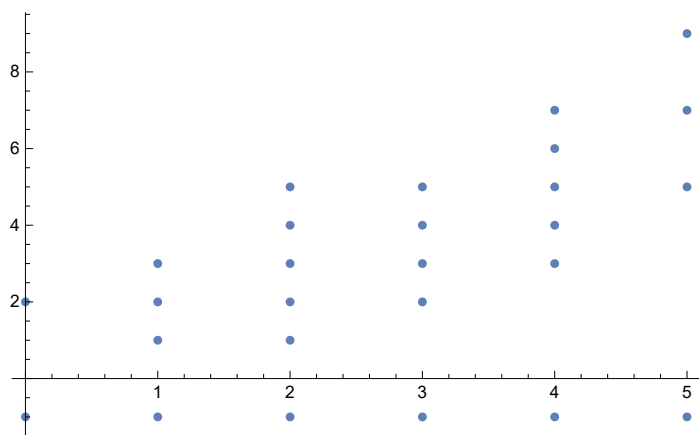
```

```

Column@Sort@DeleteDuplicatesBy[
  Table[{Exponent[ap[K], t], Exponent[ep[K], t], K}, {K, AllKs}],
  Most
]
{0, 2, Knot[11, NonAlternating, 34]}
{0, -∞, Knot[0, 1]}
{1, 1, Knot[3, 1]}
{1, 2, Knot[11, NonAlternating, 67]}
{1, 3, Knot[12, NonAlternating, 23]}
{1, -∞, Knot[4, 1]}
{2, 1, Knot[7, 7]}
{2, 2, Knot[9, 44]}
{2, 3, Knot[5, 1]}
{2, 4, Knot[12, NonAlternating, 256]}
{2, 5, Knot[12, NonAlternating, 750]}
{2, -∞, Knot[6, 3]}
{3, 2, Knot[9, 24]}
{3, 3, Knot[9, 27]}
{3, 4, Knot[11, NonAlternating, 6]}
{3, 5, Knot[7, 1]}
{3, -∞, Knot[8, 9]}
{4, 3, Knot[12, Alternating, 342]}
{4, 4, Knot[11, Alternating, 28]}
{4, 5, Knot[10, 48]}
{4, 6, Knot[12, NonAlternating, 346]}
{4, 7, Knot[9, 1]}
{4, -∞, Knot[10, 17]}
{5, 5, Knot[12, Alternating, 1250]}
{5, 7, Knot[12, Alternating, 1011]}
{5, 9, Knot[11, Alternating, 367]}
{5, -∞, Knot[12, Alternating, 819]}
{6, 11, TorusKnot[13, 2]}
{7, 13, TorusKnot[15, 2]}
{8, 15, TorusKnot[17, 2]}
{9, 17, TorusKnot[19, 2]}
{10, 19, TorusKnot[21, 2]}
{11, 21, TorusKnot[23, 2]}
{12, 23, TorusKnot[25, 2]}
{13, 25, TorusKnot[27, 2]}
{14, 27, TorusKnot[29, 2]}
{15, 29, TorusKnot[31, 2]}
{16, 31, TorusKnot[33, 2]}
{17, 33, TorusKnot[35, 2]}
{18, 35, TorusKnot[37, 2]}
{19, 37, TorusKnot[39, 2]}
{20, 39, TorusKnot[41, 2]}
{21, 41, TorusKnot[43, 2]}
{22, 43, TorusKnot[45, 2]}
{23, 45, TorusKnot[47, 2]}

```

```
ListPlot[Union[
  Table[{Exponent[ap[K], t], Exponent[ep[K], t]}, {K, AllKnots[{0, 12}]}]
] /. -∞ → -1]
```



```
ListPlot[Union[
  Table[{Exponent[ap[K], t], Exponent[ep[K], t]}, {K, AllKs}]
] /. -∞ → -1]
```

