

Dror Bar-Natan: Talks: Fields-0911:

Dror Bar-Natan: Academic Pensieve: 2009-11:

## Hilbert's 13th Problem

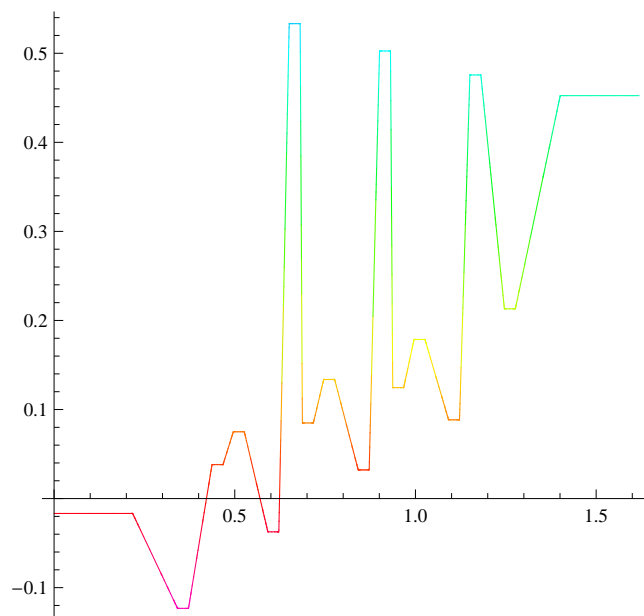
Pensieve Header: Hilbert's 13th problem - the heavy computations.

```
SetDirectory[
  $MachineName /. {
    "dror-x61" → "C:/drorbn/AcademicPensieve/2009-11"
  } /. $MachineName → "."
];
<< Hilbert13th-Program.m

phi4 := Phi[phi0, 4, 0.15, 0.5];
g = G[f, phi4];
Plot[g[z], {z, 0, 1 + λ}]
```

InterpolatingFunction::dmval:

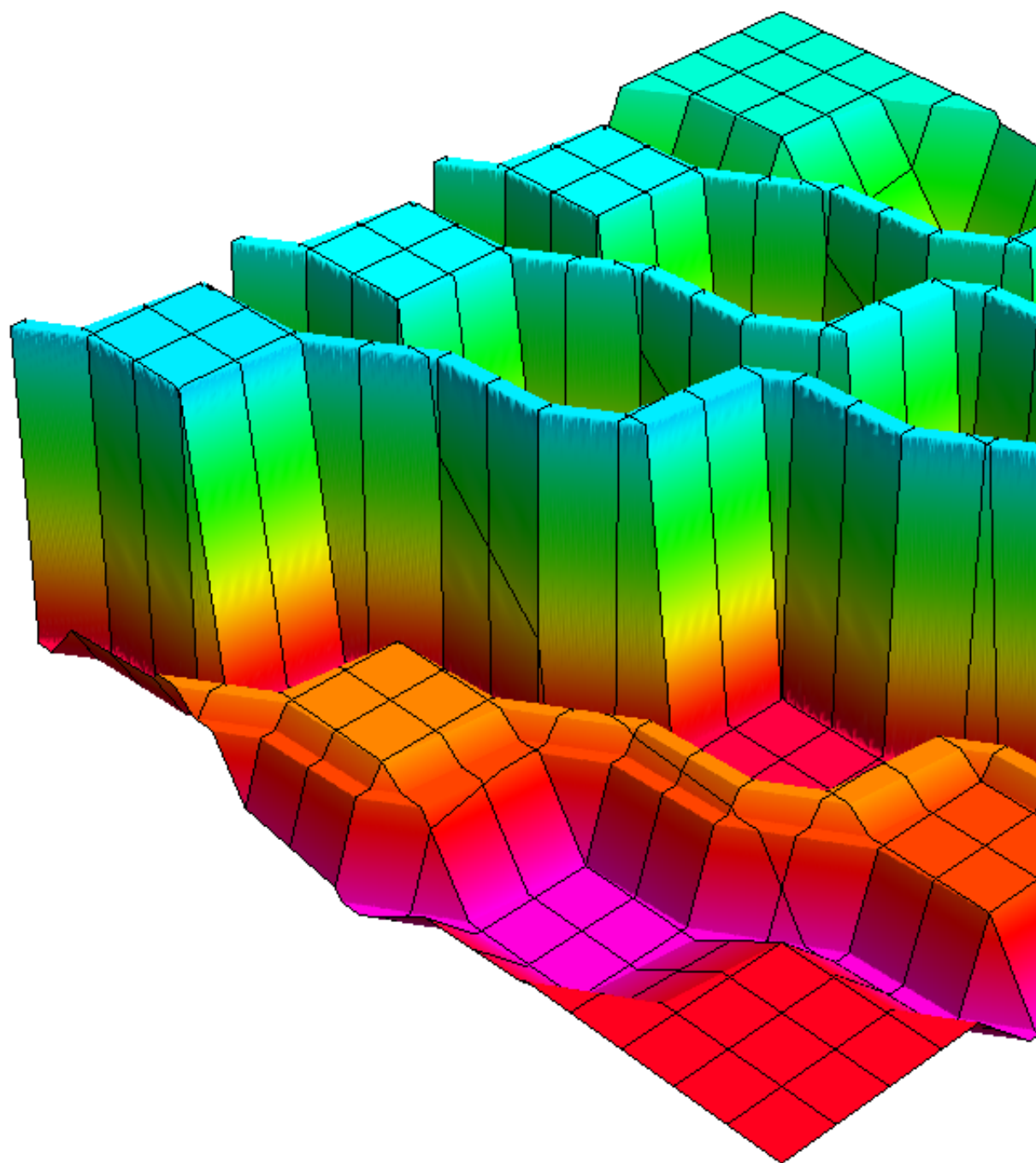
Input value {0.0000330541} lies outside the range of data in the interpolating function. Extrapolation will be used. >>



```
Step1Trenches = Rasterize[Plot3D[g[phi4[x] + λ * phi4[y]], {x, 0, 1}, {y, 0, 1},
  Mesh → 15, AxesLabel → None, PlotPoints → 500,
  ViewPoint → {-2, -2, 2}, Axes → None, Boxed → False
]]
```

InterpolatingFunction::dmval:

Input value  $\{6.00472 \times 10^{-6}\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

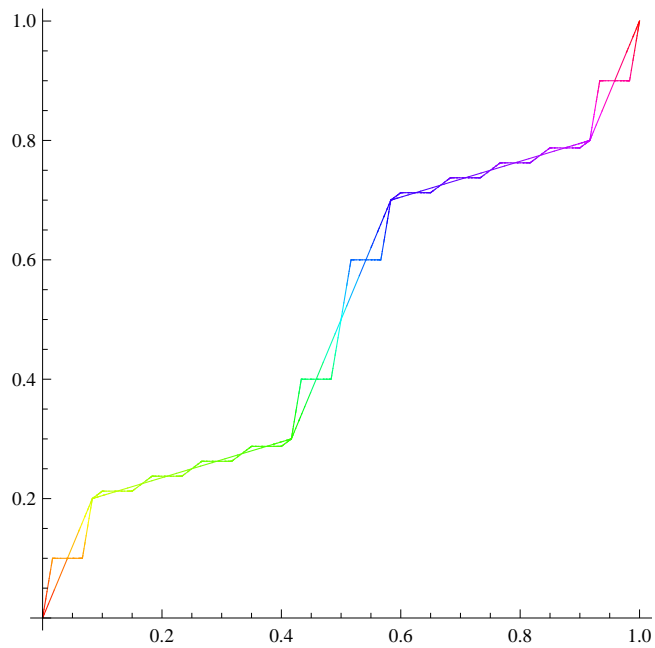


```
Export[
  "Step1Trenches.png",
  ImageCrop[Step1Trenches]
]
```

Step1Trenches.png

```
phi1 = Phi[Identity, 2, 0.3, 2/3];
phi2 = Phi[phi1, 12, 0.0001, 0.6];
phi3 = Phi[phi, phi0 -> phi1, Subdivisions -> 12, Slope -> 0.0001, FlatFraction -> 0.6];
g1 = G[f, phi1];
g2 = G[f, phi2];
```

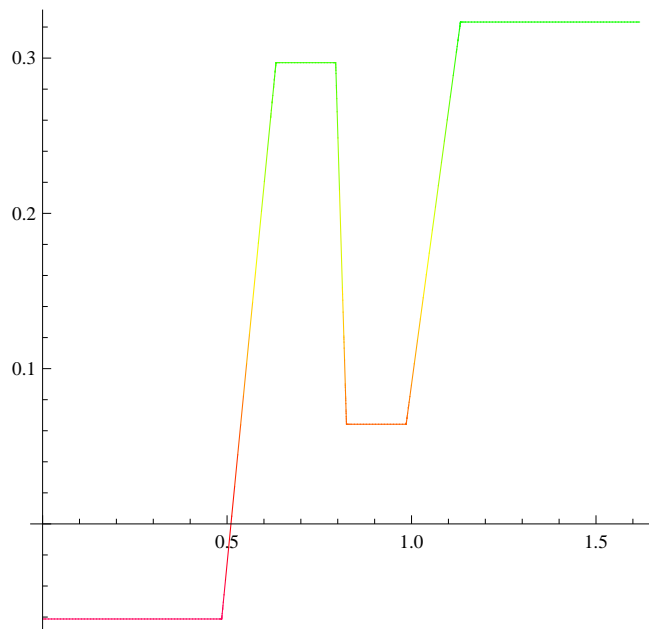
```
Plot[{phi1[x], phi2[x]}, {x, 0, 1}]
```



```
Plot[
  g1[z], {z, 0, 1 + λ},
  PlotPoints → 200
]
```

InterpolatingFunction::dmval:

Input value  $\{8.13895 \times 10^{-6}\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

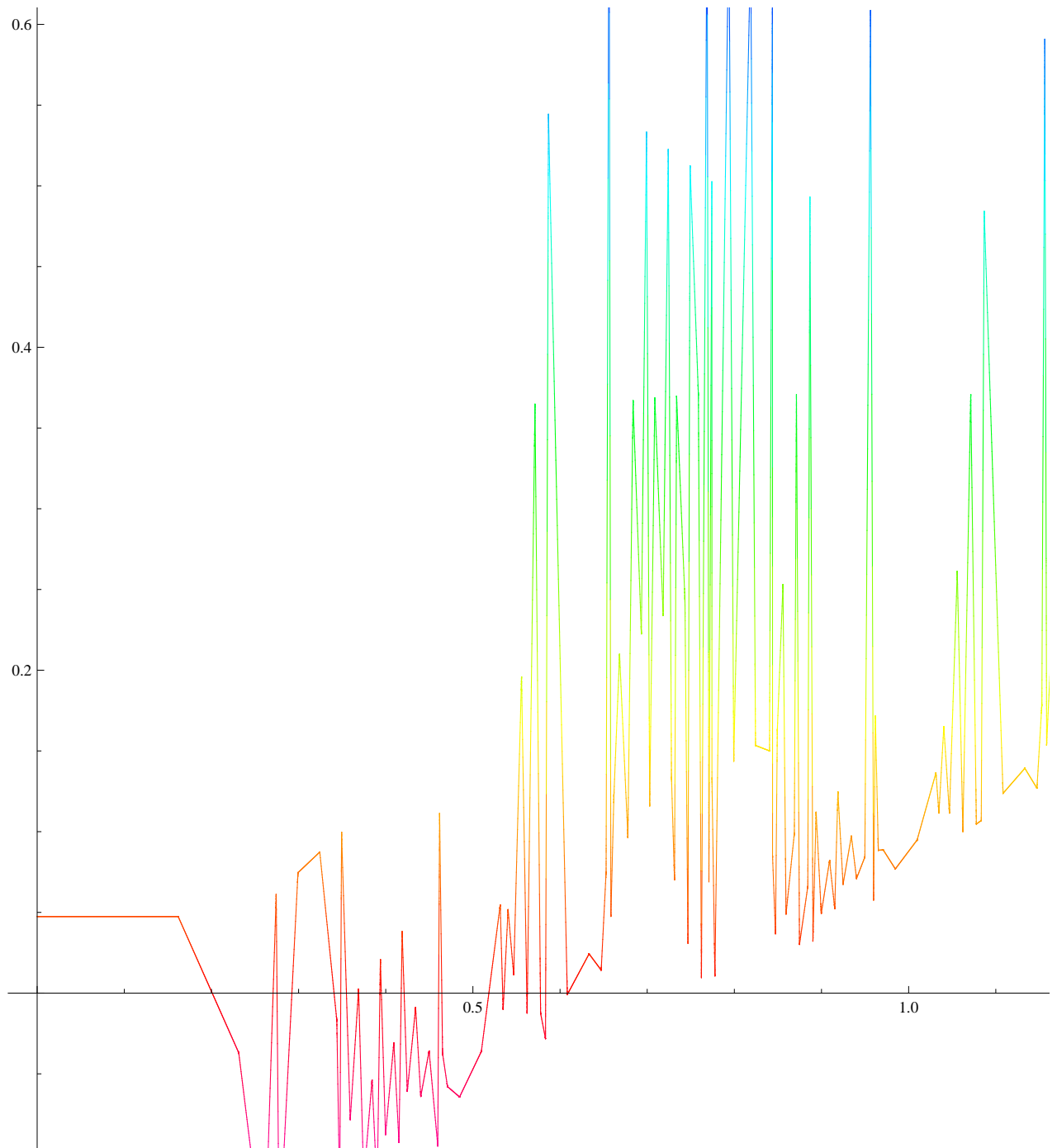


```
Plot[
  g2[z], {z, 0, 1 + λ},
  PlotPoints → 5000
]
```

InterpolatingFunction::dmval:

Input value  $\{3.23995 \times 10^{-7}\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>







```

Rasterize[
  Plot3D[phi2[x] + λ * phi2[y], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 119, Mesh → 11, ViewPoint → {-2, -2, 1}, Axes → None,
    Boxed → False, NormalsFunction → None, ColorFunction → (Hue[g2[#3]] &)
  ]
]

```

InterpolatingFunction::dmval:

Input value  $\{8.22709 \times 10^{-8}\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

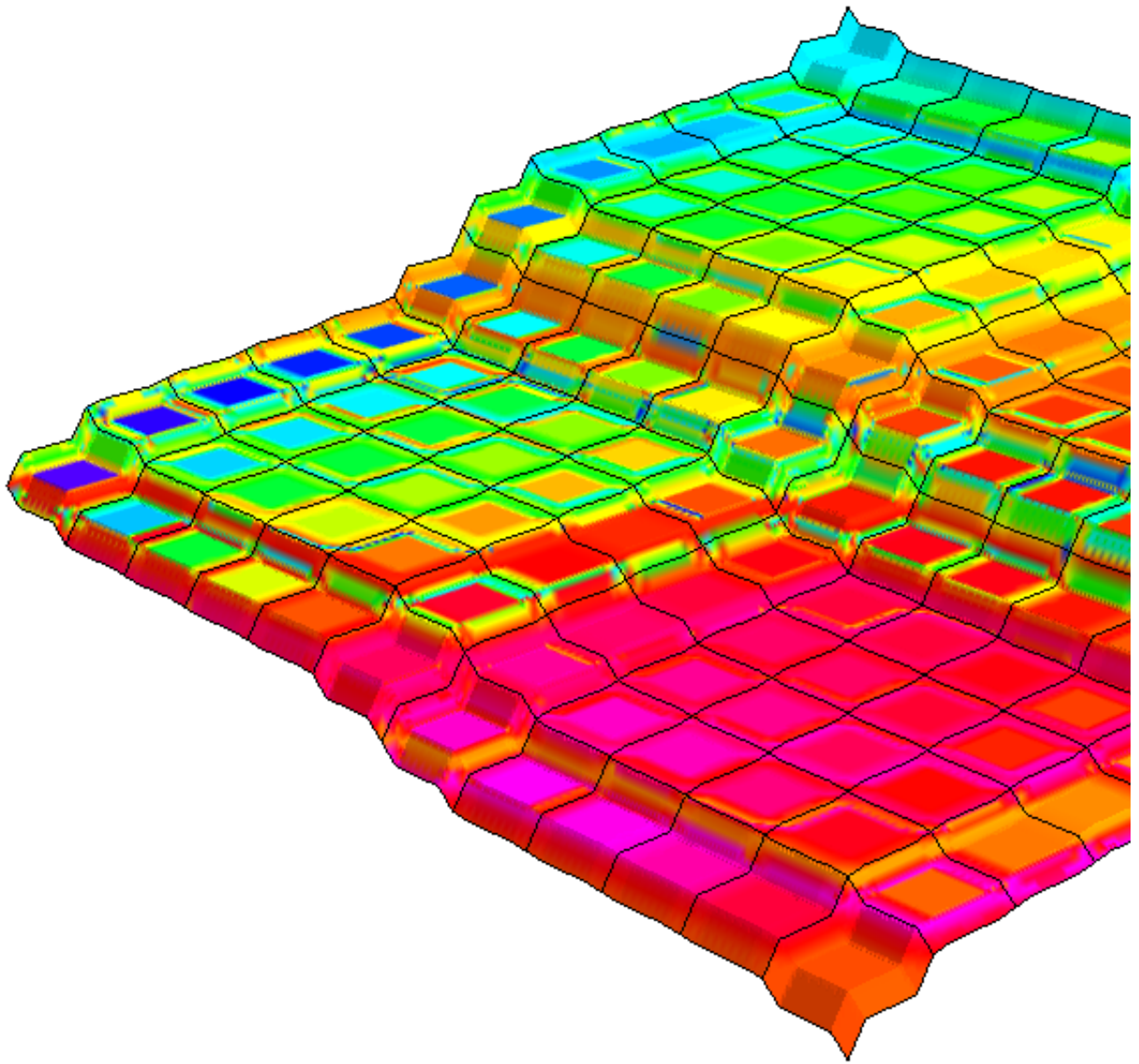
InterpolatingFunction::dmval:

Input value  $\{0.0508463\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

InterpolatingFunction::dmval:

Input value  $\{0.0999976\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

General::stop: Further output of InterpolatingFunction::dmval will be suppressed during this calculation. >>



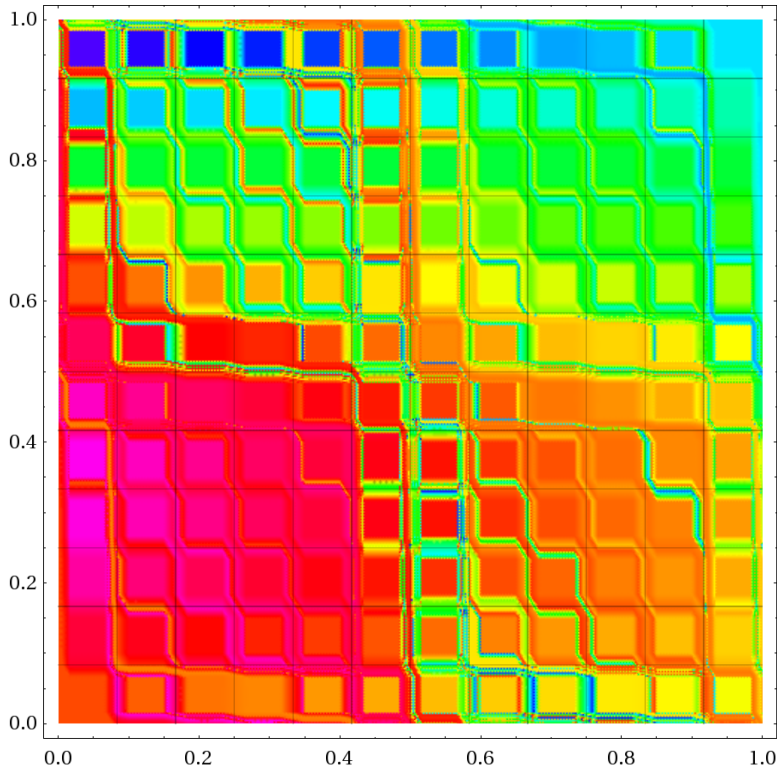
```

Rasterize[
  DensityPlot[
    g2[phi2[x] + λ * phi2[y]], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 119, Mesh → 11
  ]
]

```

InterpolatingFunction::dmval:

Input value {0.0000822709} lies outside the range of data in the interpolating function. Extrapolation will be used. >>



```

Timing[
  Step2CascadeWithG2 = Rasterize[
    Plot3D[phi2[x] + λ * phi2[y], {x, 0, 1}, {y, 0, 1},
    PlotPoints → 1199, Mesh → 11, ViewPoint → {-2, -2, 1}, NormalsFunction → None,
    ColorFunction → (Hue[g2[#3]] &), ImageSize → 1600, Axes → None, Boxed → False
  ], ImageSize → 1600, RasterSize → 1600
]
]

```

InterpolatingFunction::dmval:

Input value  $\{8.10347 \times 10^{-9}\}$  lies outside the range of data in the interpolating function. Extrapolation will be used. >>

InterpolatingFunction::dmval:

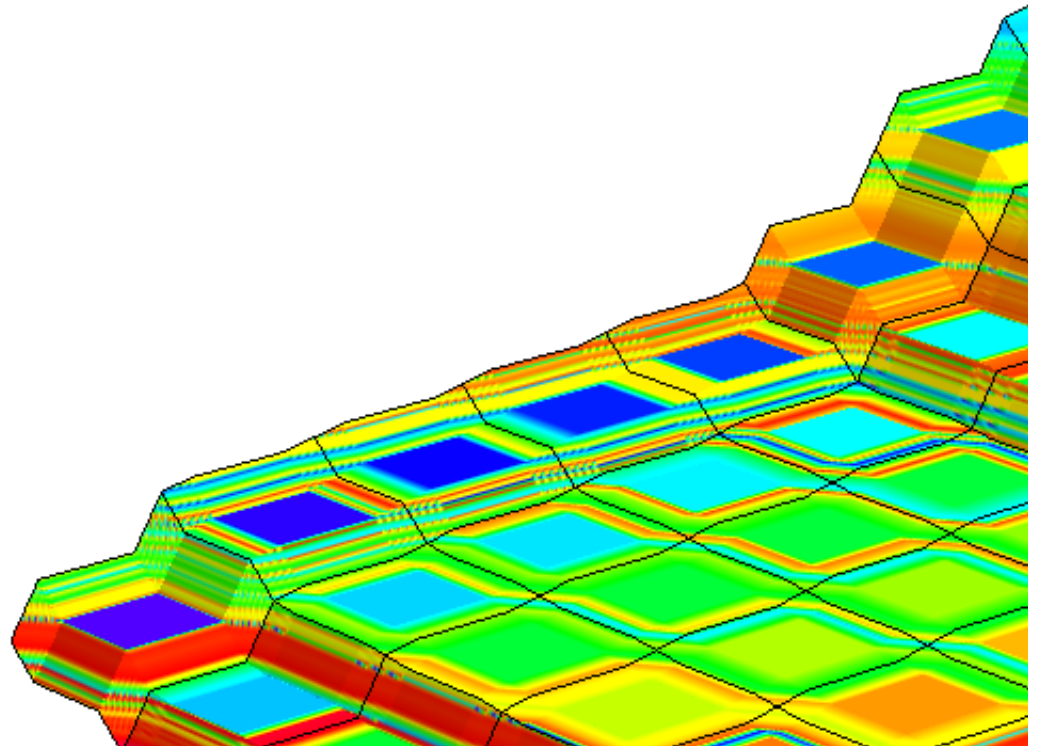
Input value {0.00500823} lies outside the range of data in the interpolating function. Extrapolation will be used. >>

InterpolatingFunction::dmval:

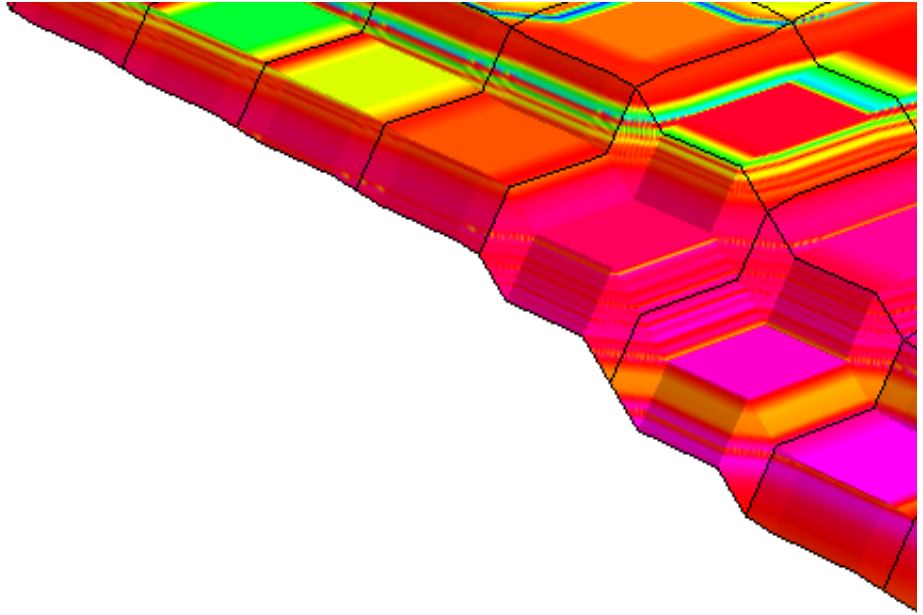
Input value {0.0100165} lies outside the range of data in the interpolating function. Extrapolation will be used. >>

General::stop: Further output of InterpolatingFunction::dmval will be suppressed during this calculation. >>





[1070] 25

$\{10/5.25,$ 


```
Export[
  "Step2CascadeWithG2.png",
  Step2CascadeWithG2
]
Step2CascadeWithG2.png

Do[
  phi2[i] = Phi[Identity, 2, 0.02, 0.95, i/5];
  g2[i] = G[f, phi2[i]],
  {i, 0, 4}
];
Do[
  phi3[i] = Phi[Identity, 3, 0.01, 0.95, i/5];
  g3[i] = G[f, phi3[i]],
  {i, 0, 4}
]
```

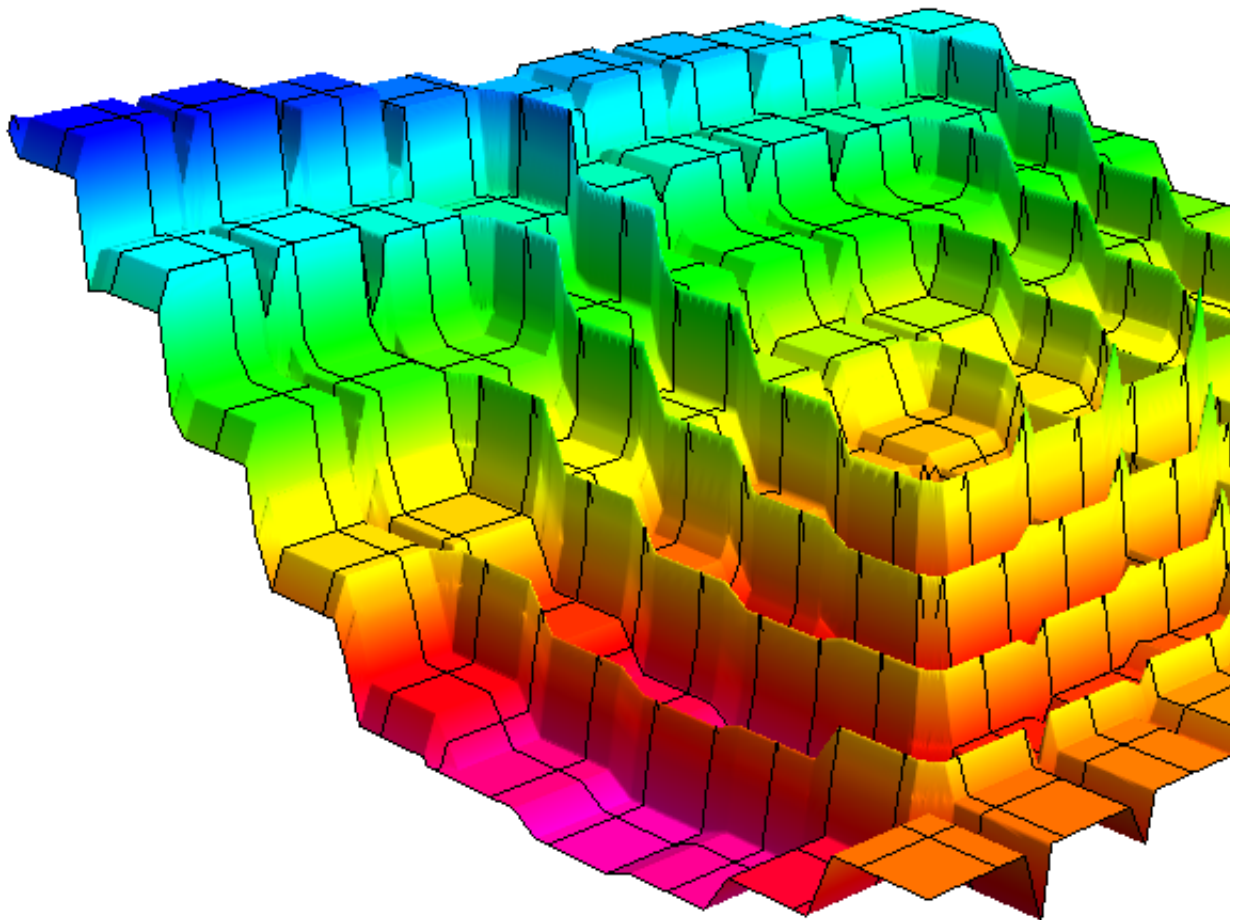
```

Step3Trenches = Rasterize[
  Plot3D[
    (1 / 5) * Sum[g2[k][phi2[k][x] + λ * phi2[k][y]], {k, 0, 4}],
    {x, 0, 1}, {y, 0, 1}, PlotPoints → 500,
    Axes → None, Boxed → False, ViewPoint → {-2, -2, 1}
  ]
]

```

InterpolatingFunction::dmval:

Input value {0.0000636825} lies outside the range of data in the interpolating function. Extrapolation will be used. >>



```
Export [  
  "Step3Trenches.png",  
  Step3Trenches  
]  
Step3Trenches.png
```