

```
pts4 = Point @@@ Permutations[Range[4]]
```

```
{Point[1, 2, 3, 4], Point[1, 2, 4, 3], Point[1, 3, 2, 4], Point[1, 3, 4, 2], Point[1, 4, 2, 3],
 Point[1, 4, 3, 2], Point[2, 1, 3, 4], Point[2, 1, 4, 3], Point[2, 3, 1, 4], Point[2, 3, 4, 1],
 Point[2, 4, 1, 3], Point[2, 4, 3, 1], Point[3, 1, 2, 4], Point[3, 1, 4, 2], Point[3, 2, 1, 4],
 Point[3, 2, 4, 1], Point[3, 4, 1, 2], Point[3, 4, 2, 1], Point[4, 1, 2, 3], Point[4, 1, 3, 2],
 Point[4, 2, 1, 3], Point[4, 2, 3, 1], Point[4, 3, 1, 2], Point[4, 3, 2, 1]}
```

```
mat = {{1, -1, 0, 0}, {0, 1, -1, 0}, {0, 0, 1, -1}};
```

```
mat = Drop[Orthogonalize[{{1, 1, 1, 1}, {1, -1, 0, 0}, {0, 1, -1, 0}, {0, 0, 1, -1}}], 1]
```

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}, 0 \right\}, \left\{ \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, -\frac{\sqrt{3}}{2} \right\} \right\}$$

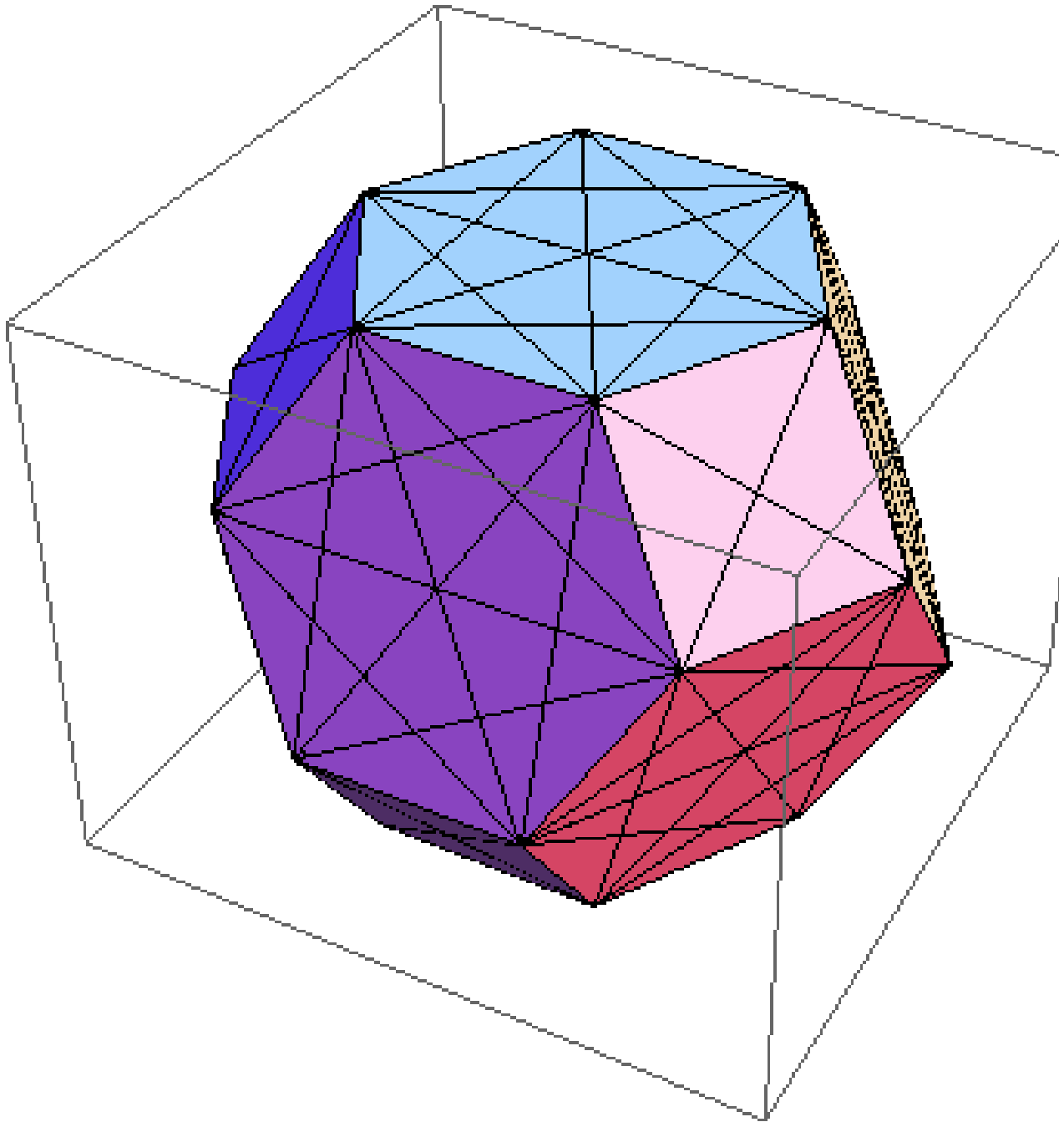
```
pts3 = pts4 /. Point[xs_] => Point @@ (
  mat.{xs}
)
```

$$\begin{aligned}
& \left\{ \text{Point} \left[\frac{1}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, -\sqrt{3} \right], \text{Point} \left[\frac{1}{\sqrt{2}} - \sqrt{2}, \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \right. \\
& \text{Point} \left[-\sqrt{2}, -2\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, -\sqrt{3} \right], \text{Point} \left[-\sqrt{2}, -4\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \\
& \text{Point} \left[\frac{1}{\sqrt{2}} - 2\sqrt{2}, \frac{1}{\sqrt{6}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \text{Point} \left[\frac{1}{\sqrt{2}} - 2\sqrt{2}, 2\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \\
& \text{Point} \left[-\frac{1}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, -\sqrt{3} \right], \text{Point} \left[-\frac{1}{\sqrt{2}} + \sqrt{2}, \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \\
& \text{Point} \left[-\frac{3}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{3}{2}}, -\sqrt{3} \right], \text{Point} \left[-\frac{3}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{3}{2}} - \sqrt{6}, \sqrt{3} \right], \\
& \text{Point} \left[-\sqrt{2}, 2\sqrt{\frac{2}{3}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \text{Point} \left[-\sqrt{2}, 0, \sqrt{3} \right], \\
& \text{Point} \left[\sqrt{2}, -2\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, -\sqrt{3} \right], \text{Point} \left[\sqrt{2}, -4\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \\
& \text{Point} \left[\frac{3}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{3}{2}}, -\sqrt{3} \right], \text{Point} \left[\frac{3}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{3}{2}} - \sqrt{6}, \sqrt{3} \right], \\
& \text{Point} \left[\frac{3}{\sqrt{2}} - 2\sqrt{2}, \sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \text{Point} \left[\frac{3}{\sqrt{2}} - 2\sqrt{2}, \sqrt{\frac{3}{2}}, \sqrt{3} \right], \\
& \text{Point} \left[-\frac{1}{\sqrt{2}} + 2\sqrt{2}, \frac{1}{\sqrt{6}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \text{Point} \left[-\frac{1}{\sqrt{2}} + 2\sqrt{2}, 2\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \\
& \text{Point} \left[\sqrt{2}, 2\sqrt{\frac{2}{3}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2} \right], \text{Point} \left[\sqrt{2}, 0, \sqrt{3} \right], \\
& \left. \text{Point} \left[-\frac{3}{\sqrt{2}} + 2\sqrt{2}, \sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2} \right], \text{Point} \left[-\frac{3}{\sqrt{2}} + 2\sqrt{2}, \sqrt{\frac{3}{2}}, \sqrt{3} \right] \right\}
\end{aligned}$$

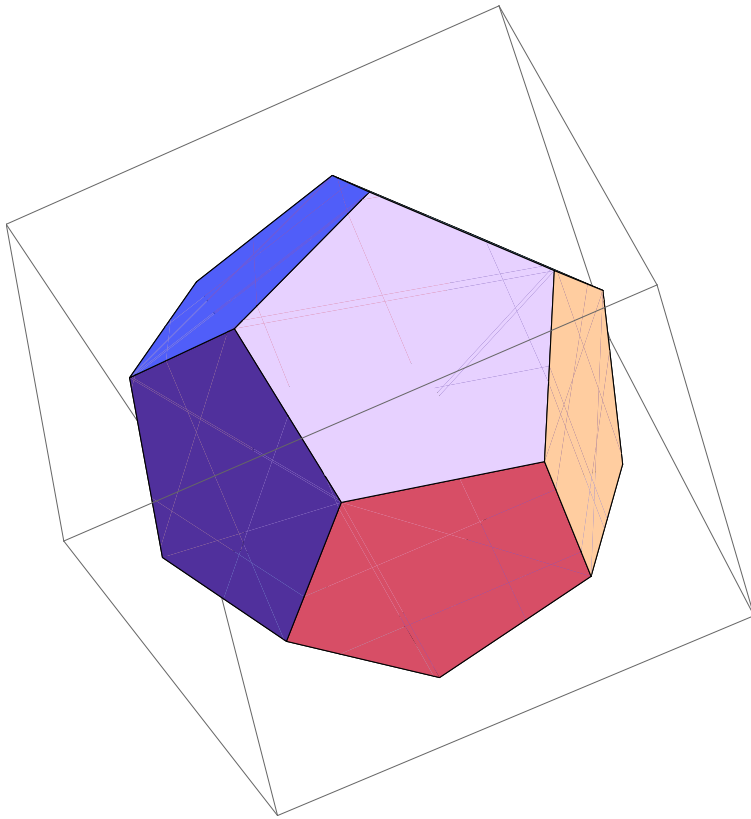
```

l = Length[pts3];
Rasterize[Graphics3D[
  Table[
    Polygon[List @@@ pts3[{{i, j, k}}]],
    {i, 1, l-2}, {j, i+1, l-1}, {k, j+1, l}
  ]
]]

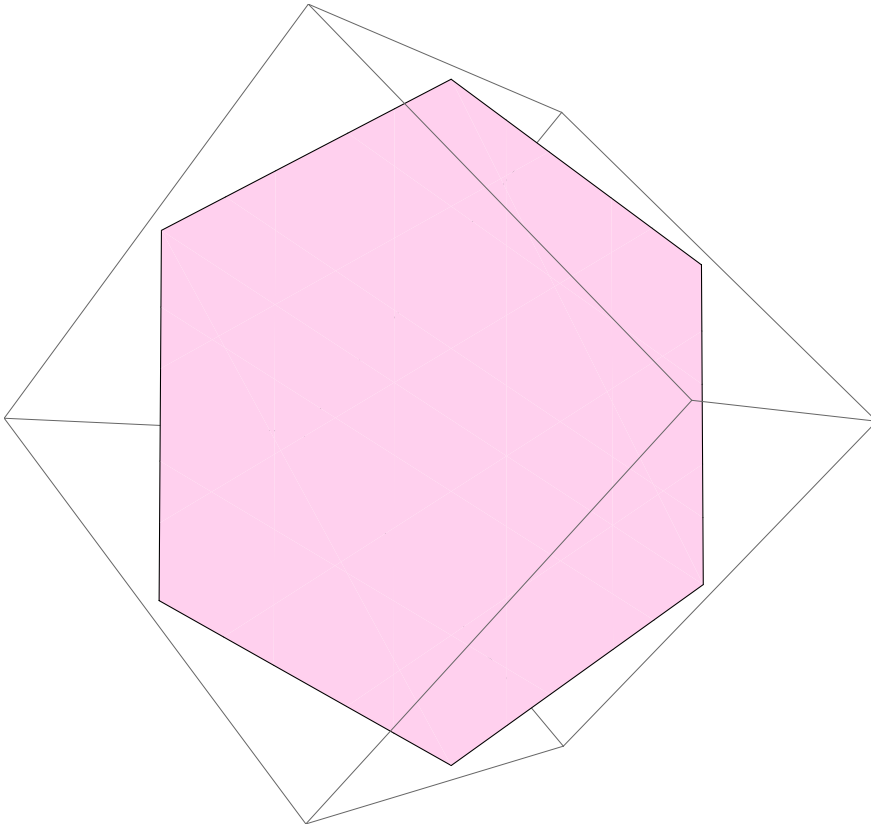
```



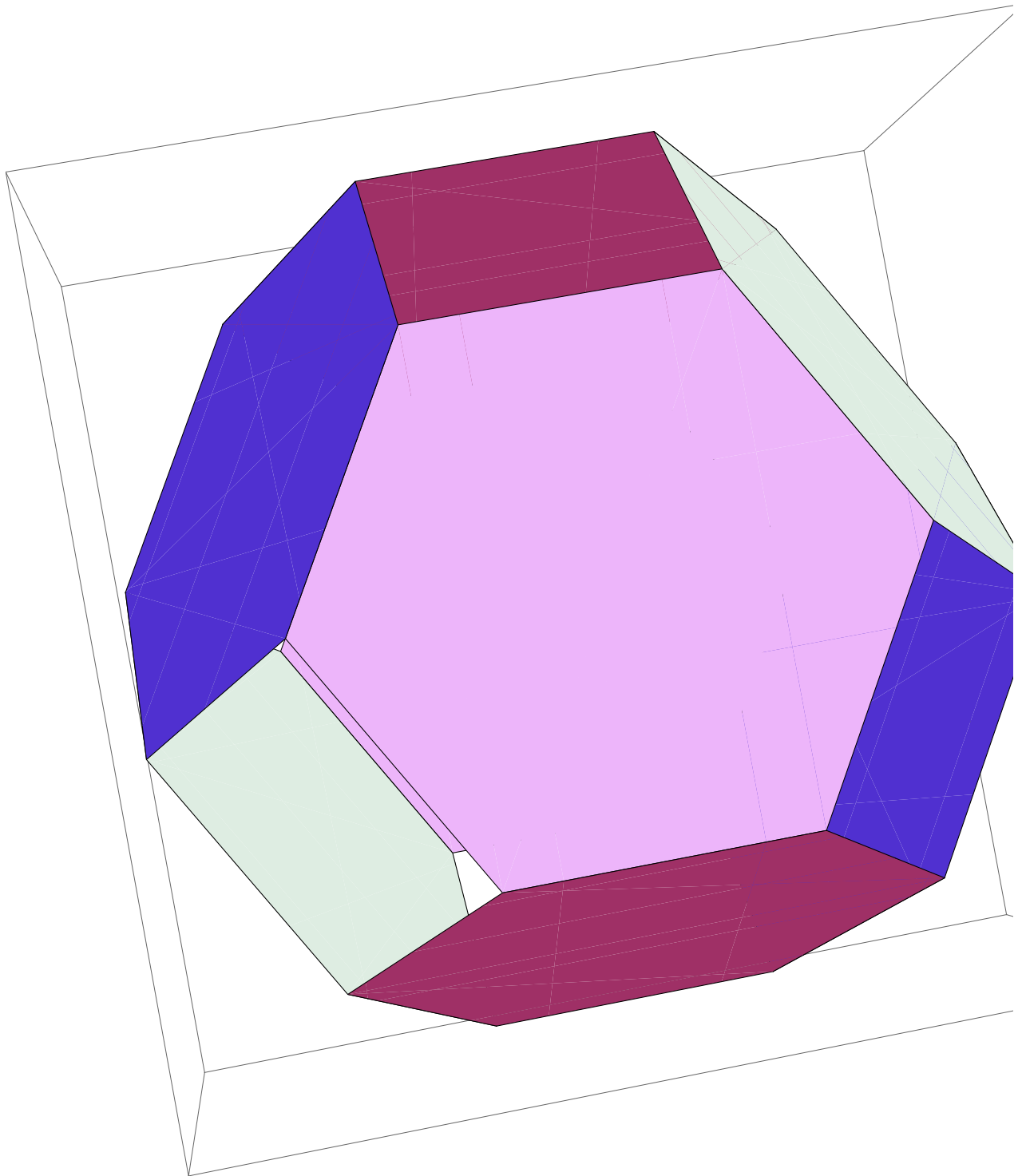
```
PolyhedronData["Dodecahedron"]
```



```
bhex = {{1, 2, 3}, {1, 3, 2}, {2, 3, 1}, {3, 2, 1}, {3, 1, 2}, {2, 1, 3}};
Graphics3D[Polygon[bhex]]
```



```
Graphics3D[
Join[
  Table[
    Polygon[Insert[#, 4, i] & /@ bhex],
    {i, 4}
  ],
  Table[
    Polygon[Insert[#, 1, i] & /@ (1 + bhex)],
    {i, 4}
  ]
] /. {x1_, x2_, x3_, x4_} -> mat.{x1, x2, x3, x4}
]
```



```

Graphics3D[{Opacity[0.7],
Join[
  Table[
    Polygon[Insert[#, 4, i] & /@ bhex],
    {i, 4}
  ],
  Table[
    Polygon[Insert[#, 1, i] & /@ (1 + bhex)],
    {i, 4}
  ],
  {
    Polygon[{{1, 2, 3, 4}, {1, 2, 4, 3}, {2, 1, 4, 3}, {2, 1, 3, 4}}],
    Polygon[{{1, 3, 2, 4}, {1, 4, 2, 3}, {2, 4, 1, 3}, {2, 3, 1, 4}}],
    Polygon[{{1, 4, 3, 2}, {1, 3, 4, 2}, {2, 3, 4, 1}, {2, 4, 3, 1}}],
    Polygon[{{3, 1, 2, 4}, {4, 1, 2, 3}, {4, 2, 1, 3}, {3, 2, 1, 4}}],
    Polygon[{{4, 2, 3, 1}, {3, 2, 4, 1}, {3, 1, 4, 2}, {4, 1, 3, 2}}],
    Polygon[{{3, 4, 1, 2}, {4, 3, 1, 2}, {4, 3, 2, 1}, {3, 4, 2, 1}}]
  ]
] /. {x1_Integer, x2_Integer, x3_Integer, x4_Integer} -> mat.{x1, x2, x3, x4}
}
]

```

