Pensieve header: Implementing ρ_1.

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Preliminaries

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This is Rho1.nb of http://drorbn.net/waco22/ap.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Waco-2203"];
```

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In[•]:=
```
Once[<< KnotTheory`; << Rot.m];
```

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```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

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```
Loading Rot.m from http://drorbn.net/waco22/ap to compute rotation numbers.
```

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The Program

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In[•]:=
```
R1[s_, i_, j_] := s (gji (gj+1,j + gj,j+1 - gij) - gii (gj,j+1 - 1) - 1 / 2);
ρ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]]) / 2) Det[A];
  G = Inverse[A];
  ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (gkk - 1 / 2);
  Factor@{Δ, Δ^2 ρ1 /. gα_,β_ :> G[[α, β]]}];
```

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The First Few Knots

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In[•]:=
```
Table[K → ρ[K], {K, AllKnots[{3, 6}]}]
```

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KnotTheory: Loading precomputed data in PD4Knots`.

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Out[•]=
$$\left\{\text{Knot}[3,1]\to\left\{\frac{1-T+T^2}{T},\ \frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\},\ \text{Knot}[4,1]\to\left\{-\frac{1-3T+T^2}{T},\ 0\right\},\right.$$
$$\text{Knot}[5,1]\to\left\{\frac{1-T+T^2-T^3+T^4}{T^2},\ \frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2T^4\right)}{T^4}\right\},$$
$$\text{Knot}[5,2]\to\left\{\frac{2-3T+2T^2}{T},\ \frac{(-1+T)^2\left(5-4T+5T^2\right)}{T^2}\right\},$$
$$\text{Knot}[6,1]\to\left\{-\frac{(-2+T)(-1+2T)}{T},\ \frac{(-1+T)^2\left(1-4T+T^2\right)}{T^2}\right\},$$
$$\text{Knot}[6,2]\to\left\{-\frac{1-3T+3T^2-3T^3+T^4}{T^2},\ \frac{(-1+T)^2\left(1-4T+4T^2-4T^3+4T^4-4T^5+T^6\right)}{T^4}\right\},$$
$$\left.\text{Knot}[6,3]\to\left\{\frac{1-3T+5T^2-3T^3+T^4}{T^2},\ 0\right\}\right\}$$

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\needspace{2in}

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Fast!

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\[\resizebox{\linewidth}{!}{\input{GST48-Marked.pdf_t}} \]

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In[•]:=
```
Timing@ρ[EPD[X14,1, X2,29, X3,40, X43,4, X26,5, X6,95, X96,7, X13,8, X9,28, X10,41, X42,11, X27,12,
    X30,15, X16,61, X17,72, X18,83, X19,34, X89,20, X21,92, X79,22, X68,23, X57,24, X25,56, X62,31,
    X73,32, X84,33, X50,35, X36,81, X37,70, X38,59, X39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
    X90,49, X51,82, X52,71, X53,60, X63,74, X64,85, X76,65, X87,66, X67,94, X75,86, X88,77, X78,93]]
```

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Out[•]=
$$\left\{86.2031,\ \left\{-\frac{\left(-1+2T-T^2-T^3+2T^4-T^5+T^8\right)\left(-1+T^3-2T^4+T^5+T^6-2T^7+T^8\right)}{T^8},\right.\right.$$
$$\frac{1}{T^{16}}(-1+T)^2\left(5-18T+33T^2-32T^3+2T^4+42T^5-62T^6-8T^7+166T^8-242T^9+108T^{10}+\right.$$
$$\left.\left.\left.132T^{11}-226T^{12}+148T^{13}-11T^{14}-36T^{15}-11T^{16}+148T^{17}-226T^{18}+132T^{19}+108T^{20}-\right.\right.\right.$$
$$\left.\left.\left.242T^{21}+166T^{22}-8T^{23}-62T^{24}+42T^{25}+2T^{26}-32T^{27}+33T^{28}-18T^{29}+5T^{30}\right)\right\}\right\}$$

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Strong!

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```
{NumberOfKnots[{3, 12}],
 Length@Union@Table[ρ[K], {K, AllKnots[{3, 12}]}],
 Length@Union@Table[{HOMFLYPT[K], Kh[K]}, {K, AllKnots[{3, 12}]}]}
```

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Out[•]= {2977, 2882, 2785}

In[•]:= **2977 - {2882, 2785}**

Out[•]= {95, 192}

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So the pair (Δ,ρ_1) attains 2,882 distinct values on the 2,977 prime knots with up to 12 crossings (a deficit of 95), whereas the pair (HOMFLYPT, Khovanov Homology) attains only 2,785 distinct values on the same knots (a deficit of 192).

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\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}

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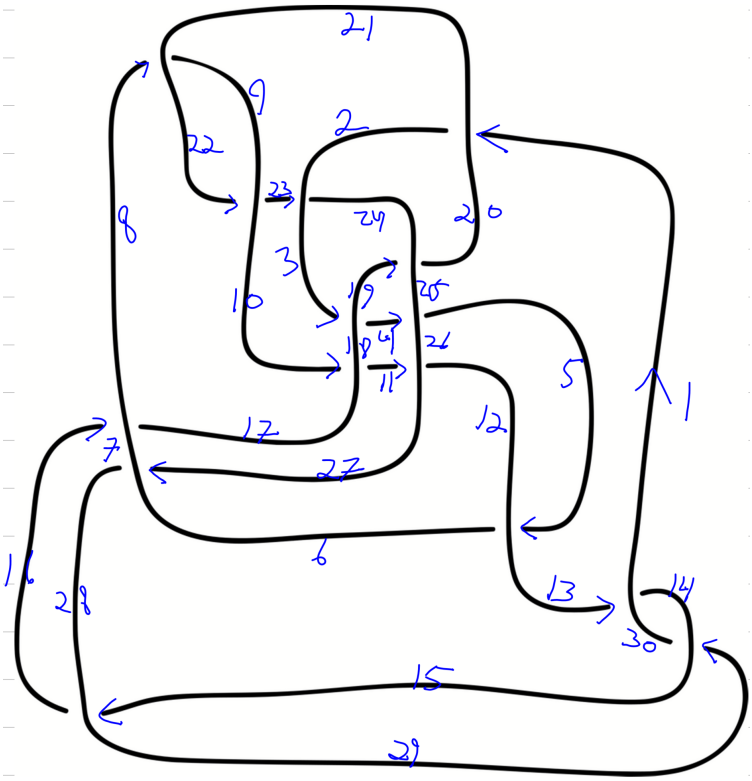
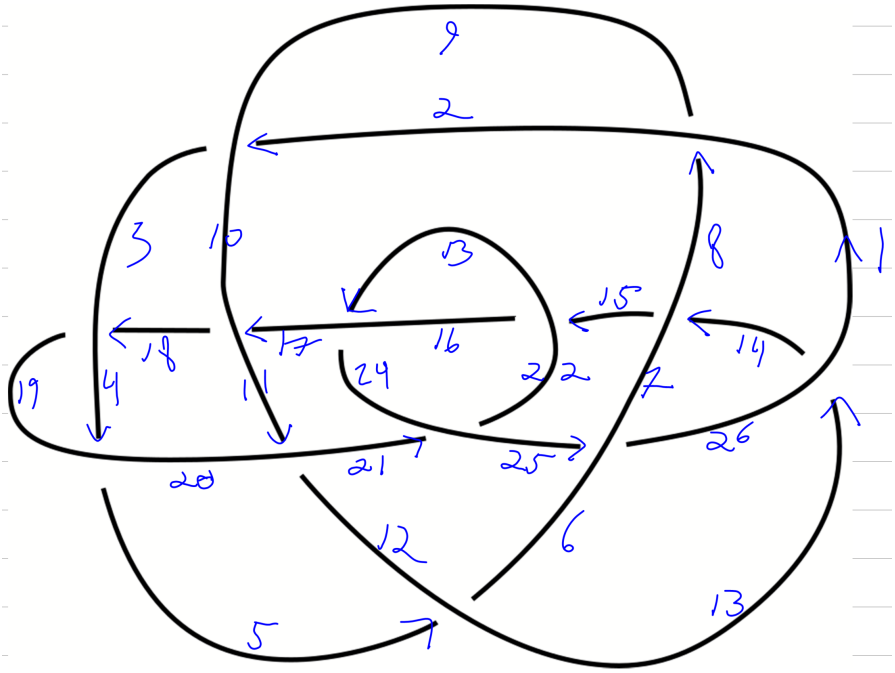
Hoste Ocneanu Millett Freyd Lickorish Yetter Przytycki Traczyk Khovanov

tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}



```
In[•]:= Timing@ρ[EPD[X_{20,1}, X̄_{18,3}, X_{25,4}, X̄_{12,5}, X_{21,8},
          X̄_{17,10}, X_{26,11}, X̄_{30,13}, X̄_{28,15}, X̄_{7,16}, X_{24,19}, X_{9,22}, X_{2,23}, X_{6,27}, X̄_{14,29}]]
Out[•]= {1.28125, {1, 0}}
```




```
In[•]:= Timing@ρ[EPD[X̄_{9,2}, X̄_{19,4}, X_{12,5}, X̄_{1,8}, X̄_{20,11}, X_{26,13}, X_{7,14}, X_{22,15}, X̄_{10,17}, X̄_{3,18}, X_{24,21}, X_{16,23}, X̄_{6,25}]]
Out[•]= {0.796875, {1, 0}}
```

```
In[•]:= K = PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]];

In[•]:= {Cs, r} = List @@ RVK[K]

  Set: Lists {Cs, r} and {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]} are not the same shape.

Out[•]= {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]}

In[•]:= n = Length[Cs]

Out[•]= 0

In[•]:= A = IdentityMatrix[2 n + 1]

Out[•]= {{1}}

In[•]:= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = ( -T^s   T^s - 1
                                                        0      -1 ), {c, Cs}]

  Do: Iterator {c, Cs} does not have appropriate bounds.

Out[•]= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = {{-T^s, T^s - 1}, {0, -1}}, {c, Cs}]
```

```
In[•]:= A // MatrixForm
```

```
Out[•]//MatrixForm=
( 1 )
```

```
In[•]:= A // MatrixForm // TeXForm
```

```
Out[•]//TeXForm=
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```

```
In[•]:= Δ = T^(-Total[r]-Total[First/@Cs])/2 Det[A]
```

```
Out[•]= T^(1/2 (-Total[Cs]-Total[r]))
```

```
In[•]:= G = Inverse[A];
```

```
In[•]:= G // MatrixForm
```

```
Out[•]//MatrixForm=
( 1 )
```

```
In[•]:= G // Simplify // MatrixForm
```

```
Out[•]//MatrixForm=
( 1 )
```

```
In[•]:= G // Simplify // MatrixForm // TeXForm
```

```
Out[•]//TeXForm=
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```