In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Waco-2203"];
<< Rho1.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/waco22/ap to compute rotation numbers.
```

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$\delta_{i\_,j\_}$:= If[$i == j$, 1, 0];
gRules$_{s_,i_,j_}$:= {$g_{i\beta\_} \mapsto \delta_{i\beta} + T^s g_{i+1,\beta} + (1 - T^s) g_{j+1,\beta}$,
$g_{j\beta\_} \mapsto \delta_{j\beta} + g_{j+1,\beta}$, $g_{\alpha\_,i} \mapsto T^{-s} (g_{\alpha,i+1} - \delta_{\alpha,i+1})$,
$g_{\alpha\_j} \mapsto g_{\alpha,j+1} - (1 - T^s) g_{\alpha i} - \delta_{\alpha,j+1}$}

Proof of Reidemeister 3:

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In[∘]:= lhs = R_1[1, 20, 30] + R_1[1, 10, 31] + R_1[1, 11, 21] //. gRules$_{1,20,30}$ ⋃ gRules$_{1,10,31}$ ⋃ gRules$_{1,11,21}$;
rhs = R_1[1, 10, 20] + R_1[1, 11, 30] + R_1[1, 21, 31] //. gRules$_{1,10,20}$ ⋃ gRules$_{1,11,30}$ ⋃ gRules$_{1,21,31}$;
Simplify[lhs == rhs]

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Out[∘]= True

tex

Next comes Reid1, where we use results from an earlier example:

In[∘]:= $\begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$ // Inverse // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & T & -T \\ 0 & 0 & 1 \end{pmatrix}$$

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In[∘]:= R_1[1, 2, 1] − 1 (g_{22} − 1 / 2) /. $g_{\alpha\_,\beta\_} \mapsto \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$⟦α, β⟧

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Out[∘]= $\frac{1}{T^2} - \frac{1}{T} - \frac{-1 + \frac{1}{T}}{T}$

tex

Invariance under the other moves is proven similarly.

Alternative R_1's:

In[∘]:= Simplify[
R_1[s, i, j] == s ($(1 - T^s) g_{ji} (g_{ji} - g_{ii}) + 2 g_{jj} g_{ji} - g_{ji} g_{ij} - g_{jj} g_{ii} - g_{ji} + g_{ii} - 1/2$) //. gRules$_{s,i,j}$]

Out[∘]= $i \neq j \,||\, s (-1 + T^s) (g_{1+i,1+i} - g_{1+j,1+i}) == 0$

In[ ]:= $\text{Simplify}\left[R_1[s, i, j] == s\left(\left(g_{j,j+1} - g_{j,j}\right)\left(g_{ji} - g_{ii}\right) + 2 g_{jj} g_{ji} - g_{ji} g_{ij} - g_{jj} g_{ii} - g_{ji} + g_{ii} - 1/2\right) \text{//. } \text{gRules}_{s,i,j}\right]$

Out[ ]= True

In[ ]:= $\text{Simplify}\left[R_1[s, i, j] == s\left(g_{j,i}\left(-1 - g_{i,j} + g_{j,j} + g_{j,1+j}\right) - g_{i,i}\left(-1 + g_{j,1+j}\right) - 1/2\right) \text{//. } \text{gRules}_{s,i,j}\right]$

Out[ ]= True

In[ ]:= $\text{Simplify}\left[R_1[s, i, j] == s\left(g_{ji}\left(g_{jj} + g_{j,j+1} - g_{ij} - 1\right) - g_{ii}\left(g_{j,j+1} - 1\right) - 1/2\right) \text{//. } \text{gRules}_{s,i,j}\right]$

Out[ ]= True

In[ ]:= $\text{Simplify}\left[R_1[s, i, j] == s\left(g_{ji}\left(g_{j+1,j} + g_{j,j+1} - g_{ij}\right) - g_{ii}\left(g_{j,j+1} - 1\right) - 1/2\right) \text{//. } \text{gRules}_{s,i,j}\right]$

Out[ ]= True

In[ ]:= $\text{Simplify}\left[\left(g_{ji}\left(g_{j+1,j} + g_{j,j+1} - g_{ij}\right) - g_{ii}\left(g_{j,j+1} - 1\right) - 1/2\right)\right]$

Out[ ]= $-\frac{1}{2} - g_{i,i}\left(-1 + g_{j,1+j}\right) + g_{j,i}\left(-g_{i,j} + g_{j,1+j} + g_{1+j,j}\right)$

In[ ]:= $\text{Simplify}\left[\left\{g_{jj}, g_{j,j+1}, g_{ij}, g_{j+1,j} + g_{j,j+1}\right\} \text{//. } \text{gRules}_{s,i,j}\right]$

Out[ ]= $\{-T^{-s}\left(-1 + T^{s}\right)\left(\text{If}[i == j, 1, 0] - g_{1+j,1+i}\right) + g_{1+j,1+j},$
$g_{1+j,1+j}, \text{If}[i == j, 1, 0] - T^{s}\,\text{If}[i == j, 1, 0] + \left(-1 + T^{s}\right)\left(-1 + g_{1+i,1+i}\right) +$
$T^{s} g_{1+i,1+j} + \left(1 - T^{s}\right)\left(-1 - T^{-s}\left(-1 + T^{s}\right)\left(\text{If}[i == j, 1, 0] - g_{1+j,1+i}\right) + g_{1+j,1+j}\right),$
$T^{-s}\left(-\left(\left(-1 + T^{s}\right)\text{If}[i == j, 1, 0]\right) + \left(-1 + T^{s}\right) g_{1+j,1+i} + T^{s}\left(-1 + 2 g_{1+j,1+j}\right)\right)\}$