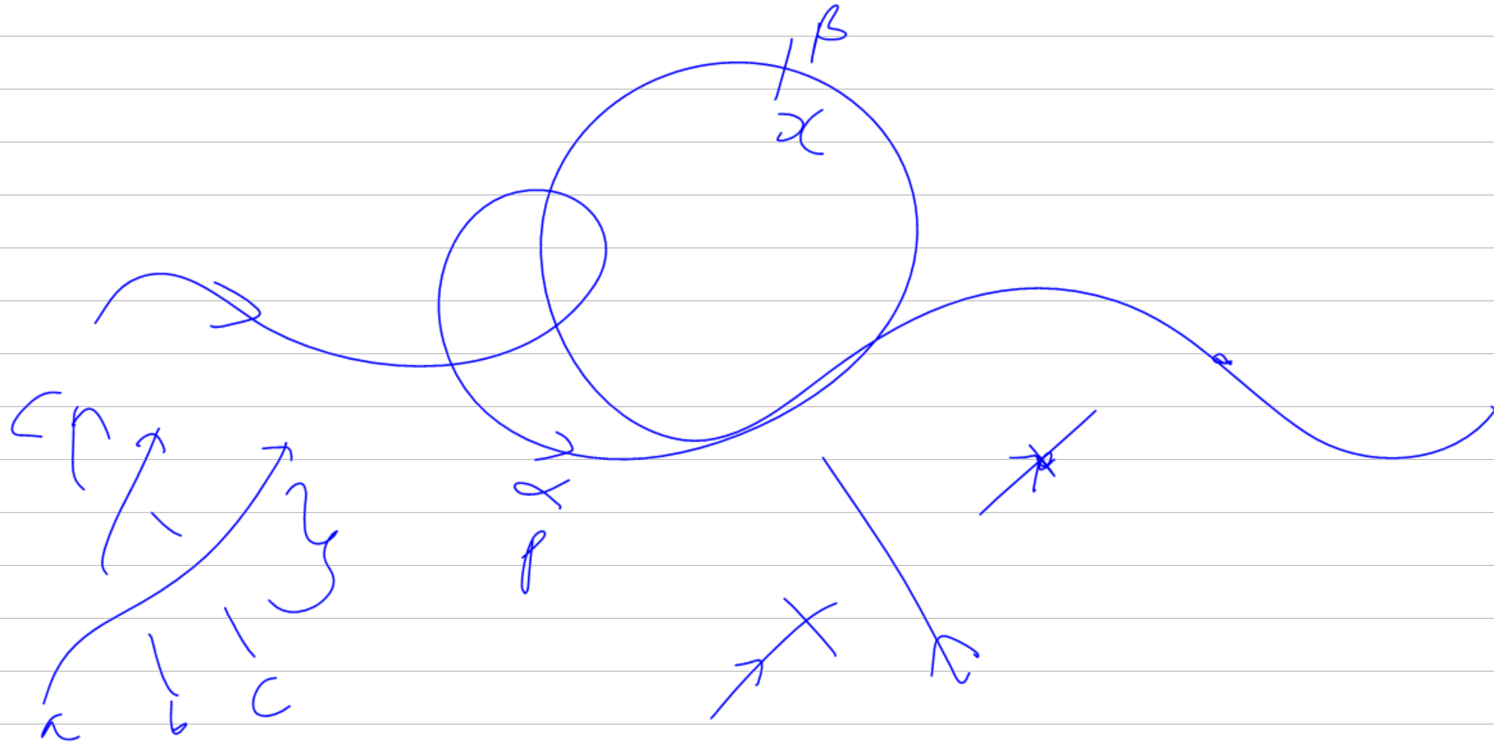
Cars, Interchanges, Traffic Counters, and a Pretty Darned Good Knot Invariant

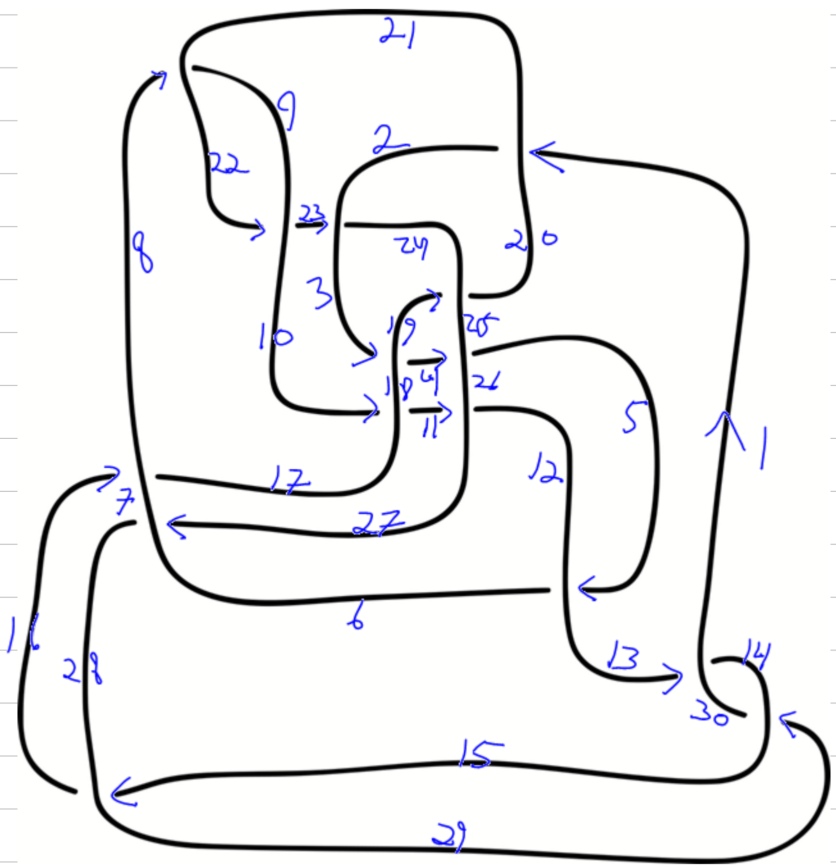
Reporting on joint work with Roland van der Veen, I'll tell you some stories about \rho_1, an easy to define, strong, fast to compute, homomorphic, and well-connected knot invariant. \rho_1 was first studied by Rozansky and Overbay, it has far-reaching generalizations, and I wish I understood it.

Sketch

1. Knots & Invariants.
2. Seek strong, fast, homomorphic invts. GST48

   Jones: Formulas stay, interpretations change.
3. ρ_1 formulas.
4. Implementation & demo: strong & fast on knots w/ 3-12 x-ing on GST48
5. Cars, interchanges, traffic counters
6. Proof of invariance.
7. Other comments.

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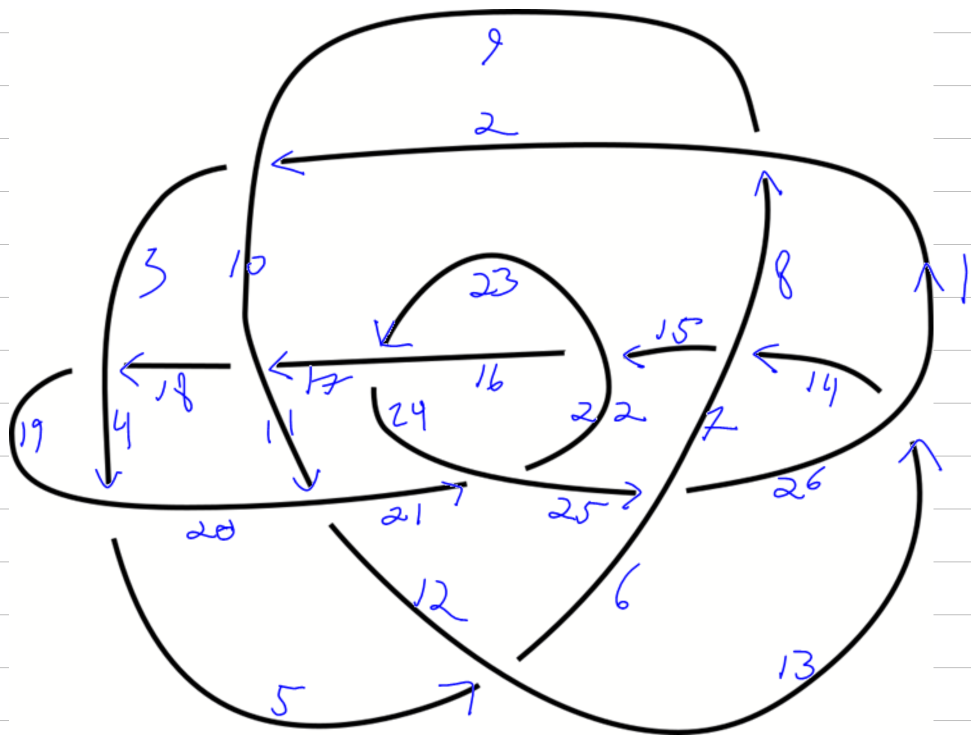
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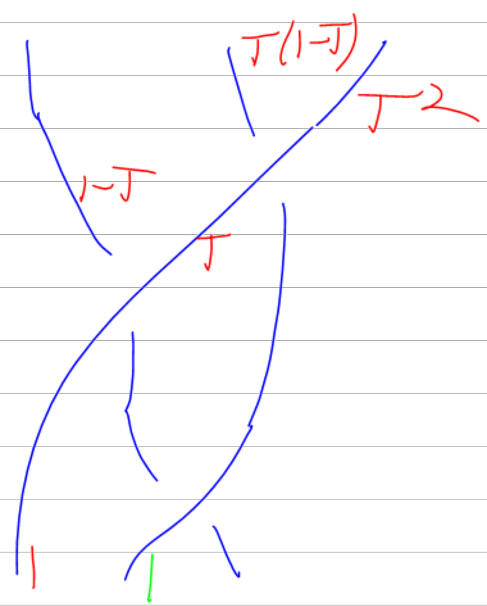
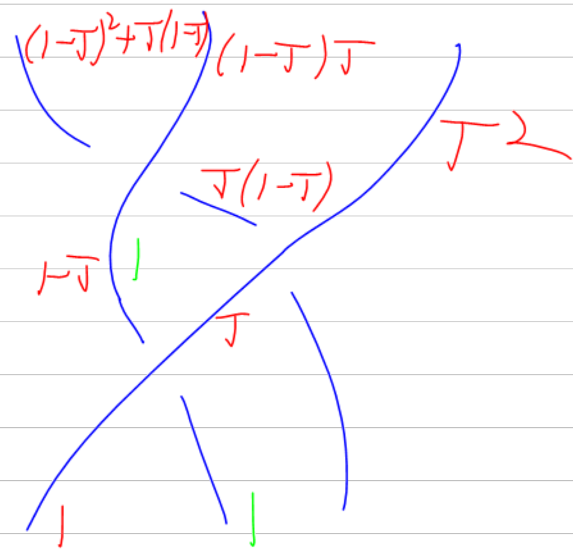
The new G:

$$
\begin{array}{cc}
 & p \xrightarrow{\quad x \quad} \\
\begin{array}{c} x \\ p \downarrow \end{array} &
\begin{pmatrix}
1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 \\
0 & 1 & \frac{T^2}{1-T+T^2} & \frac{T}{1-T+T^2} & \frac{T}{1-T+T^2} & \frac{1}{1-T+T^2} & 1 \\
0 & 0 & \frac{T^2}{1-T+T^2} & \frac{T}{1-T+T^2} & \frac{T}{1-T+T^2} & \frac{1}{1-T+T^2} & 1 \\
0 & 0 & \frac{(-1+T)\,T}{1-T+T^2} & \frac{T^2}{1-T+T^2} & \frac{T^2}{1-T+T^2} & \frac{T}{1-T+T^2} & 1 \\
0 & 0 & \frac{(-1+T)\,T}{1-T+T^2} & \frac{-1+T}{1-T+T^2} & \frac{T^2}{1-T+T^2} & \frac{T}{1-T+T^2} & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{array}{c} p \\ \downarrow x \end{array} \\
 & \xrightarrow{\quad x \quad} p
\end{array}
$$





$2-2=0$

actually the crossings on the Reidemeister 1 pictures on Page 3 top left are fine. Sorry for the confusion.

On Fri, Mar 11, 2022 at 10:19 AM Roland Mathematics <roland.mathematics@gmail.com> wrote:

Hi Dror,

Here's a link to Sjabbo's thesis in the Leiden repository:
https://scholarlypublications.universiteitleiden.nl/handle/1887/136272 [scholarlypublications.universiteitleiden.nl]
I hope you dont mind some comments on your cars handout:
Page 1 top right: It's not clear that the zeroes written next to the edges of the little trefoil are rotation numbers so I'd remove them, e.g. the 1 0 looks like ten.
Page 1 middle right: If I did not already know I'd not understand the sentence "Note. Alexander's Delta..." Especially given the fact that you already used the notation Delta for something completely different on the same page on the left when talking about strand doubling.
Page 3 top left: I believe the crossings in the Reidemeister 1 pictures are off because cars do not float up.
Page 3 top left: I find it easier to see the bonus g-rules in the form g_{alpha,j+1} = g_{alpha,j}+.... i.e. moving the traffic counter ahead.
Page 3 bottom left: The name R_1 clashes a bit with Reidemeister 1 which you call R1 here,
Page 3 bottom left: Also, the phi next to the little kink picture should be phi_2.
Page 3 top right: Maybe replace exp(t) by T in the commutation formulas with R_0

Finally the cars and counters interpretation seems similar to the random walk in Lin and Wang, see page 4 of:
https://arxiv.org/pdf/math/9812039.pdf [arxiv.org]

Best,
Roland

$[p, x] = 1$ $\quad px = xp + 1$ $\quad$ work in $\mathbb{H}/\mathbb{H}p$

Porschl

$$p_i \left(x_i (p_i - p_j) \right) = (\underline{\hspace{6em}}) \, p_i$$

$$p_j \left(x_i (p_i - p_j) \right) = (\underline{\hspace{5em}}) \, p_j$$