

Pensieve header: Examples for the UCLA FDA talk.

Startup

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\UCLA-191101"];
```

hm

hm

```
In[ ]:= {x̂ = (0 1 0), ŷ = (0 0 0), ĉ = (0 0 1)};
{ŷ.ŷ - ŷ.x̂ == ĉ, x̂.ĉ == ĉ.x̂, ŷ.ĉ == ĉ.ŷ}
```

hm

```
Out[ ]:= {True, True, True}
```

hm

```
In[ ]:= Δ = -ħ η_i ξ_j c_k + (ξ_i + ξ_j) x_k + (η_i + η_j) y_k;
Simplify@With[{E = MatrixExp},
  E[x̂ ξ_i].E[ŷ η_j].E[x̂ ξ_j].E[ŷ η_j] == E[x̂ ∂_{x_k} Δ].E[ŷ ∂_{y_k} Δ].E[ĉ ∂_{c_k} Δ]
```

hm

```
Out[ ]:= True
```

sl2

```
In[ ]:= Δ0 = HoldForm[ (η_i + (e^{-α_i - ε β_i} η_j) / (1 + ε η_j ξ_i)) y_k + (β_i + β_j + (Log[1 + ε η_j ξ_i] / ε) b_k +
  (α_i + α_j + Log[1 + ε η_j ξ_i]) a_k + (e^{-α_j - ε β_j} ξ_i / (1 + ε η_j ξ_i) + ξ_j) x_k ];
TeXForm[Δ0]
Δ = ReleaseHold[Δ0]
```

```
Out[ ]:= a_k (Log[1 + ε η_j ξ_i] + α_i + α_j) +
```

```
b_k ( (Log[1 + ε η_j ξ_i] / ε) + β_i + β_j ) + y_k ( η_i + (e^{-α_i - ε β_i} η_j) / (1 + ε η_j ξ_i) ) + x_k ( (e^{-α_j - ε β_j} ξ_i / (1 + ε η_j ξ_i) + ξ_j )
```

```
\left(\eta_i + \frac{e^{-\alpha_i - \epsilon \beta_i} \eta_j}{1 + \epsilon \eta_j \xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\log\left(1 + \epsilon \eta_j \xi_i\right)}{\epsilon} b_k + \left(\alpha_i + \alpha_j + \log\left(1 + \epsilon \eta_j \xi_i\right)\right) a_k + \left(\frac{e^{-\alpha_j - \epsilon \beta_j} \xi_i}{1 + \epsilon \eta_j \xi_i} + \xi_j\right) x_k\right);
```

sl2

```
In[ ]:= {ŷ = (0 0), b̂ = (0 0), â = (1 0), x̂ = (0 1)};
{â.x̂ - x̂.â == x̂, â.ŷ - ŷ.â == -ŷ, b̂.ŷ - ŷ.b̂ == -ε ŷ, b̂.x̂ - x̂.b̂ == ε x̂, x̂.ŷ - ŷ.x̂ == b̂ + ε â}
```

sl2

```
Out[ ]:= {True, True, True, True, True}
```

s12

```
In[ ]:= Simplify@With[{E = MatrixExp},
  E[ηi ŷ] . E[βi b̂] . E[αi â] . E[ξi x̂] . E[ηj ŷ] . E[βj b̂] . E[αj â] . E[ξj x̂] ==
  E[ŷ ∂yk Λ] . E[b̂ ∂bk Λ] . E[â ∂ak Λ] . E[x̂ ∂xk Λ]
```

s12

Out[]:= True

s12

```
In[ ]:= Series[Λ, {ε, 0, 2}]
```

s12

```
Out[ ]:= (ak (αi + αj) + yk (ηi + e-αi ηj) + bk (βi + βj + ηj ξi) + xk (e-αj ξi + ξj)) +
  (ak ηj ξi -  $\frac{1}{2}$  bk ηj2 ξi2 - e-αi yk ηj (βi + ηj ξi) - e-αj xk ξi (βj + ηj ξi)) ε +
  (- $\frac{1}{2}$  ak ηj2 ξi2 +  $\frac{1}{3}$  bk ηj3 ξi3 +  $\frac{1}{2}$  e-αi yk ηj (βi2 + 2 βi ηj ξi + 2 ηj2 ξi2) +
   $\frac{1}{2}$  e-αj xk ξi (βj2 + 2 βj ηj ξi + 2 ηj2 ξi2)) ε2 + O[ε]3
```