Dogma handout on 170507

May 7, 2017 12:17 PM

Dror Bar-Natan: Talks: Toulouse-1705:

The Dogma is Wrong

Follows Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen.

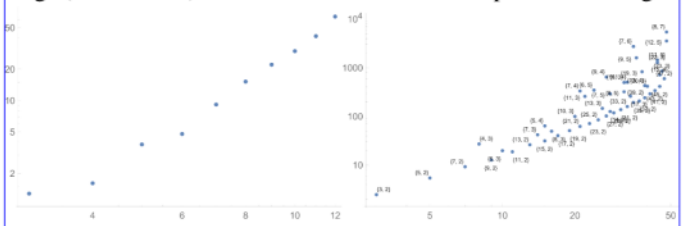
ωεβ:=http://drorbn.net/Toulouse-1705/

Thanks for the invitation!

Abstract. It has long been known that there are knot invariants associated to semi-simple Lie algebras, and there has long been a dogma as for how to extract them: "quantize and use representation theory". We present an alternative and better procedure: "centrally extend, approximate by solvable, and learn how to re-order exponentials in a universal enveloping algebra". While equivalent to the old invariants via a complicated process, our invariants are in practice stronger, faster to compute (poly-time vs. exp-time), and clearly carry topological information.

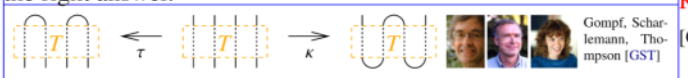
KiW XLIII Abstract. Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

Experimental Analysis (ωεβ/Exp). Log-log plots of computation time (sec) vs. crossing number, for all knots with up to 12 crossings (mean times) and for all torus knots with up to 48 crossings:



Power. On the 250 knots with at most 10 crossings, the pair (ω,ρ_1) attains 250 distinct values, while (Khovanov, HOMFLY-PT) attains only 249 distinct values. To 11 crossings the numbers are (802, 788, 772) and to 12 they are (2978, 2883, 2786).

Genus. Up to 12 xings, always ρ_1 is symmetric under $t \leftrightarrow t^{-1}$. With ρ_1^+ denoting the positive-degree part of ρ_1, always $\deg\rho_1^+ \le 2g-1$, where g is the 3-genus of K (equallity for 2530 knots). This gives a lower bound on g in terms of ρ_1 (conjectural, but undoubtedly true). This bound is often weaker than the Alexander bound, yet for 10 of the 12-xing Alexander failures it does give the right answer.



Gompf, Scharlemann, Thompson [GST]

$U \in \mathcal{T}_n \quad 1 \in \mathcal{A}_n$

$\mathcal{T}_{2n} \xrightarrow{z} \mathcal{A}_{2n}$

ribbon $K \in \mathcal{T}_1 \quad z(K) \in \mathcal{R} \subseteq \mathcal{A}_1$

[Vo]: Works for Alexander!

with $\mathcal{R} := \kappa(\tau^{-1}(1))$

$A^+ = -t^8 + 2t^7 - t^6 - 2t^4 + 5t^3 - 2t^2 - 7t + 13$

$\rho_1^+ = 5t^{15} - 18t^{14} + 33t^{13} - 32t^{12} + 2t^{11} + 42t^{10} - 62t^9 - 8t^8 + 166t^7 - 242t^6 +$

Faster is better, leaner is meaner! $108t^5 + 132t^4 - 226t^3 + 148t^2 - 11t - 36$

The moduli space of Lie algebras.

dog·ma (dôg'mə, dŏg'-) The Free Dictionary, ωεβ/TFD

n. pl. **dog·mas** or **dog·ma·ta** (-mə-tə)

1. A doctrine or a corpus of doctrines relating to matters such as morality and faith, set forth in an authoritative manner by a religion.

2. A principle or statement of ideas, or a group of such principles or statements, especially when considered to be authoritative or accepted uncritically: *"Much education consists in the instilling of unfounded dogmas in place of a spirit of inquiry" (Bertrand Russell).*

gl_n^E similar to McGill ✓

An MMR box ✓

References.

[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures*, Geom. and Top. **14** (2010) 2305–2347, arXiv:1103.1601.

[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial*, University of North Carolina PhD thesis, ωεβ/Ov.

[Ro1] L. Rozansky, *A contribution of the trivial flat connection to the Jones polynomial and Witten's invariant of 3d manifolds, I*, Comm. Math. Phys. **175-2** (1996) 275–296, arXiv:hep-th/9401061.

[Ro2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial*, Adv. Math. **134-1** (1998) 1–31, arXiv:q-alg/9604005.

[Ro3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture*, arXiv:math/0201139.

[Vo] H. Vo, University of Toronto Ph.D. thesis, in preparation.

+ Add content page 2. ✓
+ content: "Solve 1/c" mathematica session as in McGill. ✓
  Algebras to invariants similar to McGill. ✓
  Continue as in McGill ✓
  Mathematica sessions as in MIT. ✗