Pensieve header: Examples for the Da-Nang talk: Double Integration and the trefoil.

Startup

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Sydney-191002"];
```

hm

hm

In[∘]:=

$$\left\{\rho x = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \rho y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \hbar \\ 0 & 0 & 0 \end{pmatrix}, \rho c = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\};$$

```
{ρx.ρy - ρy.ρx == ℏ ρc, ρx.ρc == ρc.ρx, ρy.ρc == ρc.ρy}
```

hm

Out[∘]= {True, True, True}

hm

In[∘]:=

$$\Lambda = -\hbar\, \eta_i\, \xi_j\, c_k + (\xi_i + \xi_j)\, x_k + (\eta_i + \eta_j)\, y_k;$$

```
Simplify@With[{𝔼 = MatrixExp},
  𝔼[ξi ρx].𝔼[ηi ρy].𝔼[ξj ρx].𝔼[ηj ρy] == 𝔼[∂xk Λ ρx].𝔼[∂yk Λ ρy].𝔼[∂ck Λ ρc]]
```

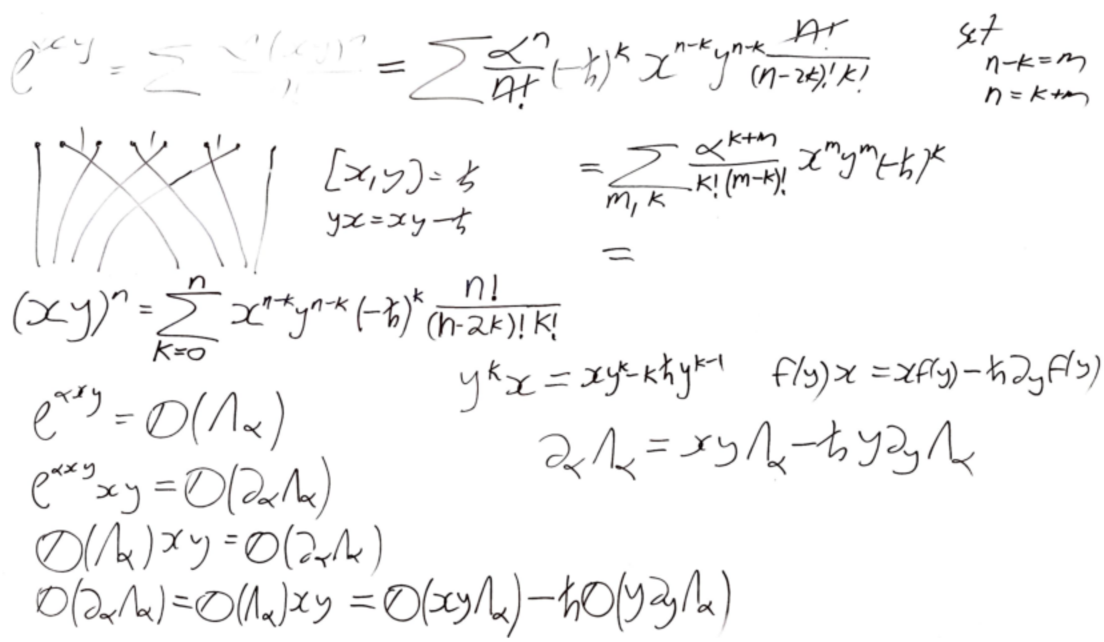
hm

Out[∘]= True

A failing attempt to figure out dilations

Dilations. In $\mathbb{H}$, $\mathbb{e}^{\alpha xy}$ is a "dilation operator". Thinking of it as in $\mathbb{H}^{\otimes\emptyset} \to \mathbb{H}_i$, it gets map by PBW to an element $d \in \mathrm{Hom}(\mathbb{Q}[] \to \mathbb{Q}[x,y][\![\alpha]\!])$.

Claim. $\mathcal{D}(d) =$



In[∘]:=
```
DSolve[{λ[0, y] == 1, ∂_α λ[α, y] == x y λ[α, y] - ℏ y ∂_y λ[α, y]}, {λ[α, y]}, {α, y}]
```

Out[∘]=
```
DSolve[{λ[0, y] == 1, λ^(1,0)[α, y] == x y λ[α, y] - y ℏ λ^(0,1)[α, y]}, {λ[α, y]}, {α, y}]
```

In[∘]:=
```
With[{λ = e^(α x y/(1-α ℏ)) (1 - α ℏ)^-1},
 Simplify[∂_α λ == x y λ - ℏ y ∂_y λ] ]
```

Out[∘]= $\frac{e^{\frac{x y \alpha}{1-\alpha \hbar}} \hbar \left(-1+\alpha \hbar + x y \alpha \left(-3+2\alpha\hbar\right)\right)}{-1+\alpha\hbar} == 0$

In[∘]:=
```
Coefficient[q - x y α + q^2 α (1 + x y α) ℏ + q x y α (2 - α ℏ), α]
```

Out[∘]= $-x y + 2 q x y + q^2 \hbar$

cm

In[∘]:=
```
Λ0 = HoldForm[(η_i + e^(-α_i - ϵ β_i) η_j/(1 + ϵ η_j ξ_i)) y_k + (β_i + β_j + Log[1 + ϵ η_j ξ_i]/ϵ) b_k +
    (α_i + α_j + Log[1 + ϵ η_j ξ_i]) a_k + (e^(-α_j - ϵ β_j) ξ_i/(1 + ϵ η_j ξ_i) + ξ_j) x_k];
TeXForm[Λ0]
Λ = ReleaseHold[Λ0]
```

Out[∘]= $a_k \left(\text{Log}[1+\epsilon\,\eta_j\,\xi_i] + \alpha_i + \alpha_j\right) + b_k \left(\frac{\text{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon} + \beta_i + \beta_j\right) + y_k \left(\eta_i + \frac{e^{-\alpha_i - \epsilon\,\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right) + x_k \left(\frac{e^{-\alpha_j - \epsilon\,\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i} + \xi_j\right)$

```
\left(\eta _i+\frac{e^{-\alpha _i-\epsilon  \beta _i} \eta _j}{1+\epsilon  \eta _j \xi _i}
   _i+\beta _j+\frac{\log \left(1+\epsilon  \eta _j \xi _i\right)}{\epsilon }\right) b_k+\
   _j+\log \left(1+\epsilon  \eta _j \xi _i\right)\right) a_k+\left(\frac{e^{-\alpha _j-\e
   _i}{1+\epsilon  \eta _j \xi _i}+\xi _j\right) x_k
```

cm

In[∘]:=
```
{ρy = (0 0; ϵ 0), ρb = (0 0; 0 -ϵ), ρa = (1 0; 0 0), ρx = (0 1; 0 0)};
{ρa.ρx - ρx.ρa == ρx, ρa.ρy - ρy.ρa == -ρy,
 ρb.ρy - ρy.ρb == -ϵ ρy, ρb.ρx - ρx.ρb == ϵ ρx, ρx.ρy - ρy.ρx == ρb + ϵ ρa}
```

cm

Out[∘]= {True, True, True, True, True}

cm

In[∘]:=
```
Simplify@With[{𝔼 = MatrixExp},
  𝔼[η_i ρy].𝔼[β_i ρb].𝔼[α_i ρa].𝔼[ξ_i ρx].𝔼[η_j ρy].𝔼[β_j ρb].𝔼[α_j ρa].𝔼[ξ_j ρx] ==
   𝔼[∂_{y_k} Λ ρy].𝔼[∂_{b_k} Λ ρb].𝔼[∂_{a_k} Λ ρa].𝔼[∂_{x_k} Λ ρx]]
```

cm

Out[∘]= True

cm

In[]:= **Series[Λ, {ϵ, 0, 2}]**

cm

Out[]=
$$\left(a_k\,(\alpha_i+\alpha_j)+y_k\left(\eta_i+e^{-\alpha_i}\,\eta_j\right)+b_k\,(\beta_i+\beta_j+\eta_j\,\xi_i)+x_k\left(e^{-\alpha_j}\,\xi_i+\xi_j\right)\right)+$$
$$\left(a_k\,\eta_j\,\xi_i-\frac{1}{2}\,b_k\,\eta_j^2\,\xi_i^2-e^{-\alpha_i}\,y_k\,\eta_j\,(\beta_i+\eta_j\,\xi_i)-e^{-\alpha_j}\,x_k\,\xi_i\,(\beta_j+\eta_j\,\xi_i)\right)\epsilon+$$
$$\left(-\frac{1}{2}\,a_k\,\eta_j^2\,\xi_i^2+\frac{1}{3}\,b_k\,\eta_j^3\,\xi_i^3+\frac{1}{2}\,e^{-\alpha_i}\,y_k\,\eta_j\left(\beta_i^2+2\,\beta_i\,\eta_j\,\xi_i+2\,\eta_j^2\,\xi_i^2\right)+\right.$$
$$\left.\frac{1}{2}\,e^{-\alpha_j}\,x_k\,\xi_i\left(\beta_j^2+2\,\beta_j\,\eta_j\,\xi_i+2\,\eta_j^2\,\xi_i^2\right)\right)\epsilon^2+O[\epsilon]^3$$