StonyBrook-1805 Ideas

December 9, 2017 7:41 AM

1. "What else can you do with solvable approximation" as in McGill, with some Wigner thrown in.
2. "Topology: "What are Quantum Groups?"".
3. "Consolidating a Portfolio: The Solvable Approximation of sl2".

Abandoned.

Colloquium Title. Computation without Representation.

Abstract. A major part of "quantum topology" (you don't have to know what's that) is the definition and computation of various knot invariants by carrying out computations in quantum groups (you don't have to know what are these). Traditionally these computations are carried out "in a representation", but this is very slow: one has to use tensor powers of these representations, and the dimensions of powers grow exponentially fast. I will describe a direct-participation method for carrying out these computations without having to choose a representation and explain why in many ways the results are better and faster. The two key points we use are a technique for composing infinite-order "perturbed Gaussian" differential operators, and the little-known fact that every semi-simple Lie algebra can be approximated by solvable Lie algebras, where computations are easier.

(Alt: A key point we use is that every semi-simple Lie algebra can be approximated by solvable ones, where computations are easier.)

Sketch.
1. (5) The knot theory portfolio.
2. (5) The (quantum) group portfolio. (Write in general-algebra language, not in universal enveloping algebra language).
3. (3) Quantum groups have "PBW" basis. Hence they are symmetric algebras, with funny products / co-products.
4. (3) The DO category.
5. (5) Example: the Abelian case.
6. (3) Example: k=0.
7. (5) Example: sl2.
8. (10) Zipping and composing. The zipping theorem.
9. (5) Docility and polynomiallity.
10. (5) Solvable Approximation.
11. (8) Mention Rozansky-Overbay, mention computations.

Hours 2-3 Sketch.
1. Review of hour 1.
2. Some remaining details from hour 1.
3. How this all relates to "Poisson geometry of moduli spaces, associators and quantum field theory".
4. A longer to-do list following hour 1.
5. What universal problem did we solve? [The dread of every mathematician - be generalized into obsolescence].
6. Rotational virtual knots.
7. Expansions and quantization.
8. VOUS-tangles.
9. The loop filtration.