

```

In[ ]:= K = Knot[3, 1], d = 2}
Out[ ]:=
{Knot[3, 1], 2}

In[ ]:= {Cs, φ} = Rot[K]
KnotTheory: Loading precomputed data in PD4Knots`.
Out[ ]:=
{{{ -1, 4, 1}, {-1, 6, 3}, {-1, 2, 5}}, {0, 0, 0, -1, 0, 0}}

In[ ]:= n = Length[Cs]
Out[ ]:=
3

In[ ]:= A = IdentityMatrix[2 n + 1]
Out[ ]:=
{{1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}}

In[ ]:= Cases[Cs, {s_, i_, j_} => (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))]
Out[ ]:=
{{{ -1/T, -1 + 1/T}, {0, -1}}, {{ -1/T, -1 + 1/T}, {0, -1}}, {{ -1/T, -1 + 1/T}, {0, -1}}}

In[ ]:= G = Inverse[A]
Out[ ]:=
{{1, 1 + 1/T^2 - 1/T, 1, 1 + 1/T^2 - 1/T, 1, 1 + 1/T^2 - 1/T, 1},
{1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T,
{0, 1 + 1/T^2 - 1/T, 1, 1 + 1/T^2 - 1/T, 1, 1 + 1/T^2 - 1/T, 1},
{0, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T,
(1/T^3 - 1/T^2 + 1/T) T, 1 + 1/T^2 - 1/T, 1},
{0, 1 - 1/T, 1/T^3 - 2/T^2 + 2/T, 1, 1, 1 + 1/T^2 - 1/T, 1},
{0, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T,
(1/T^3 - 1/T^2 + 1/T) T, 1/T^3 - 1/T^2 + 1/T, 1},
{0, 1 + 1/T^2 - 2/T, 1/T^3 - 2/T^2 + 1/T, 1 - 1/T, 1/T^3 - 2/T^2 + 2/T, 1 + 1/T^2 - 1/T, 1},
{0, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T,
1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1},
{0, 1 + 1/T^2 - 2/T, 1/T^3 - 2/T^2 + 1/T, 1 - 1/T, -1/T^2 + 1/T, 1 + 1/T^2 - 1/T, 1},
{0, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T,
1/T^3 - 1/T^2 + 1/T, 1/T^3 - 1/T^2 + 1/T, 1}, {0, 0, 0, 0, 0, 0, 1}}

```

$$\text{In[*]:= } \rho_{d1} = \text{PP}_{\text{Mod}} @ \text{Exp} \left[\text{Total} \left[\text{Cases} \left[\text{Cs}, \{s_ , i_ , j_ \} \Rightarrow \text{Sum} \left[e^{d1} r_{d1,s} [i, j], \{d1, d\} \right] \right] \right] + \right. \\ \left. \text{Sum} \left[e^{d1} \gamma_{d1, \varphi[[k]]} [k], \{k, 2n\}, \{d1, d\} \right] \right]$$

Out[*]=

$$e^{\epsilon} r_{1,-1} [2, 5] + e^{\epsilon} r_{1,-1} [4, 1] + e^{\epsilon} r_{1,-1} [6, 3] + e^{2\epsilon} r_{2,-1} [2, 5] + e^{2\epsilon} r_{2,-1} [4, 1] + e^{2\epsilon} r_{2,-1} [6, 3] + e^{\epsilon} \gamma_{1,-1} [4] + e^{\epsilon} \gamma_{1,0} [1] + e^{\epsilon} \gamma_{1,0} [2] + e^{\epsilon} \gamma_{1,0} [3] + e^{\epsilon} \gamma_{1,0} [5] + e^{\epsilon} \gamma_{1,0} [6]$$

```
In[*]:= pd2 = PPExpandedMold[
  Expand[F[{}, {}] × Normal@Series[pd1, {ε, 0, d}]] //.
  F[fs_, {es___}] × (f : (r | γ)ps_[is___])p- ⇒
  F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}]
]
```

Out[*]=

$$\begin{aligned}
 & F[{}, {}] + \epsilon F[\{r_{1,-1}[2, 5]\}, \{2, 5\}] + \epsilon F[\{r_{1,-1}[4, 1]\}, \{4, 1\}] + \\
 & \epsilon F[\{r_{1,-1}[6, 3]\}, \{6, 3\}] + \epsilon^2 F[\{r_{2,-1}[2, 5]\}, \{2, 5\}] + \epsilon^2 F[\{r_{2,-1}[4, 1]\}, \{4, 1\}] + \\
 & \epsilon^2 F[\{r_{2,-1}[6, 3]\}, \{6, 3\}] + \epsilon F[\{\gamma_{1,-1}[4]\}, \{4\}] + \epsilon F[\{\gamma_{1,0}[1]\}, \{1\}] + \\
 & \epsilon F[\{\gamma_{1,0}[2]\}, \{2\}] + \epsilon F[\{\gamma_{1,0}[3]\}, \{3\}] + \epsilon F[\{\gamma_{1,0}[5]\}, \{5\}] + \epsilon F[\{\gamma_{1,0}[6]\}, \{6\}] + \\
 & \epsilon^2 F[\{\gamma_{2,-1}[4]\}, \{4\}] + \epsilon^2 F[\{\gamma_{2,0}[1]\}, \{1\}] + \epsilon^2 F[\{\gamma_{2,0}[2]\}, \{2\}] + \epsilon^2 F[\{\gamma_{2,0}[3]\}, \{3\}] + \\
 & \epsilon^2 F[\{\gamma_{2,0}[5]\}, \{5\}] + \epsilon^2 F[\{\gamma_{2,0}[6]\}, \{6\}] + \frac{1}{2} \epsilon^2 F[\{r_{1,-1}[2, 5], r_{1,-1}[2, 5]\}, \{2, 5\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[2, 5], r_{1,-1}[4, 1]\}, \{2, 5, 4, 1\}] + \epsilon^2 F[\{r_{1,-1}[2, 5], r_{1,-1}[6, 3]\}, \{2, 5, 6, 3\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,-1}[4]\}, \{2, 5, 4\}] + \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,0}[1]\}, \{2, 5, 1\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,0}[2]\}, \{2, 5\}] + \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,0}[3]\}, \{2, 5, 3\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,0}[5]\}, \{2, 5\}] + \epsilon^2 F[\{r_{1,-1}[2, 5], \gamma_{1,0}[6]\}, \{2, 5, 6\}] + \\
 & \frac{1}{2} \epsilon^2 F[\{r_{1,-1}[4, 1], r_{1,-1}[4, 1]\}, \{4, 1\}] + \epsilon^2 F[\{r_{1,-1}[4, 1], r_{1,-1}[6, 3]\}, \{4, 1, 6, 3\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,-1}[4]\}, \{4, 1\}] + \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,0}[1]\}, \{4, 1\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,0}[2]\}, \{4, 1, 2\}] + \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,0}[3]\}, \{4, 1, 3\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,0}[5]\}, \{4, 1, 5\}] + \epsilon^2 F[\{r_{1,-1}[4, 1], \gamma_{1,0}[6]\}, \{4, 1, 6\}] + \\
 & \frac{1}{2} \epsilon^2 F[\{r_{1,-1}[6, 3], r_{1,-1}[6, 3]\}, \{6, 3\}] + \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,-1}[4]\}, \{6, 3, 4\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,0}[1]\}, \{6, 3, 1\}] + \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,0}[2]\}, \{6, 3, 2\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,0}[3]\}, \{6, 3\}] + \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,0}[5]\}, \{6, 3, 5\}] + \\
 & \epsilon^2 F[\{r_{1,-1}[6, 3], \gamma_{1,0}[6]\}, \{6, 3\}] + \frac{1}{2} \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,-1}[4]\}, \{4\}] + \\
 & \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,0}[1]\}, \{4, 1\}] + \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,0}[2]\}, \{4, 2\}] + \\
 & \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,0}[3]\}, \{4, 3\}] + \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,0}[5]\}, \{4, 5\}] + \\
 & \epsilon^2 F[\{\gamma_{1,-1}[4], \gamma_{1,0}[6]\}, \{4, 6\}] + \frac{1}{2} \epsilon^2 F[\{\gamma_{1,0}[1], \gamma_{1,0}[1]\}, \{1\}] + \\
 & \epsilon^2 F[\{\gamma_{1,0}[1], \gamma_{1,0}[2]\}, \{1, 2\}] + \epsilon^2 F[\{\gamma_{1,0}[1], \gamma_{1,0}[3]\}, \{1, 3\}] + \\
 & \epsilon^2 F[\{\gamma_{1,0}[1], \gamma_{1,0}[5]\}, \{1, 5\}] + \epsilon^2 F[\{\gamma_{1,0}[1], \gamma_{1,0}[6]\}, \{1, 6\}] + \\
 & \frac{1}{2} \epsilon^2 F[\{\gamma_{1,0}[2], \gamma_{1,0}[2]\}, \{2\}] + \epsilon^2 F[\{\gamma_{1,0}[2], \gamma_{1,0}[3]\}, \{2, 3\}] + \\
 & \epsilon^2 F[\{\gamma_{1,0}[2], \gamma_{1,0}[5]\}, \{2, 5\}] + \epsilon^2 F[\{\gamma_{1,0}[2], \gamma_{1,0}[6]\}, \{2, 6\}] + \\
 & \frac{1}{2} \epsilon^2 F[\{\gamma_{1,0}[3], \gamma_{1,0}[3]\}, \{3\}] + \epsilon^2 F[\{\gamma_{1,0}[3], \gamma_{1,0}[5]\}, \{3, 5\}] + \\
 & \epsilon^2 F[\{\gamma_{1,0}[3], \gamma_{1,0}[6]\}, \{3, 6\}] + \frac{1}{2} \epsilon^2 F[\{\gamma_{1,0}[5], \gamma_{1,0}[5]\}, \{5\}] + \\
 & \epsilon^2 F[\{\gamma_{1,0}[5], \gamma_{1,0}[6]\}, \{5, 6\}] + \frac{1}{2} \epsilon^2 F[\{\gamma_{1,0}[6], \gamma_{1,0}[6]\}, \{6\}]
 \end{aligned}$$

```

In[*]:= gPair[fs_, w_] := gPair[fs, w] = (
  Print["Running gPair on", {fs, w}];
  ZipJoin@@Table[{pα, p̄α, xα, x̄α}, {α, w}] [
    (Times@@ (V /@ fs)) Exp[Sum[gα,β (πα + π̄α) (ξβ + ξ̄β), {α, w}, {β, w}] - Sum[ξ̄α πα, {α, w}]]]
)

In[*]:= ρd3 = PPairing [
  ρd2 /. F[fs_, es_] := (gPair[
    Replace[fs, Thread[es → Range@Length@es], {2}],
    Length@es
  ] /. gα,β := ges[[α]], es[[β]])
]

```

```

Running gPair on{{}, 0}
Running gPair on{{r1,-1[1, 2]}, 2}
Running gPair on{{r2,-1[1, 2]}, 2}
Running gPair on{{γ1,-1[1]}, 1}
Running gPair on{{γ1,0[1]}, 1}
Running gPair on{{γ2,-1[1]}, 1}
Running gPair on{{γ2,0[1]}, 1}
Running gPair on{{r1,-1[1, 2], r1,-1[1, 2]}, 2}
Running gPair on{{r1,-1[1, 2], r1,-1[3, 4]}, 4}
Running gPair on{{r1,-1[1, 2], γ1,-1[3]}, 3}
Running gPair on{{r1,-1[1, 2], γ1,0[3]}, 3}
Running gPair on{{r1,-1[1, 2], γ1,0[1]}, 2}
Running gPair on{{r1,-1[1, 2], γ1,0[2]}, 2}
Running gPair on{{r1,-1[1, 2], γ1,-1[1]}, 2}
Running gPair on{{γ1,-1[1], γ1,-1[1]}, 1}
Running gPair on{{γ1,-1[1], γ1,0[2]}, 2}
Running gPair on{{γ1,0[1], γ1,0[1]}, 1}
Running gPair on{{γ1,0[1], γ1,0[2]}, 2}
    
```

Out[]=

$$\begin{aligned}
 & 1 + \dots 17 \dots + \epsilon^2 (\dots 1 \dots) + \\
 & \frac{1}{2} \epsilon^2 \left(\frac{1}{4} + g_{3,6} - 2 g_{3,3} g_{3,6} + 2 g_{3,6}^2 - \left(1 - \frac{1}{T}\right) g_{3,6}^2 - 12 g_{3,3} g_{3,6}^2 + 24 g_{3,3}^2 g_{3,6}^2 - \right. \\
 & \quad 6 \left(1 - \frac{1}{T}\right) g_{3,6}^3 + 24 \left(1 - \frac{1}{T}\right) g_{3,3} g_{3,6}^3 + 6 \left(1 - \frac{1}{T}\right)^2 g_{3,6}^4 + g_{3,6} g_{6,3} + 8 g_{3,6}^2 g_{6,3} - \\
 & \quad 24 g_{3,3} g_{3,6}^2 g_{6,3} + 12 \left(-1 + \frac{1}{T}\right) g_{3,6}^3 g_{6,3} + \dots 10 \dots + 6 \left(1 - \frac{1}{T}\right) g_{3,6}^2 g_{6,6} - \\
 & \quad 6 \left(-1 + \frac{1}{T}\right) g_{3,6}^2 g_{6,6} + 36 \left(-1 + \frac{1}{T}\right) g_{3,3} g_{3,6}^2 g_{6,6} + 12 \left(1 - \frac{1}{T}\right) \left(-1 + \frac{1}{T}\right) g_{3,6}^3 g_{6,6} - \\
 & \quad 8 g_{3,6} g_{6,3} g_{6,6} + 16 g_{3,3} g_{3,6} g_{6,3} g_{6,6} + 12 \left(1 - \frac{1}{T}\right) g_{3,6}^2 g_{6,3} g_{6,6} + 2 g_{6,6}^2 - 4 g_{3,3} g_{6,6}^2 + \\
 & \quad \left. 4 g_{3,3}^2 g_{6,6}^2 + 6 \left(-1 + \frac{1}{T}\right) g_{3,6} g_{6,6}^2 + 12 \left(1 - \frac{1}{T}\right) g_{3,3} g_{3,6} g_{6,6}^2 + 6 \left(-1 + \frac{1}{T}\right)^2 g_{3,6}^2 g_{6,6}^2 \right)
 \end{aligned}$$

Full expression not available (original memory size: 0.4 MB)

In[]:= **? gPair**

Out[]:=

Symbol
Global`gPair
Definitions
$\text{gPair}[\{\}, \emptyset] = 1$
$\text{gPair}[\{r_{1,-1}[1, 2]\}, 2] = \frac{1}{2} - g_{1,1} + g_{2,1} + \left(1 - \frac{1}{T}\right) g_{1,1} g_{2,1} + g_{1,2} g_{2,1} - \left(1 - \frac{1}{T}\right) g_{2,1}^2 + g_{1,1} g_{2,2} - 2 g_{2,1} g_{2,2}$
$\text{gPair}[\{\gamma_{1,-1}[1]\}, 1] = -\frac{1}{2} + g_{1,1}$
$\text{gPair}[\{\gamma_{1,\emptyset}[1]\}, 1] = 0$
$\text{gPair}[fs_ , w_] := \text{gPair}[fs, w] = (\text{Print}[\text{Running gPair on, } \{fs, w\}]; \text{ZipJoin}@\text{Table}[\{\{p_{\alpha}, \bar{p}_{\alpha}, x_{\alpha}, \bar{x}_{\alpha}\}, \{\alpha, w\}\} [\text{Times} @@ V / @ fs \text{Exp} [\sum_{\alpha}^w \sum_{\beta}^w g_{\alpha, \beta} (\pi_{\alpha} + \bar{\pi}_{\alpha}) (\xi_{\beta} + \bar{\xi}_{\beta}) - \sum_{\alpha}^w \bar{\xi}_{\alpha} \pi_{\alpha}]]])$
Full Name Global`gPair
^

In[]:= **{K = Knot[3, 1], d = 1}**

Out[]:=

{Knot[3, 1], 1}

In[]:= **PP"Green" [**
{Cs, φ} = Rot[K]; n = Length[Cs];
A = IdentityMatrix[2 n + 1];
Cases[Cs, {s_, i_, j_} => (A[[{i, j}, {i + 1, j + 1}]] += (
$$\begin{pmatrix} -T^S & T^S - 1 \\ 0 & -1 \end{pmatrix}$$
))];
G = Inverse[A];
]

In[]:= **ρd1 = PP_{Mold}@Exp[Total[Cases[Cs, {s_, i_, j_} => Sum[$e^{d1} r_{d1,s}[i, j]$, {d1, d}]]] + Sum[$e^{d1} \gamma_{d1, \emptyset[[k]]}[k]$, {k, 2 n}, {d1, d}]]]**

Out[]:=

$e^{\in r_{1,-1}[2,5]} + e^{\in r_{1,-1}[4,1]} + e^{\in r_{1,-1}[6,3]} + e^{\in \gamma_{1,-1}[4]} + e^{\in \gamma_{1,\emptyset}[1]} + e^{\in \gamma_{1,\emptyset}[2]} + e^{\in \gamma_{1,\emptyset}[3]} + e^{\in \gamma_{1,\emptyset}[5]} + e^{\in \gamma_{1,\emptyset}[6]}$

```
In[*]:= ρd2 = PPExpandedMold [
  Expand[F[{}, {}] × Normal@Series[ρd1, {ε, 0, d}]] //.
  F[fs_, {es___}] × (f : (r | γ)_{ps__}[is___])^{p-} :=
  F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}]
]
```

```
Out[*]:= F[{}, {}] + ε F[{r1,-1[2, 5]}, {2, 5}] + ε F[{r1,-1[4, 1]}, {4, 1}] +
ε F[{r1,-1[6, 3]}, {6, 3}] + ε F[{γ1,-1[4]}, {4}] + ε F[{γ1,0[1]}, {1}] +
ε F[{γ1,0[2]}, {2}] + ε F[{γ1,0[3]}, {3}] + ε F[{γ1,0[5]}, {5}] + ε F[{γ1,0[6]}, {6}]
```

```
In[*]:= ρd3 = Echo@PPpairing [
  ρd2 /. F[fs_, es_] := (gPair [
    Replace[fs, Thread[es → Range@Length@es], {2}],
    Length@es
  ] /. g_{α,β} := g_{es[[α], es[[β]]})
]
```

$$\begin{aligned} & \gg 1 + \epsilon \left(-\frac{1}{2} + g_{4,4} \right) + \epsilon \left(\frac{1}{2} + g_{1,4} - 2 g_{1,1} g_{1,4} - \left(1 - \frac{1}{T} \right) g_{1,4}^2 + g_{1,4} g_{4,1} - g_{4,4} + g_{1,1} g_{4,4} + \left(1 - \frac{1}{T} \right) g_{1,4} g_{4,4} \right) + \\ & \epsilon \left(\frac{1}{2} - g_{2,2} + g_{5,2} + \left(1 - \frac{1}{T} \right) g_{2,2} g_{5,2} + g_{2,5} g_{5,2} - \left(1 - \frac{1}{T} \right) g_{5,2}^2 + g_{2,2} g_{5,5} - 2 g_{5,2} g_{5,5} \right) + \\ & \epsilon \left(\frac{1}{2} + g_{3,6} - 2 g_{3,3} g_{3,6} - \left(1 - \frac{1}{T} \right) g_{3,6}^2 + g_{3,6} g_{6,3} - g_{6,6} + g_{3,3} g_{6,6} + \left(1 - \frac{1}{T} \right) g_{3,6} g_{6,6} \right) \end{aligned}$$

```
Out[*]:= 1 + ε \left( -\frac{1}{2} + g_{4,4} \right) + ε \left( \frac{1}{2} + g_{1,4} - 2 g_{1,1} g_{1,4} - \left( 1 - \frac{1}{T} \right) g_{1,4}^2 + g_{1,4} g_{4,1} - g_{4,4} + g_{1,1} g_{4,4} + \left( 1 - \frac{1}{T} \right) g_{1,4} g_{4,4} \right) +
ε \left( \frac{1}{2} - g_{2,2} + g_{5,2} + \left( 1 - \frac{1}{T} \right) g_{2,2} g_{5,2} + g_{2,5} g_{5,2} - \left( 1 - \frac{1}{T} \right) g_{5,2}^2 + g_{2,2} g_{5,5} - 2 g_{5,2} g_{5,5} \right) +
ε \left( \frac{1}{2} + g_{3,6} - 2 g_{3,3} g_{3,6} - \left( 1 - \frac{1}{T} \right) g_{3,6}^2 + g_{3,6} g_{6,3} - g_{6,6} + g_{3,3} g_{6,6} + \left( 1 - \frac{1}{T} \right) g_{3,6} g_{6,6} \right)
```

```
In[*]:= {Δ, ρd3 /. ε → ε Δ^2}
```

```
Out[*]:= {Δ, 1 + Δ^2 ε \left( -\frac{1}{2} + g_{4,4} \right) +
Δ^2 ε \left( \frac{1}{2} + g_{1,4} - 2 g_{1,1} g_{1,4} - \left( 1 - \frac{1}{T} \right) g_{1,4}^2 + g_{1,4} g_{4,1} - g_{4,4} + g_{1,1} g_{4,4} + \left( 1 - \frac{1}{T} \right) g_{1,4} g_{4,4} \right) +
Δ^2 ε \left( \frac{1}{2} - g_{2,2} + g_{5,2} + \left( 1 - \frac{1}{T} \right) g_{2,2} g_{5,2} + g_{2,5} g_{5,2} - \left( 1 - \frac{1}{T} \right) g_{5,2}^2 + g_{2,2} g_{5,5} - 2 g_{5,2} g_{5,5} \right) +
Δ^2 ε \left( \frac{1}{2} + g_{3,6} - 2 g_{3,3} g_{3,6} - \left( 1 - \frac{1}{T} \right) g_{3,6}^2 + g_{3,6} g_{6,3} - g_{6,6} + g_{3,3} g_{6,6} + \left( 1 - \frac{1}{T} \right) g_{3,6} g_{6,6} \right)}
```