Pensieve header: $\mathcal A$-calculus and the Hodge star. Continues Alpha.nb.

Γ-Calculus

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In[∘]:=
```
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_] := Module[{vs = Union@Cases[ℰ, (ξ | x)_, ∞]},
   Total[(CCF[#⟦2⟧] (Times @@ vs^#⟦1⟧)) & /@ CoefficientRules[ℰ, vs]]];
```

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In[∘]:=
```
Γ /: Γ[is1_, os1_, cs1_, ω1_, λ1_] Γ[is2_, os2_, cs2_, ω2_, λ2_] :=
  Γ[is1 ⋃ is2, os1 ⋃ os2, Join[cs1, cs2], ω1 ω2, λ1 + λ2]
Γ /: Γ[is1_, os1_, _, ω1_, λ1_] ≡ Γ[is2_, os2_, _, ω2_, λ2_] := TrueQ[
   (Sort@is1 === Sort@is2) ∧ (Sort@os1 === Sort@os2) ∧ Simplify[ω1 == ω2] ∧ CF@λ1 == CF@λ2]
```

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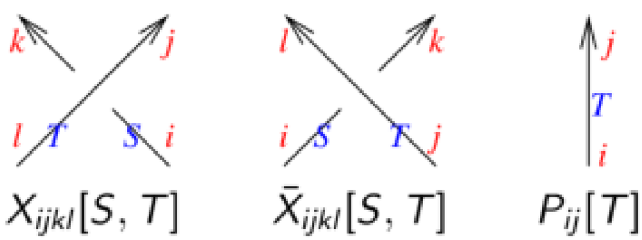
In[∘]:=
```
c_{h_,t_}@Γ[is_, os_, cs_, ω_, λ_] := Module[{α, η, y, μ},
   α = ∂_{ξ_t,x_h} λ; μ = λ /. ξ_t | x_h → 0;
   η = ∂_{x_h} λ /. ξ_t → 0; y = ∂_{ξ_t} λ /. x_h → 0;
   Γ[
     DeleteCases[is, t], DeleteCases[os, h], KeyDrop[cs, {x_h, ξ_t}],
     CCF[(1 - α) ω], CF[μ + η y / (1 - α)]
    ] /. If[MatchQ[cs[ξ_t], τ_], cs[ξ_t] → cs[x_h], cs[x_h] → cs[ξ_t]]];
c@Γ[is_, os_, cs_, ω_, λ_] := Fold[c_{#2,#2}[#1] &, Γ[is, os, cs, ω, λ], is ⋂ os]
```



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In[∘]:=
```
Γ[X_{i_,j_,k_,l_}[S_, T_]] := Γ[{l, i}, {j, k},
   ⟨|ξ_i → S, x_j → T, x_k → S, ξ_l → T|⟩, S^(-1/2), CF[{ξ_l, ξ_i}.(S  1-S
                                                                  0  1  ).{x_j, x_k}]];
Γ[X̄_{i_,j_,k_,l_}[S_, T_]] := Γ[{i, j}, {k, l},
   ⟨|ξ_i → S, ξ_j → T, x_k → S, x_l → T|⟩, S^(1/2), CF[{ξ_i, ξ_j}.(1        0
                                                                 1 - S^-1  S^-1).{x_k, x_l}]];
Γ[X_{i_,j_,k_,l_}] := Γ[X_{i,j,k,l}[τ_i, τ_l]];
Γ[X̄_{i_,j_,k_,l_}] := Γ[X̄_{i,j,k,l}[τ_i, τ_j]];
```

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In[∘]:=
```
Γ[P_{i_,j_}[T_]] := Γ[{i}, {j}, ⟨|ξ_i → T, x_j → T|⟩, 1, ξ_i x_j];
Γ[P_{i_,j_}] := Γ[P_{i,j}[τ_i]];
```

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In[∘]:=
```
Γ[{γ_Γ}] := c[γ];
Γ[{γ1_Γ, γs__Γ}] := Module[{γ2},
  γ2 = First@MaximalBy[{γs}, Length[γ1⟦1⟧ ⋂ #⟦2⟧] + Length[γ1⟦2⟧ ⋂ #⟦1⟧] &];
  Γ[Join[{c[γ1 γ2]}, DeleteCases[{γs}, γ2]]] ]
Γ[Os_List] := Γ[Γ /@ Os]
```

In[∘]:=
```
With[{n = 3},
 γ0 = Γ[{1, 2, 3}, {1, 2, 3}, ⟨|ξ_1 → T_1, x_1 → T_1, ξ_2 → T_2, x_2 → T_2, ξ_3 → T_3, x_3 → T_3|⟩,
    ω, Sum[Sum[ξ_a x_b α_{ab}, {b, 1, n}], {a, 1, n}]]]
γ0 // c_{1,2}
```

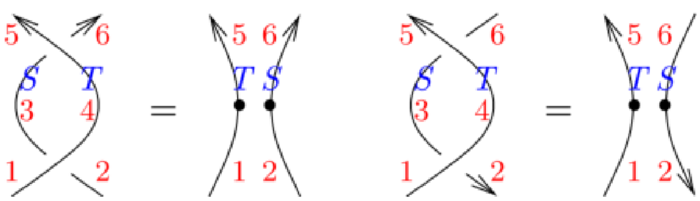
Out[∘]=

$$\Gamma[\{1,2,3\},\{1,2,3\},\langle|\xi_1\to T_1,\ x_1\to T_1,\ \xi_2\to T_2,\ x_2\to T_2,\ \xi_3\to T_3,\ x_3\to T_3|\rangle,\ \omega,$$
$$x_1\xi_1\alpha_{1,1}+x_2\xi_1\alpha_{1,2}+x_3\xi_1\alpha_{1,3}+x_1\xi_2\alpha_{2,1}+x_2\xi_2\alpha_{2,2}+x_3\xi_2\alpha_{2,3}+x_1\xi_3\alpha_{3,1}+x_2\xi_3\alpha_{3,2}+x_3\xi_3\alpha_{3,3}]$$

Out[∘]=

$$\Gamma\Big[\{1,3\},\{2,3\},\langle|\xi_1\to T_2,\ x_2\to T_2,\ \xi_3\to T_3,\ x_3\to T_3|\rangle,\ -\omega\,(-1+\alpha_{2,1}),$$
$$\frac{x_2\,\xi_1\,(-\alpha_{1,2}+\alpha_{1,2}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,2})}{-1+\alpha_{2,1}}+\frac{x_3\,\xi_1\,(-\alpha_{1,3}+\alpha_{1,3}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,3})}{-1+\alpha_{2,1}}+$$
$$\frac{x_2\,\xi_3\,(-\alpha_{2,2}\alpha_{3,1}-\alpha_{3,2}+\alpha_{2,1}\alpha_{3,2})}{-1+\alpha_{2,1}}+\frac{x_3\,\xi_3\,(-\alpha_{2,3}\alpha_{3,1}-\alpha_{3,3}+\alpha_{2,1}\alpha_{3,3})}{-1+\alpha_{2,1}}\Big]$$

Reidemeister 2



In[∘]:=
```
{Γ@{X_{2,4,3,1}[S, T], X̄_{3,4,6,5}} ≡ Γ@{P_{1,5}[T], P_{2,6}[S]},
 Γ@{X̄_{3,1,2,4}[S, T], X_{6,5,3,4}} ≡ Γ@{P_{1,5}[T], P_{6,2}[S]}}
```

Out[∘]=

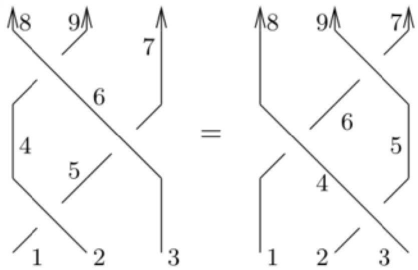
```
{True, True}
```

In[∘]:=
```
{Γ@{X_{2,4,3,1}[T, T], X̄_{3,4,6,5}} ≡ Γ@{P_{1,5}[T], P_{2,6}[T]},
 Γ@{X̄_{3,1,2,4}[T, T], X_{6,5,3,4}} ≡ Γ@{P_{1,5}[T], P_{6,2}[T]}}
```

Out[∘]=

```
{True, True}
```

Reidemeister 3



In[∘]:=
```
Γ@{X2,5,4,1[T2, T1], X3,7,6,5[T3, T1], X6,9,8,4} ≡ Γ@{X3,5,4,2[T3, T2], X4,6,8,1[T3, T1], X5,7,9,6}
```
Out[∘]=
```
True
```

In[∘]:=
```
Γ@{X2,5,4,1[T, T], X3,7,6,5[T, T], X6,9,8,4} ≡ Γ@{X3,5,4,2[T, T], X4,6,8,1[T, T], X5,7,9,6}
```
Out[∘]=
```
True
```

𝒜-Calculus

In[∘]:=
```
WP[Wedge[u___], Wedge[v___]] := Signature[{u, v}] * Wedge @@ Sort[{u, v}];
WP[0, _] = WP[_, 0] = 0;
WP[A_, B_] := Expand[Distribute[Expand[A] ** Expand[B]] /.
    (a_. * u_Wedge) ** (b_. * v_Wedge) :→ a b WP[u, v]];
```

In[∘]:=
```
WP[Wedge[] + Wedge[a] - 2 b ∧ a, Wedge[] - 3 Wedge[b] + 7 c ∧ d]
```
Out[∘]=
```
Wedge[] + Wedge[a] - 3 Wedge[b] - a ∧ b + 7 c ∧ d + 7 a ∧ c ∧ d + 14 a ∧ b ∧ c ∧ d
```

In[∘]:=
```
WExp[A_] := Module[{s = Wedge[], t = Wedge[], k = 0},
   While[t =!= 0, s += (t = Expand[WP[t, A] / (++k)])]; s];
WExpQ[B_] := Module[{ω, A},
   ω = Coefficient[B, Wedge[]];
   A = B /. w_Wedge /; Length[w] ≠ 2 :→ 0;
   Expand[B == ω WExp[A / ω]]
  ] // Simplify
```

In[∘]:=
```
WExp[a ∧ b + c ∧ d + e ∧ f]
```
Out[∘]=
```
Wedge[] + a ∧ b + c ∧ d + e ∧ f + a ∧ b ∧ c ∧ d + a ∧ b ∧ e ∧ f + c ∧ d ∧ e ∧ f + a ∧ b ∧ c ∧ d ∧ e ∧ f
```

In[∘]:=
```
c_{x_,y_}[w_Wedge] := Module[{i, j},
  {i} = FirstPosition[w, x, {0}]; {j} = FirstPosition[w, y, {0}];
  { w                                          (i == 0) ∧ (j == 0)
  { (-1)^(i+j+If[i>j,0,1]) Delete[w, {{i}, {j}}]   (i > 0) ∧ (j > 0)
];
c_{x_,y_}[ℰ_] := ℰ /. w_Wedge :> c_{x,y}[w]
```

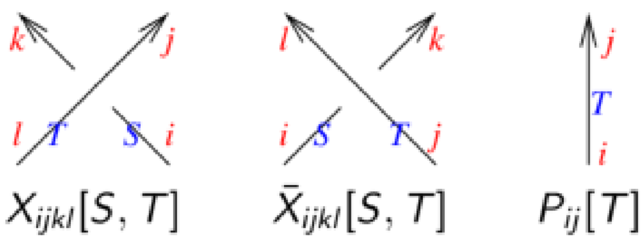
In[∘]:=
```
WExp[a ∧ b + 2 c ∧ d]
c_{d,c}@WExp[a ∧ b + 2 c ∧ d]
```

Out[∘]=

$\text{Wedge}[\,] + a \wedge b + 2\, c \wedge d + 2\, a \wedge b \wedge c \wedge d$

Out[∘]=

$-\text{Wedge}[\,] - a \wedge b$



In[∘]:=
```
𝒜[X_{i_,j_,k_,l_}[S_, T_]] := 𝒜[{l, i}, {j, k}, ⟨|ξ_i → S, x_j → T, x_k → S, ξ_l → T|⟩,
   Expand[S^(-1/2) WExp[Expand[{ξ_l, ξ_i}.( S  1-S
                                            0   1  ).{x_j, x_k}] /. ξ_a_ x_b_ :> ξ_a ∧ x_b]]];
𝒜[X_{i_,j_,k_,l_}] := 𝒜[X_{i,j,k,l}[τ_i, τ_l]];
```

In[∘]:=
```
𝒜[X̄_{i_,j_,k_,l_}[S_, T_]] := 𝒜[{i, j}, {k, l}, ⟨|ξ_i → S, ξ_j → T, x_k → S, x_l → T|⟩,
   Expand[S^(1/2) WExp[Expand[{ξ_i, ξ_j}.(   1      0
                                          1-S^-1  S^-1 ).{x_k, x_l}] /. ξ_a_ x_b_ :> ξ_a ∧ x_b]]];
𝒜[X̄_{i_,j_,k_,l_}] := 𝒜[X̄_{i,j,k,l}[τ_i, τ_j]];
```

In[∘]:=
```
𝒜[P_{i_,j_}[T_]] := 𝒜[{i}, {j}, ⟨|ξ_i → T, x_j → T|⟩, WExp[ξ_i ∧ x_j]];
𝒜[P_{i_,j_}] := 𝒜[P_{i,j}[τ_i]]
```

In[∘]:=
```
𝒜[X_{1,2,3,4}[u, v]]
```

Out[∘]=

$$\mathcal{A}\Big[\{4, 1\}, \{2, 3\}, \langle| \xi_1 \to u, x_2 \to v, x_3 \to u, \xi_4 \to v |\rangle,\ \frac{\text{Wedge}[\,]}{\sqrt{u}} - \sqrt{u}\, x_2 \wedge \xi_4 - \frac{x_3 \wedge \xi_1}{\sqrt{u}} - \frac{x_3 \wedge \xi_4}{\sqrt{u}} + \sqrt{u}\, x_3 \wedge \xi_4 + \sqrt{u}\, x_2 \wedge x_3 \wedge \xi_1 \wedge \xi_4\Big]$$

Linearity:

In[∘]:=
```
𝒜 /: α_ × 𝒜[is_, os_, cs_, w_] := 𝒜[is, os, cs, Expand[α w]]
𝒜 /: 𝒜[is1_, os1_, cs1_, w1_] + 𝒜[is2_, os2_, cs2_, w2_] /;
  (Sort@is1 == Sort@is2) ∧ (Sort@os1 == Sort@os2) ∧
   (Sort@Normal@cs1 == Sort@Normal@cs2) := 𝒜[is1, os1, cs1, w1 + w2]
```

Deciding if two $\calA$’s are equal:

In[∘]:=
```
𝒜 /: 𝒜[is1_, os1_, _, w1_] ≡ 𝒜[is2_, os2_, _, w2_] :=
 TrueQ[(Sort@is1 === Sort@is2) ∧ (Sort@os1 === Sort@os2) ∧ PowerExpand[w1 == w2]]
```

Disjoint unions:

In[∘]:=
```
𝒜 /: 𝒜[is1_, os1_, cs1_, w1_] 𝒜[is2_, os2_, cs2_, w2_] :=
 𝒜[is1 ⋃ is2, os1 ⋃ os2, Join[cs1, cs2], WP[w1, w2]]
```

In[∘]:= Short$\left[\mathcal{A}[X_{2,4,3,1}[S, T]]\ \mathcal{A}[\overline{X}_{3,4,6,5}], 5\right]$

Out[∘]//Short=

$$\mathcal{A}\Big[\{1, 2, 3, 4\}, \{3, 4, 5, 6\}, \langle| \xi_2 \to S, x_4 \to T, x_3 \to S, \xi_1 \to T, \xi_3 \to \tau_3, \xi_4 \to \tau_4, x_6 \to \tau_3, x_5 \to \tau_4 |\rangle,$$
$$\frac{\sqrt{\tau_3}\ \text{Wedge}[]}{\sqrt{S}} - \frac{\sqrt{\tau_3}\ x_3 \wedge \xi_1}{\sqrt{S}} + \ll 49 \gg + \frac{\sqrt{S}\ x_3 \wedge x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_3 \wedge \xi_4}{\sqrt{\tau_3}} -$$
$$\frac{x_3 \wedge x_5 \wedge x_6 \wedge \xi_2 \wedge \xi_3 \wedge \xi_4}{\sqrt{S}\ \sqrt{\tau_3}} - \frac{\sqrt{S}\ x_4 \wedge x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_3 \wedge \xi_4}{\sqrt{\tau_3}} + \frac{\sqrt{S}\ x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3 \wedge \xi_4}{\sqrt{\tau_3}}\Big]$$

In[∘]:=
```
c_{h_,t_}@𝒜[is_, os_, cs_, w_] := 𝒜[
   DeleteCases[is, t], DeleteCases[os, h], KeyDrop[cs, {x_h, ξ_t}], c_{x_h,ξ_t}[w]
  ] /. If[MatchQ[cs[ξ_t], τ_], cs[ξ_t] → cs[x_h], cs[x_h] → cs[ξ_t]];
```

In[∘]:= $c_{4,4}\left[\mathcal{A}[X_{2,4,3,1}[S, T]]\ \mathcal{A}[\overline{X}_{3,4,6,5}]\right]$

Out[∘]=

$$\mathcal{A}\Big[\{1, 2, 3\}, \{3, 5, 6\}, \langle| \xi_2 \to S, x_3 \to S, \xi_1 \to T, \xi_3 \to \tau_3, x_6 \to \tau_3, x_5 \to T |\rangle,$$
$$\frac{\sqrt{\tau_3}\ \text{Wedge}[]}{\sqrt{S}} - \frac{\sqrt{\tau_3}\ x_3 \wedge \xi_1}{\sqrt{S}} + \sqrt{S}\ \sqrt{\tau_3}\ x_3 \wedge \xi_1 - \frac{\sqrt{\tau_3}\ x_3 \wedge \xi_2}{\sqrt{S}} - \frac{\sqrt{S}\ x_5 \wedge \xi_1}{\sqrt{\tau_3}} +$$
$$\frac{\sqrt{S}\ x_6 \wedge \xi_1}{\sqrt{\tau_3}} - \sqrt{S}\ \sqrt{\tau_3}\ x_6 \wedge \xi_1 - \frac{\sqrt{\tau_3}\ x_6 \wedge \xi_3}{\sqrt{S}} + \frac{\sqrt{S}\ x_3 \wedge x_5 \wedge \xi_1 \wedge \xi_2}{\sqrt{\tau_3}} - \frac{\sqrt{S}\ x_3 \wedge x_6 \wedge \xi_1 \wedge \xi_2}{\sqrt{\tau_3}} +$$
$$\sqrt{S}\ \sqrt{\tau_3}\ x_3 \wedge x_6 \wedge \xi_1 \wedge \xi_2 - \frac{\sqrt{\tau_3}\ x_3 \wedge x_6 \wedge \xi_1 \wedge \xi_3}{\sqrt{S}} + \sqrt{S}\ \sqrt{\tau_3}\ x_3 \wedge x_6 \wedge \xi_1 \wedge \xi_3 -$$
$$\frac{\sqrt{\tau_3}\ x_3 \wedge x_6 \wedge \xi_2 \wedge \xi_3}{\sqrt{S}} - \frac{\sqrt{S}\ x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_3}{\sqrt{\tau_3}} - \frac{\sqrt{S}\ x_3 \wedge x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3}{\sqrt{\tau_3}}\Big]$$

In[∘]:=
```
c@𝒜[is_, os_, cs_, w_] := Fold[c#2,#2[#1] &, 𝒜[is, os, cs, w], is ⋂ os]
𝒜[{A_𝒜}] := c[A];
𝒜[{A1_𝒜, As__𝒜}] := Module[{A2},
  A2 = First@MaximalBy[{As}, Length[A1⟦1⟧ ⋂ #⟦2⟧] + Length[A1⟦2⟧ ⋂ #⟦1⟧] &];
  𝒜[Join[{c[A1 A2]}, DeleteCases[{As}, A2]]] ]
𝒜[𝒪s_List] := 𝒜[𝒜 /@ 𝒪s]
```

In[∘]:=
```
c[𝒜[X2,4,3,1[S, T]] 𝒜[X̄3,4,6,5]]
```

Out[∘]=

$\mathcal{A}[\{1,2\},\{5,6\},\langle|\xi_2\to S,\ \xi_1\to T,\ x_6\to S,\ x_5\to T|\rangle,\ \mathrm{Wedge}[\,]-x_5\wedge\xi_1-x_6\wedge\xi_2-x_5\wedge x_6\wedge\xi_1\wedge\xi_2]$

In[∘]:=
```
𝒜@{𝒜[X2,4,3,1[S, T]], 𝒜[X̄3,4,6,5]}
```

Out[∘]=

$\mathcal{A}[\{1,2\},\{5,6\},\langle|\xi_2\to S,\ \xi_1\to T,\ x_6\to S,\ x_5\to T|\rangle,\ \mathrm{Wedge}[\,]-x_5\wedge\xi_1-x_6\wedge\xi_2-x_5\wedge x_6\wedge\xi_1\wedge\xi_2]$

Γ ↔ 𝒜 Conversions

pdf

In[∘]:=
```
Γ@𝒜[is_, os_, cs_, w_] := Module[{i, j, ω = Coefficient[w, Wedge[]]},
   Γ[is, os, cs, ω, Sum[Cancel[-Coefficient[w, xj ∧ ξi] ξi xj / ω], {i, is}, {j, os}]]
  ];
𝒜@Γ[is_, os_, cs_, ω_, λ_] := 𝒜[is, os, cs, Expand[ω WExp[Expand[λ] /. ξa_ xb_ :→ ξa ∧ xb]]];
```

tex

The conversions are inverses of each other:

pdf

In[∘]:=
```
γ = Γ[{1, 2, 3}, {1, 2, 3}, {x1 → τ1, x2 → τ2, x3 → τ3, ξ1 → τ1, ξ2 → τ2, ξ3 → τ3}, ω,
   a11 x1 ξ1 + a12 x2 ξ1 + a13 x3 ξ1 + a21 x1 ξ2 + a22 x2 ξ2 + a23 x3 ξ2 + a31 x1 ξ3 + a32 x2 ξ3 + a33 x3 ξ3]
Γ@𝒜@γ == γ
```

Out[∘]=
pdf

$\Gamma[\{1,2,3\},\{1,2,3\},\{x_1\to\tau_1,\ x_2\to\tau_2,\ x_3\to\tau_3,\ \xi_1\to\tau_1,\ \xi_2\to\tau_2,\ \xi_3\to\tau_3\},\ \omega,$
$a_{11}x_1\xi_1+a_{12}x_2\xi_1+a_{13}x_3\xi_1+a_{21}x_1\xi_2+a_{22}x_2\xi_2+a_{23}x_3\xi_2+a_{31}x_1\xi_3+a_{32}x_2\xi_3+a_{33}x_3\xi_3]$

Out[∘]=
pdf

True

tex

The conversions commute with contractions:

pdf

In[∘]:=
```
Γ@c3,3@𝒜@γ ≡ c3,3@γ
```

Out[∘]=
pdf

True

tex

Compatibility on the generators:

In[∘]:= $\Gamma[X_{i,j,k,l}]$

Out[∘]=

$$\Gamma\left[\{l, i\}, \{j, k\}, \langle| \xi_i \to \tau_i, x_j \to \tau_l, x_k \to \tau_i, \xi_l \to \tau_l |\rangle, \frac{1}{\sqrt{\tau_i}}, x_k \xi_i + x_k \xi_l (1 - \tau_i) + x_j \xi_l \tau_i\right]$$

In[∘]:= $\mathcal{A}[X_{i,j,k,l}]$

Out[∘]=

$$\mathcal{A}\left[\{l, i\}, \{j, k\}, \langle| \xi_i \to \tau_i, x_j \to \tau_l, x_k \to \tau_i, \xi_l \to \tau_l |\rangle, \frac{\text{Wedge}[]}{\sqrt{\tau_i}} - \sqrt{\tau_i}\, x_j \wedge \xi_l - \frac{x_k \wedge \xi_i}{\sqrt{\tau_i}} - \frac{x_k \wedge \xi_l}{\sqrt{\tau_i}} + \sqrt{\tau_i}\, x_k \wedge \xi_l + \sqrt{\tau_i}\, x_j \wedge x_k \wedge \xi_i \wedge \xi_l\right]$$

In[∘]:= $\mathcal{A}@\Gamma[X_{i,j,k,l}]$

Out[∘]=

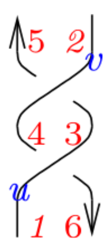
$$\mathcal{A}\left[\{l, i\}, \{j, k\}, \langle| \xi_i \to \tau_i, x_j \to \tau_l, x_k \to \tau_i, \xi_l \to \tau_l |\rangle, \frac{\text{Wedge}[]}{\sqrt{\tau_i}} - \sqrt{\tau_i}\, x_j \wedge \xi_l - \frac{x_k \wedge \xi_i}{\sqrt{\tau_i}} - \frac{x_k \wedge \xi_l}{\sqrt{\tau_i}} + \sqrt{\tau_i}\, x_k \wedge \xi_l + \sqrt{\tau_i}\, x_j \wedge x_k \wedge \xi_i \wedge \xi_l\right]$$

In[∘]:= $\mathcal{A}[X_{i,j,k,l}] \equiv \mathcal{A}@\Gamma[X_{i,j,k,l}]$

Out[∘]=

True

Skein Relations

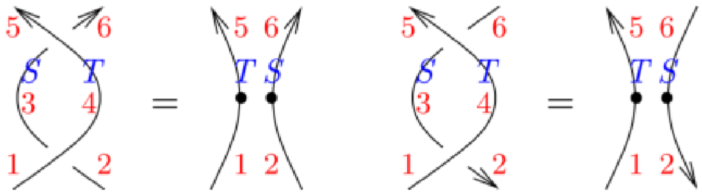


In[∘]:= $\mathcal{A}@\{\overline{X}_{4,1,6,3}[v, u], \overline{X}_{3,2,5,4}\}$

Out[∘]=

$$\mathcal{A}\left[\{1, 2\}, \{5, 6\}, \langle| \xi_2 \to v, x_5 \to u, \xi_1 \to u, x_6 \to v |\rangle, \sqrt{u}\,\sqrt{v}\,\text{Wedge}[] - \frac{\sqrt{u}\, x_5 \wedge \xi_1}{\sqrt{v}} + \frac{\sqrt{v}\, x_5 \wedge \xi_2}{\sqrt{u}} - \sqrt{u}\,\sqrt{v}\, x_5 \wedge \xi_2 + \frac{\sqrt{u}\, x_6 \wedge \xi_1}{\sqrt{v}} - \sqrt{u}\,\sqrt{v}\, x_6 \wedge \xi_1 - \frac{\sqrt{v}\, x_6 \wedge \xi_2}{\sqrt{u}} - \frac{\sqrt{u}\, x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_2}{\sqrt{v}} - \frac{\sqrt{v}\, x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_2}{\sqrt{u}} + \sqrt{u}\,\sqrt{v}\, x_5 \wedge x_6 \wedge \xi_1 \wedge \xi_2\right]$$

Reidemeister 2

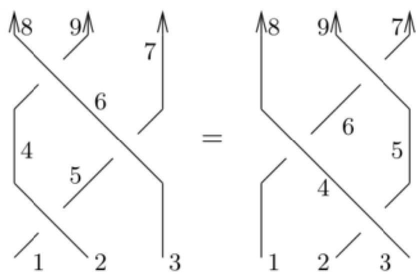


In[∘]:= $\{\mathcal{A}@\{X_{2,4,3,1}[S, T], \overline{X}_{3,4,6,5}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{2,6}[S]\},$
$\mathcal{A}@\{\overline{X}_{3,1,2,4}[S, T], X_{6,5,3,4}\} \equiv \mathcal{A}@\{P_{1,5}[T], P_{6,2}[S]\}\}$

Out[∘]=

{True, True}

Reidemeister 3

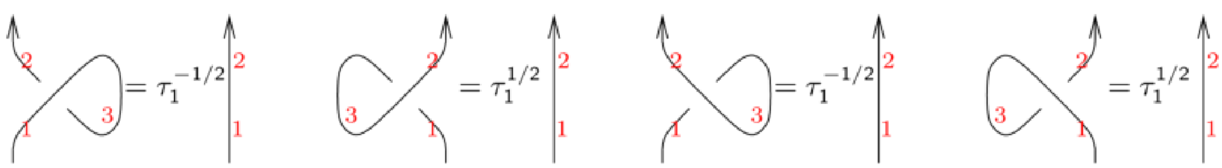


In[∘]:= $\mathcal{A}@\{X_{2,5,4,1}[T_2, T_1], X_{3,7,6,5}[T_3, T_1], X_{6,9,8,4}\} \equiv \mathcal{A}@\{X_{3,5,4,2}[T_3, T_2], X_{4,6,8,1}[T_3, T_1], X_{5,7,9,6}\}$

Out[∘]=

True

Reidemeister 1

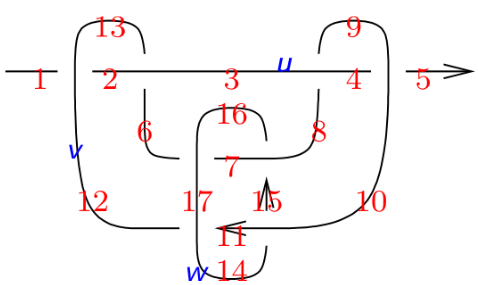


In[∘]:= $\{\mathcal{A}@\{X_{3,3,2,1}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{X_{1,2,3,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\},$
$\mathcal{A}@\{\overline{X}_{1,3,3,2}\} \equiv \tau_1^{-1/2}\,\mathcal{A}@\{P_{1,2}\},\ \mathcal{A}@\{\overline{X}_{3,1,2,3}\} \equiv \tau_1^{1/2}\,\mathcal{A}@\{P_{1,2}\}\}$

Out[∘]=

{True, True, True, True}

The Relation with the Multivariable Alexander Polynomial



In[ ]:= $\text{MVA} = u^{-1/2}\, v^{-1/2}\, w^{-1/2}\, (u-1)\, (v-1)\, (w-1);$

In[ ]:= $A = \{\overline{X}_{1,12,2,13}[u, v], \overline{X}_{13,2,6,3}, X_{8,4,9,3}, X_{4,10,5,9}, X_{6,17,7,16}[v, w], X_{15,8,16,7}, \overline{X}_{14,10,15,11}, \overline{X}_{11,17,12,14}\}$ // $\mathcal{A}$ // Last // Factor

Out[ ]=

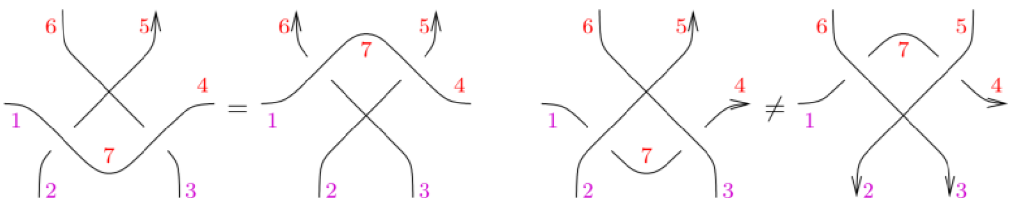
$$\frac{(-1+u)^2\,(-1+v)\,(-1+w)\,(\text{Wedge}[\,] - x_5 \wedge \xi_1)}{u\, v}$$

In[ ]:= $A == u^{-1/2}\,(u-1)\,u^0\, v^{-1/2}\, w^{1/2}\,\text{MVA}\,(\text{Wedge}[\,] - x_5 \wedge \xi_1)$

Out[ ]=

True

Overcrossings don't commute but undercrossings do

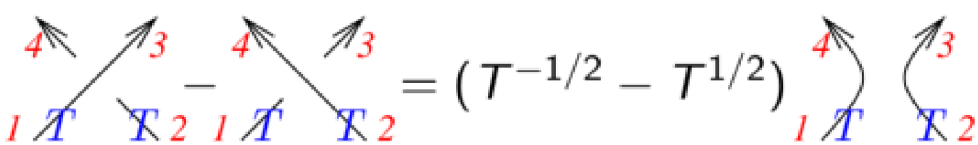


In[ ]:= $\{\mathcal{A}@\{X_{2,7,5,1}, X_{3,4,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,1}, X_{2,4,5,7}\},\ \mathcal{A}@\{\overline{X}_{1,2,7,5}, \overline{X}_{7,3,4,6}\} \equiv \mathcal{A}@\{\overline{X}_{1,3,7,6}, \overline{X}_{7,2,4,5}\}\}$

Out[ ]=

{False, True}

The Conway Relation

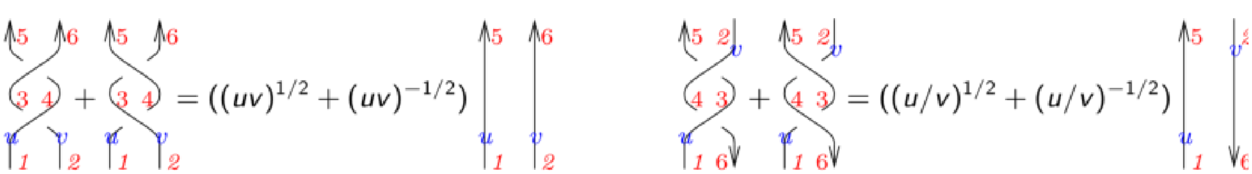


In[ ]:= $\mathcal{A}@\{X_{2,3,4,1}[T, T]\} - \mathcal{A}@\{\overline{X}_{1,2,3,4}[T, T]\} \equiv (T^{-1/2} - T^{1/2})\ \mathcal{A}@\{P_{1,4}[T], P_{2,3}[T]\}$

Out[ ]=

True

Conway's Second Set of Identities

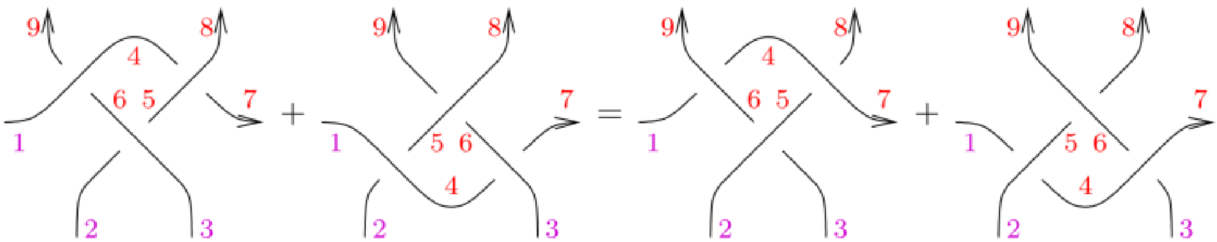


In[∘]:= $\{\mathcal{A}@\{X_{2,4,3,1}[v,u], X_{4,6,5,3}\} + \mathcal{A}@\{\overline{X}_{1,2,4,3}[u,v], \overline{X}_{3,4,6,5}\} \equiv (u^{1/2} v^{1/2} + u^{-1/2} v^{-1/2})\, \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\},$
$\mathcal{A}@\{\overline{X}_{4,1,6,3}[v,u], \overline{X}_{3,2,5,4}\} + \mathcal{A}@\{X_{1,6,3,4}[u,v], X_{2,5,4,3}\} \equiv (u^{1/2} v^{-1/2} + u^{-1/2} v^{1/2})\, \mathcal{A}@\{P_{1,5}[u], P_{2,6}[v]\}\}$

Out[∘]=

{True, True}

Conway’s Third Identity

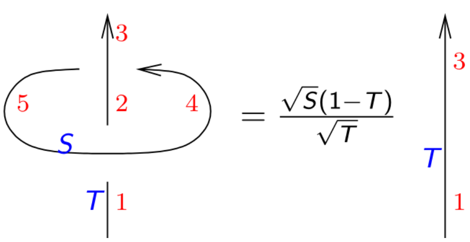


In[∘]:= $\mathcal{A}@\{X_{6,4,9,1}, \overline{X}_{4,5,7,8}, \overline{X}_{2,3,5,6}\} + \mathcal{A}@\{X_{2,4,5,1}, \overline{X}_{4,3,7,6}, X_{6,8,9,5}\} \equiv$
$\mathcal{A}@\{\overline{X}_{1,6,4,9}, X_{5,7,8,4}, X_{3,5,6,2}\} + \mathcal{A}@\{\overline{X}_{1,2,4,5}, X_{3,7,6,4}, \overline{X}_{5,6,8,9}\}$

Out[∘]=

True

Jun Murakami’s Fifth Axiom

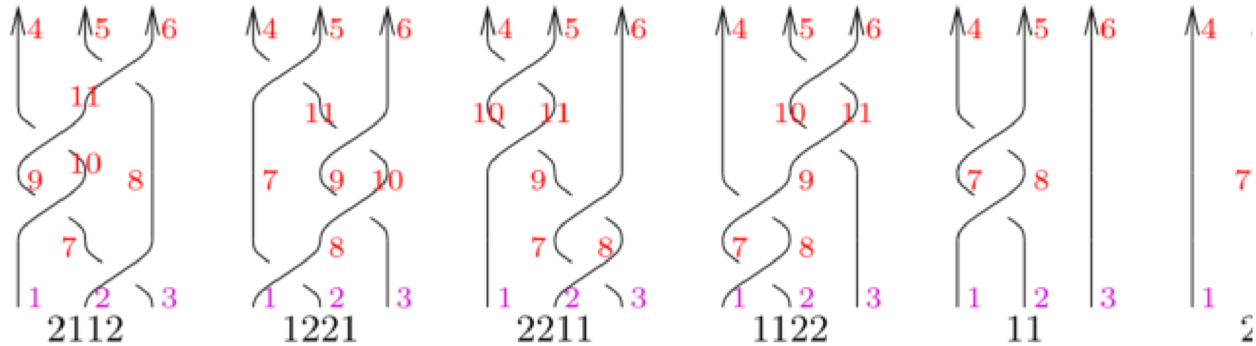


In[∘]:= $\mathcal{A}@\{X_{1,4,2,5}[T,S], X_{4,3,5,2}\} \equiv \frac{\sqrt{S}\;(1-T)}{\sqrt{T}}\, \mathcal{A}@\{P_{1,3}[T]\}$

Out[∘]=

True

Jun Murakami’s Third Axiom



In[∘]:=

$\mathcal{A}_{2112} = \mathcal{A}@\{X_{3,8,7,2},\ X_{7,10,9,1},\ X_{10,11,4,9},\ X_{8,6,5,11}\};$

$\mathcal{A}_{1221} = \mathcal{A}@\{X_{2,8,7,1},\ X_{3,10,9,8},\ X_{10,6,11,9},\ X_{11,5,4,7}\};$

$\mathcal{A}_{2211} = \mathcal{A}@\{X_{3,8,7,2},\ X_{8,6,9,7},\ X_{9,11,10,1},\ X_{11,5,4,10}\};$

$\mathcal{A}_{1122} = \mathcal{A}@\{X_{2,8,7,1},\ X_{8,9,4,7},\ X_{3,11,10,9},\ X_{11,6,5,10}\};$

$\mathcal{A}_{11} = \mathcal{A}@\{X_{2,8,7,1},\ X_{8,5,4,7},\ P_{3,6}\};\quad \mathcal{A}_{22} = \mathcal{A}@\{X_{3,8,7,2},\ X_{8,6,5,7},\ P_{1,4}\};$

$\mathcal{A}_{\phi} = \mathcal{A}@\{P_{1,4},\ P_{2,5},\ P_{3,6}\};$

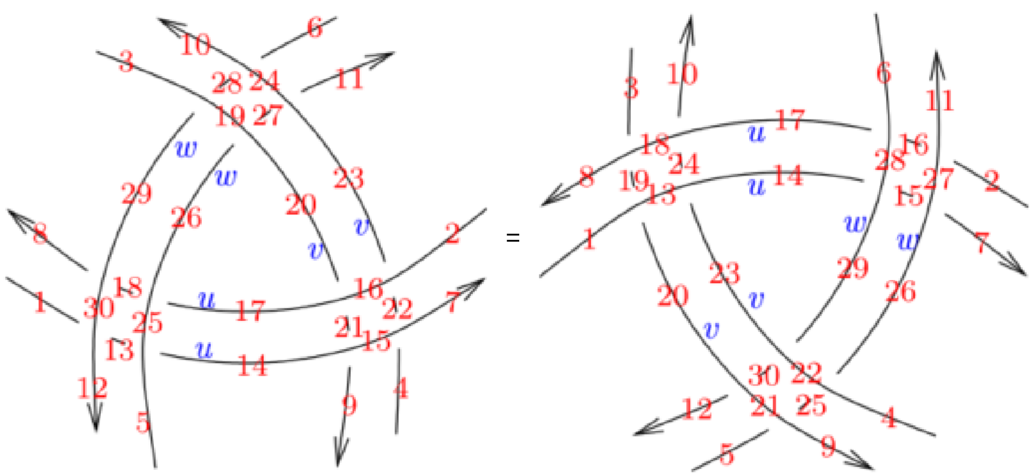
$g_+[z\_] := z^{1/2} + z^{-1/2};\ \ g_-[z\_] := z^{1/2} - z^{-1/2};$

$g_+[\tau_1]\ g_-[\tau_2]\ \mathcal{A}_{2112} - g_-[\tau_2]\ g_+[\tau_3]\ \mathcal{A}_{1221} - g_-[\tau_3 / \tau_1]\ (\mathcal{A}_{2211} + \mathcal{A}_{1122}) +$
$\quad g_-[\tau_2\,\tau_3 / \tau_1]\ g_+[\tau_3]\ \mathcal{A}_{11} - g_+[\tau_1]\ g_-[\tau_1\,\tau_2 / \tau_3]\ \mathcal{A}_{22} \equiv g_-\left[\tau_3^2 \big/ \tau_1^2\right]\ \mathcal{A}_{\phi}$

Out[∘]=

True

The Naik-Stanford Double Delta Move and Virtual Versions (Archibald)



In[∘]:=

$\text{Timing}\Big[\mathcal{A}@\big\{X_{6,10,28,24}[w, v],\ \overline{X}_{28,3,29,19}[w, v],\ X_{26,20,27,19}[w, v],\ \overline{X}_{27,23,11,24}[w, v],$
$\quad X_{1,12,13,30}[u, w],\ \overline{X}_{13,5,14,25}[u, w],\ X_{17,26,18,25}[u, w],\ \overline{X}_{18,29,8,30}[u, w],$
$\quad X_{4,7,22,15}[v, u],\ \overline{X}_{22,2,23,16}[v, u],\ X_{20,17,21,16}[v, u],\ \overline{X}_{21,14,9,15}[v, u]\big\} \equiv$
$\ \mathcal{A}@\big\{X_{5,9,25,21}[w, v],\ \overline{X}_{25,4,26,22}[w, v],\ X_{29,23,30,22}[w, v],\ \overline{X}_{30,20,12,21}[w, v],$
$\quad X_{2,11,16,27}[u, w],\ \overline{X}_{16,6,17,28}[u, w],\ X_{14,29,15,28}[u, w],\ \overline{X}_{15,26,7,27}[u, w],$
$\quad X_{3,8,19,18}[v, u],\ \overline{X}_{19,1,20,13}[v, u],\ X_{23,14,24,13}[v, u],\ \overline{X}_{24,17,10,18}[v, u]\big\}\Big]$

Out[∘]=

{298.063, True}

The Slant Product

In[∘]:=
```
Unprotect[Conjugate];
(T^p_.)* := T^-p;
Protect[Conjugate];
```

In[∘]:=
```
ep[a_, b_] := If[a === b, 1, 0];
```

In[∘]:=
```
Slant_p_ [u_Wedge, v_Wedge] :=
  Module[{n = Length@u, k = Length@v, u1 = Sort[u], s = Signature[u]},
   If[n < k, 0,
    Sum[s * Signature[σ] *
      Times @@ MapThread[p, {List @@ u1⟦σ⟧, List @@ v}] * Delete[u1, List /@ σ],
     {σ, Permutations[Range@n, {k}]}]
   ]
  ];
Slant_ [0, _] = Slant_ [_, 0] = 0;
Slant_p_ [A_, B_] :=
  Expand[Distribute[A ** B] /. (a_. * u_Wedge) ** (b_. * v_Wedge) :> a b* Slant_p[u, v]];
```

In[∘]:=
```
Slant_ep[Wedge[1, 2, 3, 4], Wedge[4, 2] + 7 Wedge[4, 1, 2] + T Wedge[2]]
```

Out[∘]=

$$7\,\text{Wedge}[3] - 1\wedge 3 + \frac{1\wedge 3\wedge 4}{T}$$

In[∘]:=
```
Slant_ep[Wedge[a, b, c, d, e, f], WExp[2 a∧b + 3 c∧d + 7 e∧f]] // WExpQ
```

Out[∘]=

True

In[∘]:=
```
With[{n = 8}, {
  A = Sum[RandomInteger[{-9, 9}] e_i ∧ e_j, {i, 1, n - 1}, {j, 2, n}],
  B = WExp[A],
  SB = Slant_ep[Wedge @@ Table[e_i, {i, n}], B] /. Conjugate[x_] :> x,
  WExpQ[SB] // Simplify // Echo
 }]
```

» True

Out[∘]=

$$\{-6 e_1\wedge e_2 + 2 e_1\wedge e_3 - 9 e_1\wedge e_4 - 2 e_1\wedge e_5 - 4 e_1\wedge e_6 + 9 e_1\wedge e_7 + 2 e_1\wedge e_8 - 5 e_2\wedge e_2 - 2 e_2\wedge e_3 + 7 e_2\wedge e_5 + 5 e_2\wedge e_6 - 9 e_2\wedge e_7 + 5 e_2\wedge e_8 - 3 e_3\wedge e_2 - 2 e_3\wedge e_3 + 3 e_3\wedge e_4 - 2 e_3\wedge e_5 + 3 e_3\wedge e_6 + 7 e_3\wedge e_7 + 9 e_3\wedge e_8 + 8 e_4\wedge e_3 + e_4\wedge e_4 + 6 e_4\wedge e_5 + 4 e_4\wedge e_6 + 6 e_4\wedge e_7 - 2 e_4\wedge e_8 - 9 e_5\wedge e_2 + 2 e_5\wedge e_3 + 9 e_5\wedge e_4 + 9 e_5\wedge e_5 + 9 e_5\wedge e_6 + 4 e_5\wedge e_7 - 8 e_5\wedge e_8 - 4 e_6\wedge e_2 - 2 e_6\wedge e_3 - 8 e_6\wedge e_4 + 2 e_6\wedge e_5 - 9 e_6\wedge e_6 + 4 e_6\wedge e_7 - 8 e_6\wedge e_8 + 5 e_7\wedge e_2 - 9 e_7\wedge e_3 + 4 e_7\wedge e_6 - 2 e_7\wedge e_7 + 5 e_7\wedge e_8,$$

$$\text{Wedge}[\,] - 6 e_1\wedge e_2 + 2 e_1\wedge e_3 - 9 e_1\wedge e_4 - 2 e_1\wedge e_5 - 4 e_1\wedge e_6 + 9 e_1\wedge e_7 + 2 e_1\wedge e_8 + e_2\wedge e_3 +$$

$16 e_2\wedge e_5 + 9 e_2\wedge e_6 - 14 e_2\wedge e_7 + 5 e_2\wedge e_8 - 5 e_3\wedge e_4 - 4 e_3\wedge e_5 + 5 e_3\wedge e_6 + 16 e_3\wedge e_7 +$
$9 e_3\wedge e_8 - 3 e_4\wedge e_5 + 12 e_4\wedge e_6 + 6 e_4\wedge e_7 - 2 e_4\wedge e_8 + 7 e_5\wedge e_6 + 4 e_5\wedge e_7 - 8 e_5\wedge e_8 - 8 e_6\wedge e_8 +$
$5 e_7\wedge e_8 + 21 e_1\wedge e_2\wedge e_3\wedge e_4 - 10 e_1\wedge e_2\wedge e_3\wedge e_5 - 52 e_1\wedge e_2\wedge e_3\wedge e_6 - 59 e_1\wedge e_2\wedge e_3\wedge e_7 -$
$62 e_1\wedge e_2\wedge e_3\wedge e_8 + 162 e_1\wedge e_2\wedge e_4\wedge e_5 + 9 e_1\wedge e_2\wedge e_4\wedge e_6 - 162 e_1\wedge e_2\wedge e_4\wedge e_7 + 57 e_1\wedge e_2\wedge e_4\wedge e_8 -$
$88 e_1\wedge e_2\wedge e_5\wedge e_6 + 92 e_1\wedge e_2\wedge e_5\wedge e_7 + 90 e_1\wedge e_2\wedge e_5\wedge e_8 + 25 e_1\wedge e_2\wedge e_6\wedge e_7 + 86 e_1\wedge e_2\wedge e_6\wedge e_8 -$
$103 e_1\wedge e_2\wedge e_7\wedge e_8 - 32 e_1\wedge e_3\wedge e_4\wedge e_5 + 89 e_1\wedge e_3\wedge e_4\wedge e_6 + 111 e_1\wedge e_3\wedge e_4\wedge e_7 +$
$67 e_1\wedge e_3\wedge e_4\wedge e_8 + 40 e_1\wedge e_3\wedge e_5\wedge e_6 + 4 e_1\wedge e_3\wedge e_5\wedge e_7 - 6 e_1\wedge e_3\wedge e_5\wedge e_8 + 109 e_1\wedge e_3\wedge e_6\wedge e_7 +$
$30 e_1\wedge e_3\wedge e_6\wedge e_8 - 39 e_1\wedge e_3\wedge e_7\wedge e_8 - 27 e_1\wedge e_4\wedge e_5\wedge e_6 - 51 e_1\wedge e_4\wedge e_5\wedge e_7 + 62 e_1\wedge e_4\wedge e_5\wedge e_8 +$
$132 e_1\wedge e_4\wedge e_6\wedge e_7 + 88 e_1\wedge e_4\wedge e_6\wedge e_8 - 15 e_1\wedge e_4\wedge e_7\wedge e_8 + 79 e_1\wedge e_5\wedge e_6\wedge e_7 - 2 e_1\wedge e_5\wedge e_6\wedge e_8 +$
$70 e_1\wedge e_5\wedge e_7\wedge e_8 + 52 e_1\wedge e_6\wedge e_7\wedge e_8 - 83 e_2\wedge e_3\wedge e_4\wedge e_5 - 33 e_2\wedge e_3\wedge e_4\wedge e_6 + 76 e_2\wedge e_3\wedge e_4\wedge e_7 -$
$27 e_2\wedge e_3\wedge e_4\wedge e_8 - 109 e_2\wedge e_3\wedge e_5\wedge e_6 - 196 e_2\wedge e_3\wedge e_5\wedge e_7 - 172 e_2\wedge e_3\wedge e_5\wedge e_8 -$
$214 e_2\wedge e_3\wedge e_6\wedge e_7 - 64 e_2\wedge e_3\wedge e_6\wedge e_8 + 211 e_2\wedge e_3\wedge e_7\wedge e_8 - 219 e_2\wedge e_4\wedge e_5\wedge e_6 -$
$54 e_2\wedge e_4\wedge e_5\wedge e_7 + 17 e_2\wedge e_4\wedge e_5\wedge e_8 - 222 e_2\wedge e_4\wedge e_6\wedge e_7 + 78 e_2\wedge e_4\wedge e_6\wedge e_8 + 2 e_2\wedge e_4\wedge e_7\wedge e_8 -$
$134 e_2\wedge e_5\wedge e_6\wedge e_7 - 21 e_2\wedge e_5\wedge e_6\wedge e_8 - 12 e_2\wedge e_5\wedge e_7\wedge e_8 - 67 e_2\wedge e_6\wedge e_7\wedge e_8 - 2 e_3\wedge e_4\wedge e_5\wedge e_6 -$
$44 e_3\wedge e_4\wedge e_5\wedge e_7 + 5 e_3\wedge e_4\wedge e_5\wedge e_8 + 162 e_3\wedge e_4\wedge e_6\wedge e_7 + 158 e_3\wedge e_4\wedge e_6\wedge e_8 + 61 e_3\wedge e_4\wedge e_7\wedge e_8 +$
$92 e_3\wedge e_5\wedge e_6\wedge e_7 + 135 e_3\wedge e_5\wedge e_6\wedge e_8 + 144 e_3\wedge e_5\wedge e_7\wedge e_8 + 153 e_3\wedge e_6\wedge e_7\wedge e_8 -$
$6 e_4\wedge e_5\wedge e_6\wedge e_7 + 106 e_4\wedge e_5\wedge e_6\wedge e_8 + 25 e_4\wedge e_5\wedge e_7\wedge e_8 + 108 e_4\wedge e_6\wedge e_7\wedge e_8 +$
$67 e_5\wedge e_6\wedge e_7\wedge e_8 + 1697 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_6 + 1541 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_7 +$
$1264 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_8 + 1405 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_6\wedge e_7 - 702 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_6\wedge e_8 -$
$1874 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_7\wedge e_8 - 1621 e_1\wedge e_2\wedge e_3\wedge e_5\wedge e_6\wedge e_7 - 1546 e_1\wedge e_2\wedge e_3\wedge e_5\wedge e_6\wedge e_8 -$
$106 e_1\wedge e_2\wedge e_3\wedge e_5\wedge e_7\wedge e_8 - 1480 e_1\wedge e_2\wedge e_3\wedge e_6\wedge e_7\wedge e_8 - 2913 e_1\wedge e_2\wedge e_4\wedge e_5\wedge e_6\wedge e_7 -$
$1351 e_1\wedge e_2\wedge e_4\wedge e_5\wedge e_6\wedge e_8 - 523 e_1\wedge e_2\wedge e_4\wedge e_5\wedge e_7\wedge e_8 - 2405 e_1\wedge e_2\wedge e_4\wedge e_6\wedge e_7\wedge e_8 -$
$567 e_1\wedge e_2\wedge e_5\wedge e_6\wedge e_7\wedge e_8 + 298 e_1\wedge e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_7 + 1127 e_1\wedge e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_8 +$
$1091 e_1\wedge e_3\wedge e_4\wedge e_5\wedge e_7\wedge e_8 + 251 e_1\wedge e_3\wedge e_4\wedge e_6\wedge e_7\wedge e_8 - 1167 e_1\wedge e_3\wedge e_5\wedge e_6\wedge e_7\wedge e_8 -$
$1453 e_1\wedge e_4\wedge e_5\wedge e_6\wedge e_7\wedge e_8 + 3010 e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_7 + 2579 e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_8 +$
$851 e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_7\wedge e_8 + 3679 e_2\wedge e_3\wedge e_4\wedge e_6\wedge e_7\wedge e_8 + 1265 e_2\wedge e_3\wedge e_5\wedge e_6\wedge e_7\wedge e_8 -$
$49 e_2\wedge e_4\wedge e_5\wedge e_6\wedge e_7\wedge e_8 - 1528 e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_7\wedge e_8 - 15\,356 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_6\wedge e_7\wedge e_8,$
$-15\,356\,\mathrm{Wedge}[\,] - 1528 e_1\wedge e_2 - 49 e_1\wedge e_3 + 1265 e_1\wedge e_4 + 3679 e_1\wedge e_5 + 851 e_1\wedge e_6 + 2579 e_1\wedge e_7 +$
$3010 e_1\wedge e_8 - 1453 e_2\wedge e_3 - 1167 e_2\wedge e_4 + 251 e_2\wedge e_5 + 1091 e_2\wedge e_6 + 1127 e_2\wedge e_7 + 298 e_2\wedge e_8 -$
$567 e_3\wedge e_4 - 2405 e_3\wedge e_5 - 523 e_3\wedge e_6 - 1351 e_3\wedge e_7 - 2913 e_3\wedge e_8 - 1480 e_4\wedge e_5 - 106 e_4\wedge e_6 -$
$1546 e_4\wedge e_7 - 1621 e_4\wedge e_8 - 1874 e_5\wedge e_6 - 702 e_5\wedge e_7 + 1405 e_5\wedge e_8 + 1264 e_6\wedge e_7 + 1541 e_6\wedge e_8 +$
$1697 e_7\wedge e_8 + 67 e_1\wedge e_2\wedge e_3\wedge e_4 + 108 e_1\wedge e_2\wedge e_3\wedge e_5 + 25 e_1\wedge e_2\wedge e_3\wedge e_6 + 106 e_1\wedge e_2\wedge e_3\wedge e_7 -$
$6 e_1\wedge e_2\wedge e_3\wedge e_8 + 153 e_1\wedge e_2\wedge e_4\wedge e_5 + 144 e_1\wedge e_2\wedge e_4\wedge e_6 + 135 e_1\wedge e_2\wedge e_4\wedge e_7 + 92 e_1\wedge e_2\wedge e_4\wedge e_8 +$
$61 e_1\wedge e_2\wedge e_5\wedge e_6 + 158 e_1\wedge e_2\wedge e_5\wedge e_7 + 162 e_1\wedge e_2\wedge e_5\wedge e_8 + 5 e_1\wedge e_2\wedge e_6\wedge e_7 - 44 e_1\wedge e_2\wedge e_6\wedge e_8 -$
$2 e_1\wedge e_2\wedge e_7\wedge e_8 - 67 e_1\wedge e_3\wedge e_4\wedge e_5 - 12 e_1\wedge e_3\wedge e_4\wedge e_6 - 21 e_1\wedge e_3\wedge e_4\wedge e_7 - 134 e_1\wedge e_3\wedge e_4\wedge e_8 +$
$2 e_1\wedge e_3\wedge e_5\wedge e_6 + 78 e_1\wedge e_3\wedge e_5\wedge e_7 - 222 e_1\wedge e_3\wedge e_5\wedge e_8 + 17 e_1\wedge e_3\wedge e_6\wedge e_7 - 54 e_1\wedge e_3\wedge e_6\wedge e_8 -$
$219 e_1\wedge e_3\wedge e_7\wedge e_8 + 211 e_1\wedge e_4\wedge e_5\wedge e_6 - 64 e_1\wedge e_4\wedge e_5\wedge e_7 - 214 e_1\wedge e_4\wedge e_5\wedge e_8 -$
$172 e_1\wedge e_4\wedge e_6\wedge e_7 - 196 e_1\wedge e_4\wedge e_6\wedge e_8 - 109 e_1\wedge e_4\wedge e_7\wedge e_8 - 27 e_1\wedge e_5\wedge e_6\wedge e_7 +$
$76 e_1\wedge e_5\wedge e_6\wedge e_8 - 33 e_1\wedge e_5\wedge e_7\wedge e_8 - 83 e_1\wedge e_6\wedge e_7\wedge e_8 + 52 e_2\wedge e_3\wedge e_4\wedge e_5 + 70 e_2\wedge e_3\wedge e_4\wedge e_6 -$
$2 e_2\wedge e_3\wedge e_4\wedge e_7 + 79 e_2\wedge e_3\wedge e_4\wedge e_8 - 15 e_2\wedge e_3\wedge e_5\wedge e_6 + 88 e_2\wedge e_3\wedge e_5\wedge e_7 + 132 e_2\wedge e_3\wedge e_5\wedge e_8 +$
$62 e_2\wedge e_3\wedge e_6\wedge e_7 - 51 e_2\wedge e_3\wedge e_6\wedge e_8 - 27 e_2\wedge e_3\wedge e_7\wedge e_8 - 39 e_2\wedge e_4\wedge e_5\wedge e_6 + 30 e_2\wedge e_4\wedge e_5\wedge e_7 +$
$109 e_2\wedge e_4\wedge e_5\wedge e_8 - 6 e_2\wedge e_4\wedge e_6\wedge e_7 + 4 e_2\wedge e_4\wedge e_6\wedge e_8 + 40 e_2\wedge e_4\wedge e_7\wedge e_8 + 67 e_2\wedge e_5\wedge e_6\wedge e_7 +$
$111 e_2\wedge e_5\wedge e_6\wedge e_8 + 89 e_2\wedge e_5\wedge e_7\wedge e_8 - 32 e_2\wedge e_6\wedge e_7\wedge e_8 - 103 e_3\wedge e_4\wedge e_5\wedge e_6 +$
$86 e_3\wedge e_4\wedge e_5\wedge e_7 + 25 e_3\wedge e_4\wedge e_5\wedge e_8 + 90 e_3\wedge e_4\wedge e_6\wedge e_7 + 92 e_3\wedge e_4\wedge e_6\wedge e_8 - 88 e_3\wedge e_4\wedge e_7\wedge e_8 +$
$57 e_3\wedge e_5\wedge e_6\wedge e_7 - 162 e_3\wedge e_5\wedge e_6\wedge e_8 + 9 e_3\wedge e_5\wedge e_7\wedge e_8 + 162 e_3\wedge e_6\wedge e_7\wedge e_8 -$
$62 e_4\wedge e_5\wedge e_6\wedge e_7 - 59 e_4\wedge e_5\wedge e_6\wedge e_8 - 52 e_4\wedge e_5\wedge e_7\wedge e_8 - 10 e_4\wedge e_6\wedge e_7\wedge e_8 + 21 e_5\wedge e_6\wedge e_7\wedge e_8 +$
$5 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_6 - 8 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_5\wedge e_7 - 8 e_1\wedge e_2\wedge e_3\wedge e_4\wedge e_6\wedge e_7 +$

$4\, e_1 \wedge e_2 \wedge e_3 \wedge e_4 \wedge e_6 \wedge e_8 + 7\, e_1 \wedge e_2 \wedge e_3 \wedge e_4 \wedge e_7 \wedge e_8 - 2\, e_1 \wedge e_2 \wedge e_3 \wedge e_5 \wedge e_6 \wedge e_7 +$
$6\, e_1 \wedge e_2 \wedge e_3 \wedge e_5 \wedge e_6 \wedge e_8 + 12\, e_1 \wedge e_2 \wedge e_3 \wedge e_5 \wedge e_7 \wedge e_8 - 3\, e_1 \wedge e_2 \wedge e_3 \wedge e_6 \wedge e_7 \wedge e_8 +$
$9\, e_1 \wedge e_2 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 + 16\, e_1 \wedge e_2 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_8 + 5\, e_1 \wedge e_2 \wedge e_4 \wedge e_5 \wedge e_7 \wedge e_8 -$
$4\, e_1 \wedge e_2 \wedge e_4 \wedge e_6 \wedge e_7 \wedge e_8 - 5\, e_1 \wedge e_2 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8 + 5\, e_1 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 -$
$14\, e_1 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_8 + 9\, e_1 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_7 \wedge e_8 + 16\, e_1 \wedge e_3 \wedge e_4 \wedge e_6 \wedge e_7 \wedge e_8 +$
$e_1 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8 + 2\, e_2 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 + 9\, e_2 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_8 -$
$4\, e_2 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_7 \wedge e_8 - 2\, e_2 \wedge e_3 \wedge e_4 \wedge e_6 \wedge e_7 \wedge e_8 - 9\, e_2 \wedge e_3 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8 +$
$2\, e_2 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8 - 6\, e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8 + e_1 \wedge e_2 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6 \wedge e_7 \wedge e_8$, True}

The Pairings

In[∘]:=
```
p_ς_[c_, d_] := Expand@Cancel[T^(1/2) (T - 1)^-1 Module[{e, f},
      Expand[c (d /. {T → T*, ξ_i_ :→ ξ_i*, x_i_ :→ x_i*})] /.
       {t_i_* t_i_ → α2 (T* - T), x_i_* x_i_ → α1 (T - T*),
        (f : ξ | x)_j_* (e : ξ | x)_i_ :→ If[Position[ς, e_i][[1, 1]] < Position[ς, f_j][[1, 1]],
          α1 (T - 1) + α2 (T* - 1),
          α1 (1 - T)* + α2 (1 - T)
         ]}
     ]];
```

In[∘]:=
```
WExp[2 ξ_1 ∧ x_1 + 3 ξ_2 ∧ x_2 + 7 ξ_3 ∧ x_3]
```

Out[∘]=

$\text{Wedge}[\,] - 2\, x_1 \wedge \xi_1 - 3\, x_2 \wedge \xi_2 - 7\, x_3 \wedge \xi_3 - 6\, x_1 \wedge x_2 \wedge \xi_1 \wedge \xi_2 -$
$14\, x_1 \wedge x_3 \wedge \xi_1 \wedge \xi_3 - 21\, x_2 \wedge x_3 \wedge \xi_2 \wedge \xi_3 + 42\, x_1 \wedge x_2 \wedge x_3 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3$

In[∘]:=
```
Slant_ep[Wedge[ξ_1, ξ_3, ξ_2, x_1, x_2, x_3], WExp[2 ξ_1 ∧ x_1 + 3 ξ_2 ∧ x_2 + 7 ξ_3 ∧ x_3]]
```

Out[∘]=

$42\, \text{Wedge}[\,] - 21\, x_1 \wedge \xi_1 - 14\, x_2 \wedge \xi_2 - 6\, x_3 \wedge \xi_3 - 7\, x_1 \wedge x_2 \wedge \xi_1 \wedge \xi_2 -$
$3\, x_1 \wedge x_3 \wedge \xi_1 \wedge \xi_3 - 2\, x_2 \wedge x_3 \wedge \xi_2 \wedge \xi_3 + x_1 \wedge x_2 \wedge x_3 \wedge \xi_1 \wedge \xi_2 \wedge \xi_3$

In[∘]:= $\mathbf{Slant}_{p_{\{\xi_1,\xi_2,\xi_3,x_1,x_2,x_3\}}}[\mathbf{Wedge}[\xi_1, \xi_2, \xi_3, x_1, x_2, x_3], \mathbf{WExp}[2\,\xi_1\wedge x_1 + 3\,\xi_2\wedge x_2 + 7\,\xi_3\wedge x_3]]$

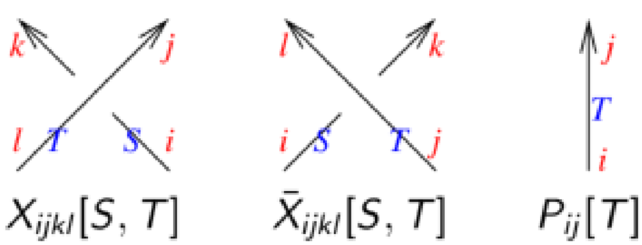
Out[∘]=

$$-\frac{42\,\mathrm{Wedge}[\,]}{T^3}+\frac{84\,\mathrm{Wedge}[\,]}{T^2}-\frac{42\,\mathrm{Wedge}[\,]}{T}-21\,x_1\wedge x_2+42\,T\,x_1\wedge x_2-21\,T^2\,x_1\wedge x_2-42\,x_1\wedge x_3+$$
$$\frac{21\,x_1\wedge x_3}{T}+21\,T\,x_1\wedge x_3+14\,x_1\wedge\xi_1+\frac{35\,x_1\wedge\xi_1}{T^2}-\frac{49\,x_1\wedge\xi_1}{T}+26\,x_1\wedge\xi_2+\frac{14\,x_1\wedge\xi_2}{T^2}-\frac{34\,x_1\wedge\xi_2}{T}-$$
$$6\,T\,x_1\wedge\xi_2+12\,x_1\wedge\xi_3-\frac{6\,x_1\wedge\xi_3}{T}-27\,T\,x_1\wedge\xi_3+21\,T^2\,x_1\wedge\xi_3+\frac{14\,x_2\wedge\xi_1}{T^2}-\frac{14\,x_2\wedge\xi_1}{T}+6\,x_2\wedge\xi_2+$$
$$\frac{20\,x_2\wedge\xi_2}{T^2}-\frac{26\,x_2\wedge\xi_2}{T}+27\,x_2\wedge\xi_3+\frac{6\,x_2\wedge\xi_3}{T^2}-\frac{12\,x_2\wedge\xi_3}{T}-42\,T\,x_2\wedge\xi_3+21\,T^2\,x_2\wedge\xi_3+$$
$$\frac{6\,x_3\wedge\xi_2}{T^2}-\frac{6\,x_3\wedge\xi_2}{T}+42\,x_3\wedge\xi_3+\frac{6\,x_3\wedge\xi_3}{T^2}-\frac{27\,x_3\wedge\xi_3}{T}-21\,T\,x_3\wedge\xi_3-14\,\xi_1\wedge\xi_3-\frac{21\,\xi_1\wedge\xi_3}{T^2}+$$
$$\frac{35\,\xi_1\wedge\xi_3}{T}-20\,\xi_2\wedge\xi_3+\frac{14\,\xi_2\wedge\xi_3}{T}+6\,T\,\xi_2\wedge\xi_3+7\,x_1\wedge x_2\wedge x_3\wedge\xi_1-7\,T\,x_1\wedge x_2\wedge x_3\wedge\xi_1+$$
$$10\,x_1\wedge x_2\wedge x_3\wedge\xi_2-10\,T\,x_1\wedge x_2\wedge x_3\wedge\xi_2+3\,x_1\wedge x_2\wedge x_3\wedge\xi_3-3\,T\,x_1\wedge x_2\wedge x_3\wedge\xi_3-5\,x_1\wedge x_2\wedge\xi_1\wedge\xi_2+$$
$$\frac{12\,x_1\wedge x_2\wedge\xi_1\wedge\xi_2}{T}-5\,x_1\wedge x_2\wedge\xi_1\wedge\xi_3+\frac{5\,x_1\wedge x_2\wedge\xi_1\wedge\xi_3}{T}+7\,T\,x_1\wedge x_2\wedge\xi_1\wedge\xi_3-2\,x_1\wedge x_2\wedge\xi_2\wedge\xi_3+$$
$$\frac{2\,x_1\wedge x_2\wedge\xi_2\wedge\xi_3}{T}+7\,T\,x_1\wedge x_2\wedge\xi_2\wedge\xi_3-2\,x_1\wedge x_3\wedge\xi_1\wedge\xi_2+\frac{5\,x_1\wedge x_3\wedge\xi_1\wedge\xi_2}{T}-9\,x_1\wedge x_3\wedge\xi_1\wedge\xi_3+$$
$$\frac{5\,x_1\wedge x_3\wedge\xi_1\wedge\xi_3}{T}+7\,T\,x_1\wedge x_3\wedge\xi_1\wedge\xi_3-9\,x_1\wedge x_3\wedge\xi_2\wedge\xi_3+\frac{2\,x_1\wedge x_3\wedge\xi_2\wedge\xi_3}{T}+10\,T\,x_1\wedge x_3\wedge\xi_2\wedge\xi_3+$$
$$3\,x_1\wedge\xi_1\wedge\xi_2\wedge\xi_3-\frac{7\,x_1\wedge\xi_1\wedge\xi_2\wedge\xi_3}{T}+\frac{2\,x_2\wedge x_3\wedge\xi_1\wedge\xi_2}{T}-7\,x_2\wedge x_3\wedge\xi_1\wedge\xi_3+\frac{2\,x_2\wedge x_3\wedge\xi_1\wedge\xi_3}{T}+$$
$$7\,T\,x_2\wedge x_3\wedge\xi_1\wedge\xi_3-10\,x_2\wedge x_3\wedge\xi_2\wedge\xi_3+\frac{2\,x_2\wedge x_3\wedge\xi_2\wedge\xi_3}{T}+10\,T\,x_2\wedge x_3\wedge\xi_2\wedge\xi_3+5\,x_2\wedge\xi_1\wedge\xi_2\wedge\xi_3-$$
$$\frac{10\,x_2\wedge\xi_1\wedge\xi_2\wedge\xi_3}{T}+2\,x_3\wedge\xi_1\wedge\xi_2\wedge\xi_3-\frac{3\,x_3\wedge\xi_1\wedge\xi_2\wedge\xi_3}{T}-x_1\wedge x_2\wedge x_3\wedge\xi_1\wedge\xi_2\wedge\xi_3$$

In[∘]:= $\mathbf{Slant}_{p_{\{\xi_1,\xi_2,\xi_3,x_1,x_2,x_3\}}}[\mathbf{Wedge}[\xi_1, \xi_2, \xi_3, x_1, x_2, x_3],$
$\quad\mathbf{WExp}[2\,\xi_1\wedge x_1 + x_1\wedge x_2 + 3\,\xi_2\wedge x_2 + 7\,\xi_3\wedge x_3]]$ // **WExpQ** // **Simplify**

Out[∘]=

True



In[∘]:= $\mathcal{A}[X_{1,2,3,4}[T, T]]$

Out[∘]=

$$\mathcal{A}\Big[\{4, 1\}, \{2, 3\}, \langle|\,\xi_1\to T,\ x_2\to T,\ x_3\to T,\ \xi_4\to T\,|\rangle,$$
$$\frac{\mathrm{Wedge}[\,]}{\sqrt{T}}-\sqrt{T}\;x_2\wedge\xi_4-\frac{x_3\wedge\xi_1}{\sqrt{T}}-\frac{x_3\wedge\xi_4}{\sqrt{T}}+\sqrt{T}\;x_3\wedge\xi_4+\sqrt{T}\;x_2\wedge x_3\wedge\xi_1\wedge\xi_4\Big]$$

In[∘]:=
```
ς0 = {ξ1, x2, x3, ξ4};
Slant_{p_{ς0}}[Wedge @@ ς0, Last@𝒜[X_{1,2,3,4}[T, T]]]
```

Out[∘]=

$$\frac{\text{Wedge}[\,]}{T^{5/2}} - \frac{\text{Wedge}[\,]}{T^{3/2}} + \frac{x_2\wedge x_3}{\sqrt{T}} - \sqrt{T}\, x_2\wedge x_3 - \sqrt{T}\, x_2\wedge \xi_1 - \frac{x_2\wedge \xi_4}{\sqrt{T}} - \frac{x_3\wedge \xi_1}{T^{3/2}} + \frac{x_3\wedge \xi_1}{\sqrt{T}} - \sqrt{T}\, x_3\wedge \xi_1 - \frac{x_3\wedge \xi_4}{\sqrt{T}} + \sqrt{T}\, x_2\wedge x_3\wedge \xi_1\wedge \xi_4$$

In[∘]:=
```
WExpQ[Slant_{p_{ς0}}[Wedge @@ ς0, Last@𝒜[X_{1,2,3,4}[T, T]]]] // Simplify
```

Out[∘]=

True

In[∘]:=
```
WExpQ[Slant_{ep}[Wedge @@ ς0, Last@𝒜[X_{1,2,3,4}[T, T]]]] // Simplify
```

Out[∘]=

True

In[∘]:=
```
ς0 = {ξ1, x2};
A0 = Last[𝒜@{P_{1,2}[T]}]
A1 = Slant_{p_{ς0}}[Wedge @@ ς0, A0]
A1 // WExpQ
```

Out[∘]=

$$\text{Wedge}[\,] - x_2\wedge \xi_1$$

Out[∘]=

$$\frac{\alpha 1^2\,\text{Wedge}[\,]}{T} - \alpha 1\,\alpha 2\,\text{Wedge}[\,] + \frac{\alpha 1\,\alpha 2\,\text{Wedge}[\,]}{T} - \alpha 2^2\,\text{Wedge}[\,] - x_2\wedge \xi_1$$

Out[∘]=

True

In[∘]:=
```
Coefficient[A1, Wedge[]] // Simplify
```

Out[∘]=

$$\frac{(\alpha 1 + \alpha 2)\,(\alpha 1 - T\,\alpha 2)}{T}$$

In[∘]:=
```
ς0 = {ξ1, x2, ξ3, x4};
A0 = Last[𝒜@{P1,2[T], P3,4[T]}]
A1 = Slantpς0[Wedge @@ ς0, A0]
A1 // WExpQ
```

Out[∘]=

$$\text{Wedge}[\,] - x_2\wedge\xi_1 - x_4\wedge\xi_3 - x_2\wedge x_4\wedge\xi_1\wedge\xi_3$$

Out[∘]=

$$\frac{\alpha1^4\,\text{Wedge}[\,]}{T^2} - \frac{\alpha1^4\,\text{Wedge}[\,]}{T} + \alpha1^3\,\alpha2\,\text{Wedge}[\,] + \frac{3\,\alpha1^3\,\alpha2\,\text{Wedge}[\,]}{T^2} - \frac{4\,\alpha1^3\,\alpha2\,\text{Wedge}[\,]}{T} +$$
$$3\,\alpha1^2\,\alpha2^2\,\text{Wedge}[\,] + \frac{3\,\alpha1^2\,\alpha2^2\,\text{Wedge}[\,]}{T^2} - \frac{6\,\alpha1^2\,\alpha2^2\,\text{Wedge}[\,]}{T} + 3\,\alpha1\,\alpha2^3\,\text{Wedge}[\,] + \frac{\alpha1\,\alpha2^3\,\text{Wedge}[\,]}{T^2} -$$
$$\frac{4\,\alpha1\,\alpha2^3\,\text{Wedge}[\,]}{T} + \alpha2^4\,\text{Wedge}[\,] - \frac{\alpha2^4\,\text{Wedge}[\,]}{T} + 2\,\alpha1^2\,x_2\wedge x_4 - \frac{\alpha1^2\,x_2\wedge x_4}{T} - T\,\alpha1^2\,x_2\wedge x_4 +$$
$$4\,\alpha1\,\alpha2\,x_2\wedge x_4 - \frac{2\,\alpha1\,\alpha2\,x_2\wedge x_4}{T} - 2\,T\,\alpha1\,\alpha2\,x_2\wedge x_4 + 2\,\alpha2^2\,x_2\wedge x_4 - \frac{\alpha2^2\,x_2\wedge x_4}{T} - T\,\alpha2^2\,x_2\wedge x_4 +$$
$$\frac{\alpha1^2\,x_2\wedge\xi_1}{T} - \alpha1\,\alpha2\,x_2\wedge\xi_1 + \frac{\alpha1\,\alpha2\,x_2\wedge\xi_1}{T} - \alpha2^2\,x_2\wedge\xi_1 - \alpha1^2\,x_2\wedge\xi_3 + \frac{\alpha1^2\,x_2\wedge\xi_3}{T} + T\,\alpha1^2\,x_2\wedge\xi_3 -$$
$$3\,\alpha1\,\alpha2\,x_2\wedge\xi_3 + \frac{\alpha1\,\alpha2\,x_2\wedge\xi_3}{T} + 2\,T\,\alpha1\,\alpha2\,x_2\wedge\xi_3 - 2\,\alpha2^2\,x_2\wedge\xi_3 + T\,\alpha2^2\,x_2\wedge\xi_3 + 2\,\alpha1^2\,x_4\wedge\xi_1 -$$
$$T\,\alpha1^2\,x_4\wedge\xi_1 + 3\,\alpha1\,\alpha2\,x_4\wedge\xi_1 - \frac{\alpha1\,\alpha2\,x_4\wedge\xi_1}{T} - 2\,T\,\alpha1\,\alpha2\,x_4\wedge\xi_1 + \alpha2^2\,x_4\wedge\xi_1 - \frac{\alpha2^2\,x_4\wedge\xi_1}{T} -$$
$$T\,\alpha2^2\,x_4\wedge\xi_1 + \frac{\alpha1^2\,x_4\wedge\xi_3}{T} - \alpha1\,\alpha2\,x_4\wedge\xi_3 + \frac{\alpha1\,\alpha2\,x_4\wedge\xi_3}{T} - \alpha2^2\,x_4\wedge\xi_3 + \alpha1^2\,\xi_1\wedge\xi_3 -$$
$$T\,\alpha1^2\,\xi_1\wedge\xi_3 + 2\,\alpha1\,\alpha2\,\xi_1\wedge\xi_3 - 2\,T\,\alpha1\,\alpha2\,\xi_1\wedge\xi_3 + \alpha2^2\,\xi_1\wedge\xi_3 - T\,\alpha2^2\,\xi_1\wedge\xi_3 - x_2\wedge x_4\wedge\xi_1\wedge\xi_3$$

Out[∘]=

True

In[∘]:=
```
Coefficient[A1, Wedge[]] // Simplify
```

Out[∘]=

$$\frac{(-1+T)\,(\alpha1+\alpha2)^3\,(-\alpha1+T\,\alpha2)}{T^2}$$

In[∘]:=
```
Coefficient[A1, ξ1 ∧ ξ3] // Simplify
```

Out[∘]=

$$-\left((-1+T)\,(\alpha1+\alpha2)^2\right)$$

In[∘]:=
```
Coefficient[A1, x2 ∧ x4] // Simplify
```

Out[∘]=

$$-\frac{(-1+T)^2\,(\alpha1+\alpha2)^2}{T}$$

In[∘]:=
```
ς0 = {ξ1, x2, ξ3, x4, ξ5, x6};
A0 = Last[𝒜@{P1,2[T], P3,4[T], P5,6[T]}]
A1 = Slantpς0[Wedge @@ ς0, A0]
A1 // WExpQ
```

Out[∘]=

$$\text{Wedge}[\,] - x_2\wedge\xi_1 - x_4\wedge\xi_3 - x_6\wedge\xi_5 - x_2\wedge x_4\wedge\xi_1\wedge\xi_3 - x_2\wedge x_6\wedge\xi_1\wedge\xi_5 - x_4\wedge x_6\wedge\xi_3\wedge\xi_5 + x_2\wedge x_4\wedge x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5$$

Out[∘]=

$$\frac{\alpha 1^6\,\text{Wedge}[\,]}{T^3} - \frac{2\,\alpha 1^6\,\text{Wedge}[\,]}{T^2} + \frac{\alpha 1^6\,\text{Wedge}[\,]}{T} - \alpha 1^5\,\alpha 2\,\text{Wedge}[\,] + \frac{5\,\alpha 1^5\,\alpha 2\,\text{Wedge}[\,]}{T^3} - \frac{11\,\alpha 1^5\,\alpha 2\,\text{Wedge}[\,]}{T^2} +$$
$$\frac{7\,\alpha 1^5\,\alpha 2\,\text{Wedge}[\,]}{T} - 5\,\alpha 1^4\,\alpha 2^2\,\text{Wedge}[\,] + \frac{10\,\alpha 1^4\,\alpha 2^2\,\text{Wedge}[\,]}{T^3} - \frac{25\,\alpha 1^4\,\alpha 2^2\,\text{Wedge}[\,]}{T^2} +$$
$$\frac{20\,\alpha 1^4\,\alpha 2^2\,\text{Wedge}[\,]}{T} - 10\,\alpha 1^3\,\alpha 2^3\,\text{Wedge}[\,] + \frac{10\,\alpha 1^3\,\alpha 2^3\,\text{Wedge}[\,]}{T^3} - \frac{30\,\alpha 1^3\,\alpha 2^3\,\text{Wedge}[\,]}{T^2} +$$
$$\frac{30\,\alpha 1^3\,\alpha 2^3\,\text{Wedge}[\,]}{T} - 10\,\alpha 1^2\,\alpha 2^4\,\text{Wedge}[\,] + \frac{5\,\alpha 1^2\,\alpha 2^4\,\text{Wedge}[\,]}{T^3} - \frac{20\,\alpha 1^2\,\alpha 2^4\,\text{Wedge}[\,]}{T^2} +$$
$$\frac{25\,\alpha 1^2\,\alpha 2^4\,\text{Wedge}[\,]}{T} - 5\,\alpha 1\,\alpha 2^5\,\text{Wedge}[\,] + \frac{\alpha 1\,\alpha 2^5\,\text{Wedge}[\,]}{T^3} - \frac{7\,\alpha 1\,\alpha 2^5\,\text{Wedge}[\,]}{T^2} + \frac{11\,\alpha 1\,\alpha 2^5\,\text{Wedge}[\,]}{T} -$$
$$\alpha 2^6\,\text{Wedge}[\,] - \frac{\alpha 2^6\,\text{Wedge}[\,]}{T^2} + \frac{2\,\alpha 2^6\,\text{Wedge}[\,]}{T} - \alpha 1^4\,x_2\wedge x_4 - \frac{\alpha 1^4\,x_2\wedge x_4}{T^2} + \frac{2\,\alpha 1^4\,x_2\wedge x_4}{T} -$$
$$5\,\alpha 1^3\,\alpha 2\,x_2\wedge x_4 - \frac{3\,\alpha 1^3\,\alpha 2\,x_2\wedge x_4}{T^2} + \frac{7\,\alpha 1^3\,\alpha 2\,x_2\wedge x_4}{T} + T\,\alpha 1^3\,\alpha 2\,x_2\wedge x_4 - 9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_4 -$$
$$\frac{3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_4}{T^2} + \frac{9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_4}{T} + 3\,T\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_4 - 7\,\alpha 1\,\alpha 2^3\,x_2\wedge x_4 - \frac{\alpha 1\,\alpha 2^3\,x_2\wedge x_4}{T^2} +$$
$$\frac{5\,\alpha 1\,\alpha 2^3\,x_2\wedge x_4}{T} + 3\,T\,\alpha 1\,\alpha 2^3\,x_2\wedge x_4 - 2\,\alpha 2^4\,x_2\wedge x_4 + \frac{\alpha 2^4\,x_2\wedge x_4}{T} + T\,\alpha 2^4\,x_2\wedge x_4 + 2\,\alpha 1^4\,x_2\wedge x_6 -$$
$$\frac{\alpha 1^4\,x_2\wedge x_6}{T} - T\,\alpha 1^4\,x_2\wedge x_6 + 7\,\alpha 1^3\,\alpha 2\,x_2\wedge x_6 + \frac{\alpha 1^3\,\alpha 2\,x_2\wedge x_6}{T^2} - \frac{5\,\alpha 1^3\,\alpha 2\,x_2\wedge x_6}{T} - 3\,T\,\alpha 1^3\,\alpha 2\,x_2\wedge x_6 +$$
$$9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_6 + \frac{3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_6}{T^2} - \frac{9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_6}{T} - 3\,T\,\alpha 1^2\,\alpha 2^2\,x_2\wedge x_6 + 5\,\alpha 1\,\alpha 2^3\,x_2\wedge x_6 +$$
$$\frac{3\,\alpha 1\,\alpha 2^3\,x_2\wedge x_6}{T^2} - \frac{7\,\alpha 1\,\alpha 2^3\,x_2\wedge x_6}{T} - T\,\alpha 1\,\alpha 2^3\,x_2\wedge x_6 + \alpha 2^4\,x_2\wedge x_6 + \frac{\alpha 2^4\,x_2\wedge x_6}{T^2} - \frac{2\,\alpha 2^4\,x_2\wedge x_6}{T} -$$
$$\frac{\alpha 1^4\,x_2\wedge\xi_1}{T^2} + \frac{\alpha 1^4\,x_2\wedge\xi_1}{T} - \alpha 1^3\,\alpha 2\,x_2\wedge\xi_1 - \frac{3\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_1}{T^2} + \frac{4\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_1}{T} - 3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_1 -$$
$$\frac{3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_1}{T^2} + \frac{6\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_1}{T} - 3\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_1 - \frac{\alpha 1\,\alpha 2^3\,x_2\wedge\xi_1}{T^2} + \frac{4\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_1}{T} - \alpha 2^4\,x_2\wedge\xi_1 +$$
$$\frac{\alpha 2^4\,x_2\wedge\xi_1}{T} - \alpha 1^4\,x_2\wedge\xi_3 - \frac{\alpha 1^4\,x_2\wedge\xi_3}{T^2} + \frac{2\,\alpha 1^4\,x_2\wedge\xi_3}{T} - 5\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_3 - \frac{3\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_3}{T^2} +$$

$$\frac{7\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_3}{T}+T\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_3-9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_3-\frac{3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_3}{T^2}+\frac{9\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_3}{T}+$$

$$3\,T\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_3-7\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_3-\frac{\alpha 1\,\alpha 2^3\,x_2\wedge\xi_3}{T^2}+\frac{5\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_3}{T}+3\,T\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_3-$$

$$2\,\alpha 2^4\,x_2\wedge\xi_3+\frac{\alpha 2^4\,x_2\wedge\xi_3}{T}+T\,\alpha 2^4\,x_2\wedge\xi_3+\alpha 1^4\,x_2\wedge\xi_5-T\,\alpha 1^4\,x_2\wedge\xi_5+4\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_5-$$

$$\frac{\alpha 1^3\,\alpha 2\,x_2\wedge\xi_5}{T}-3\,T\,\alpha 1^3\,\alpha 2\,x_2\wedge\xi_5+6\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_5-\frac{3\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_5}{T}-3\,T\,\alpha 1^2\,\alpha 2^2\,x_2\wedge\xi_5+$$

$$4\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_5-\frac{3\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_5}{T}-T\,\alpha 1\,\alpha 2^3\,x_2\wedge\xi_5+\alpha 2^4\,x_2\wedge\xi_5-\frac{\alpha 2^4\,x_2\wedge\xi_5}{T}-\alpha 1^4\,x_4\wedge x_6-$$

$$\frac{\alpha 1^4\,x_4\wedge x_6}{T^2}+\frac{2\,\alpha 1^4\,x_4\wedge x_6}{T}-5\,\alpha 1^3\,\alpha 2\,x_4\wedge x_6-\frac{3\,\alpha 1^3\,\alpha 2\,x_4\wedge x_6}{T^2}+\frac{7\,\alpha 1^3\,\alpha 2\,x_4\wedge x_6}{T}+T\,\alpha 1^3\,\alpha 2\,x_4\wedge x_6-$$

$$9\,\alpha 1^2\,\alpha 2^2\,x_4\wedge x_6-\frac{3\,\alpha 1^2\,\alpha 2^2\,x_4\wedge x_6}{T^2}+\frac{9\,\alpha 1^2\,\alpha 2^2\,x_4\wedge x_6}{T}+3\,T\,\alpha 1^2\,\alpha 2^2\,x_4\wedge x_6-7\,\alpha 1\,\alpha 2^3\,x_4\wedge x_6-$$

$$\frac{\alpha 1\,\alpha 2^3\,x_4\wedge x_6}{T^2}+\frac{5\,\alpha 1\,\alpha 2^3\,x_4\wedge x_6}{T}+3\,T\,\alpha 1\,\alpha 2^3\,x_4\wedge x_6-2\,\alpha 2^4\,x_4\wedge x_6+\frac{\alpha 2^4\,x_4\wedge x_6}{T}+T\,\alpha 2^4\,x_4\wedge x_6+$$

$$\alpha 1^4\,x_4\wedge\xi_1-\frac{\alpha 1^4\,x_4\wedge\xi_1}{T}+4\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_1-\frac{3\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_1}{T}-T\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_1+6\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_1-$$

$$\frac{3\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_1}{T}-3\,T\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_1+4\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_1-\frac{\alpha 1\,\alpha 2^3\,x_4\wedge\xi_1}{T}-3\,T\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_1+$$

$$\alpha 2^4\,x_4\wedge\xi_1-T\,\alpha 2^4\,x_4\wedge\xi_1-\frac{\alpha 1^4\,x_4\wedge\xi_3}{T^2}+\frac{\alpha 1^4\,x_4\wedge\xi_3}{T}-\alpha 1^3\,\alpha 2\,x_4\wedge\xi_3-\frac{3\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_3}{T^2}+$$

$$\frac{4\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_3}{T}-3\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_3-\frac{3\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_3}{T^2}+\frac{6\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_3}{T}-3\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_3-$$

$$\frac{\alpha 1\,\alpha 2^3\,x_4\wedge\xi_3}{T^2}+\frac{4\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_3}{T}-\alpha 2^4\,x_4\wedge\xi_3+\frac{\alpha 2^4\,x_4\wedge\xi_3}{T}-\alpha 1^4\,x_4\wedge\xi_5-\frac{\alpha 1^4\,x_4\wedge\xi_5}{T^2}+\frac{2\,\alpha 1^4\,x_4\wedge\xi_5}{T}-$$

$$5\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_5-\frac{3\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_5}{T^2}+\frac{7\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_5}{T}+T\,\alpha 1^3\,\alpha 2\,x_4\wedge\xi_5-9\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_5-$$

$$\frac{3\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_5}{T^2}+\frac{9\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_5}{T}+3\,T\,\alpha 1^2\,\alpha 2^2\,x_4\wedge\xi_5-7\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_5-\frac{\alpha 1\,\alpha 2^3\,x_4\wedge\xi_5}{T^2}+$$

$$\frac{5\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_5}{T}+3\,T\,\alpha 1\,\alpha 2^3\,x_4\wedge\xi_5-2\,\alpha 2^4\,x_4\wedge\xi_5+\frac{\alpha 2^4\,x_4\wedge\xi_5}{T}+T\,\alpha 2^4\,x_4\wedge\xi_5-2\,\alpha 1^4\,x_6\wedge\xi_1+$$

$$\frac{\alpha 1^4\,x_6\wedge\xi_1}{T}+T\,\alpha 1^4\,x_6\wedge\xi_1-7\,\alpha 1^3\,\alpha 2\,x_6\wedge\xi_1-\frac{\alpha 1^3\,\alpha 2\,x_6\wedge\xi_1}{T^2}+\frac{5\,\alpha 1^3\,\alpha 2\,x_6\wedge\xi_1}{T}+3\,T\,\alpha 1^3\,\alpha 2\,x_6\wedge\xi_1-$$

$$9\,\alpha 1^2\,\alpha 2^2\,x_6\wedge\xi_1-\frac{3\,\alpha 1^2\,\alpha 2^2\,x_6\wedge\xi_1}{T^2}+\frac{9\,\alpha 1^2\,\alpha 2^2\,x_6\wedge\xi_1}{T}+3\,T\,\alpha 1^2\,\alpha 2^2\,x_6\wedge\xi_1-5\,\alpha 1\,\alpha 2^3\,x_6\wedge\xi_1-$$

$$\frac{3\,\alpha 1\,\alpha 2^3\,x_6\wedge\xi_1}{T^2}+\frac{7\,\alpha 1\,\alpha 2^3\,x_6\wedge\xi_1}{T}+T\,\alpha 1\,\alpha 2^3\,x_6\wedge\xi_1-\alpha 2^4\,x_6\wedge\xi_1-\frac{\alpha 2^4\,x_6\wedge\xi_1}{T^2}+\frac{2\,\alpha 2^4\,x_6\wedge\xi_1}{T}+$$

$$\alpha 1^4\, x_6 \wedge \xi_3 - \frac{\alpha 1^4\, x_6 \wedge \xi_3}{T} + 4\, \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_3 - \frac{3\, \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_3}{T} - T\, \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_3 + 6\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_3 -$$

$$\frac{3\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_3}{T} - 3\, T\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_3 + 4\, \alpha 1\, \alpha 2^3\, x_6 \wedge \xi_3 - \frac{\alpha 1\, \alpha 2^3\, x_6 \wedge \xi_3}{T} - 3\, T\, \alpha 1\, \alpha 2^3\, x_6 \wedge \xi_3 +$$

$$\alpha 2^4\, x_6 \wedge \xi_3 - T\, \alpha 2^4\, x_6 \wedge \xi_3 - \frac{\alpha 1^4\, x_6 \wedge \xi_5}{T^2} + \frac{\alpha 1^4\, x_6 \wedge \xi_5}{T} - \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_5 - \frac{3\, \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_5}{T^2} +$$

$$\frac{4\, \alpha 1^3\, \alpha 2\, x_6 \wedge \xi_5}{T} - 3\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_5 - \frac{3\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_5}{T^2} + \frac{6\, \alpha 1^2\, \alpha 2^2\, x_6 \wedge \xi_5}{T} - 3\, \alpha 1\, \alpha 2^3\, x_6 \wedge \xi_5 -$$

$$\frac{\alpha 1\, \alpha 2^3\, x_6 \wedge \xi_5}{T^2} + \frac{4\, \alpha 1\, \alpha 2^3\, x_6 \wedge \xi_5}{T} - \alpha 2^4\, x_6 \wedge \xi_5 + \frac{\alpha 2^4\, x_6 \wedge \xi_5}{T} - \alpha 1^4\, \xi_1 \wedge \xi_3 + \frac{\alpha 1^4\, \xi_1 \wedge \xi_3}{T} -$$

$$4\, \alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_3 + \frac{3\, \alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_3}{T} + T\, \alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_3 - 6\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_3 + \frac{3\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_3}{T} +$$

$$3\, T\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_3 - 4\, \alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_3 + \frac{\alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_3}{T} + 3\, T\, \alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_3 - \alpha 2^4\, \xi_1 \wedge \xi_3 + T\, \alpha 2^4\, \xi_1 \wedge \xi_3 +$$

$$\alpha 1^4\, \xi_1 \wedge \xi_5 - T\, \alpha 1^4\, \xi_1 \wedge \xi_5 + 4\, \alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_5 - \frac{\alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_5}{T} - 3\, T\, \alpha 1^3\, \alpha 2\, \xi_1 \wedge \xi_5 + 6\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_5 -$$

$$\frac{3\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_5}{T} - 3\, T\, \alpha 1^2\, \alpha 2^2\, \xi_1 \wedge \xi_5 + 4\, \alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_5 - \frac{3\, \alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_5}{T} - T\, \alpha 1\, \alpha 2^3\, \xi_1 \wedge \xi_5 + \alpha 2^4\, \xi_1 \wedge \xi_5 -$$

$$\frac{\alpha 2^4\, \xi_1 \wedge \xi_5}{T} - \alpha 1^4\, \xi_3 \wedge \xi_5 + \frac{\alpha 1^4\, \xi_3 \wedge \xi_5}{T} - 4\, \alpha 1^3\, \alpha 2\, \xi_3 \wedge \xi_5 + \frac{3\, \alpha 1^3\, \alpha 2\, \xi_3 \wedge \xi_5}{T} + T\, \alpha 1^3\, \alpha 2\, \xi_3 \wedge \xi_5 -$$

$$6\, \alpha 1^2\, \alpha 2^2\, \xi_3 \wedge \xi_5 + \frac{3\, \alpha 1^2\, \alpha 2^2\, \xi_3 \wedge \xi_5}{T} + 3\, T\, \alpha 1^2\, \alpha 2^2\, \xi_3 \wedge \xi_5 - 4\, \alpha 1\, \alpha 2^3\, \xi_3 \wedge \xi_5 + \frac{\alpha 1\, \alpha 2^3\, \xi_3 \wedge \xi_5}{T} +$$

$$3\, T\, \alpha 1\, \alpha 2^3\, \xi_3 \wedge \xi_5 - \alpha 2^4\, \xi_3 \wedge \xi_5 + T\, \alpha 2^4\, \xi_3 \wedge \xi_5 - 2\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 + \frac{\alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1}{T} +$$

$$T\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 - 4\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 + \frac{2\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1}{T} + 2\, T\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 -$$

$$2\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 + \frac{\alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1}{T} + T\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_1 - 2\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 +$$

$$\frac{\alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3}{T} + T\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 - 4\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 + \frac{2\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3}{T} +$$

$$2\, T\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 - 2\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 + \frac{\alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3}{T} + T\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_3 -$$

$$2\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 + \frac{\alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5}{T} + T\, \alpha 1^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 - 4\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 +$$

$$\frac{2\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5}{T} + 2\, T\, \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 - 2\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 + \frac{\alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5}{T} +$$

$$T\, \alpha 2^2\, x_2 \wedge x_4 \wedge x_6 \wedge \xi_5 - \frac{\alpha 1^2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_3}{T} + \alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_3 - \frac{\alpha 1\, \alpha 2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_3}{T} +$$

$$\alpha 2^2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_3 + \alpha 1^2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_5 - \frac{\alpha 1^2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_5}{T} - T\, \alpha 1^2\, x_2 \wedge x_4 \wedge \xi_1 \wedge \xi_5 +$$

$$3\,\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_1\wedge\xi_5-\frac{\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_1\wedge\xi_5}{T}-2\,T\,\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_1\wedge\xi_5+2\,\alpha 2^2\,x_2\wedge x_4\wedge\xi_1\wedge\xi_5-$$
$$T\,\alpha 2^2\,x_2\wedge x_4\wedge\xi_1\wedge\xi_5+\alpha 1^2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5-\frac{\alpha 1^2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5}{T}-T\,\alpha 1^2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5+$$
$$3\,\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5-\frac{\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5}{T}-2\,T\,\alpha 1\,\alpha 2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5+2\,\alpha 2^2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5-$$
$$T\,\alpha 2^2\,x_2\wedge x_4\wedge\xi_3\wedge\xi_5-2\,\alpha 1^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3+T\,\alpha 1^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3-3\,\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3+$$
$$\frac{\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3}{T}+2\,T\,\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3-\alpha 2^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3+\frac{\alpha 2^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3}{T}+$$
$$T\,\alpha 2^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_3-\frac{\alpha 1^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_5}{T}+\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_5-\frac{\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_5}{T}+$$
$$\alpha 2^2\,x_2\wedge x_6\wedge\xi_1\wedge\xi_5+\alpha 1^2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5-\frac{\alpha 1^2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5}{T}-T\,\alpha 1^2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5+$$
$$3\,\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5-\frac{\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5}{T}-2\,T\,\alpha 1\,\alpha 2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5+2\,\alpha 2^2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5-$$
$$T\,\alpha 2^2\,x_2\wedge x_6\wedge\xi_3\wedge\xi_5-\alpha 1^2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5+T\,\alpha 1^2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5-2\,\alpha 1\,\alpha 2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5+$$
$$2\,T\,\alpha 1\,\alpha 2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5-\alpha 2^2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5+T\,\alpha 2^2\,x_2\wedge\xi_1\wedge\xi_3\wedge\xi_5-2\,\alpha 1^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3+$$
$$T\,\alpha 1^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3-3\,\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3+\frac{\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3}{T}+2\,T\,\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3-$$
$$\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3+\frac{\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3}{T}+T\,\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_3-2\,\alpha 1^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5+$$
$$T\,\alpha 1^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5-3\,\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5+\frac{\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5}{T}+2\,T\,\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5-$$
$$\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5+\frac{\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5}{T}+T\,\alpha 2^2\,x_4\wedge x_6\wedge\xi_1\wedge\xi_5-\frac{\alpha 1^2\,x_4\wedge x_6\wedge\xi_3\wedge\xi_5}{T}+$$
$$\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_3\wedge\xi_5-\frac{\alpha 1\,\alpha 2\,x_4\wedge x_6\wedge\xi_3\wedge\xi_5}{T}+\alpha 2^2\,x_4\wedge x_6\wedge\xi_3\wedge\xi_5-\alpha 1^2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5+$$
$$T\,\alpha 1^2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5-2\,\alpha 1\,\alpha 2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5+2\,T\,\alpha 1\,\alpha 2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5-\alpha 2^2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5+$$
$$T\,\alpha 2^2\,x_4\wedge\xi_1\wedge\xi_3\wedge\xi_5-\alpha 1^2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5+T\,\alpha 1^2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5-2\,\alpha 1\,\alpha 2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5+$$
$$2\,T\,\alpha 1\,\alpha 2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5-\alpha 2^2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5+T\,\alpha 2^2\,x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5+x_2\wedge x_4\wedge x_6\wedge\xi_1\wedge\xi_3\wedge\xi_5$$

Out[∘]=

```
True
```

In[∘]:=
```
Coefficient[A1, Wedge[]] // Simplify
```

Out[∘]=

$$\frac{(-1+T)^2\,(\alpha 1+\alpha 2)^5\,(\alpha 1-T\,\alpha 2)}{T^3}$$

In[∘]:= **Coefficient[A1, $x_2\wedge x_4$] // Simplify**

Out[∘]=

$$\frac{(-1+T)^2\,(\alpha 1+\alpha 2)^3\,(-\alpha 1+T\,\alpha 2)}{T^2}$$

In[•]:= **Coefficient[A1, $x_4 \wedge x_6$] // Simplify**

Out[•]=

$$\frac{(-1+T)^2\,(\alpha 1+\alpha 2)^3\,(-\alpha 1+T\,\alpha 2)}{T^2}$$

In[•]:= **Coefficient[A1, $x_2 \wedge x_6$] // Simplify**

Out[•]=

$$-\frac{(-1+T)^2\,(T\,\alpha 1-\alpha 2)\,(\alpha 1+\alpha 2)^3}{T^2}$$