



Chern-Simons-Witten Theory Near The Co-Commutative Limit

Abstract. Perhaps every algebra meeting should have one analysis talk (and vice versa), lest we forget that the other exists. In my role as the outsider, I will tell you today about the other – perturbative – evaluation of path integrals, where instead of hoping that nature will help us compute faster, we approximate nature by things we already can compute quickly.



van der Veen

Specifically I will tell you how in the Chern-Simons-Witten theory you can perturb the base Lie algebra from where it's easy towards where it's strong, leading to the strongest genuinely computable knot invariant we presently have.

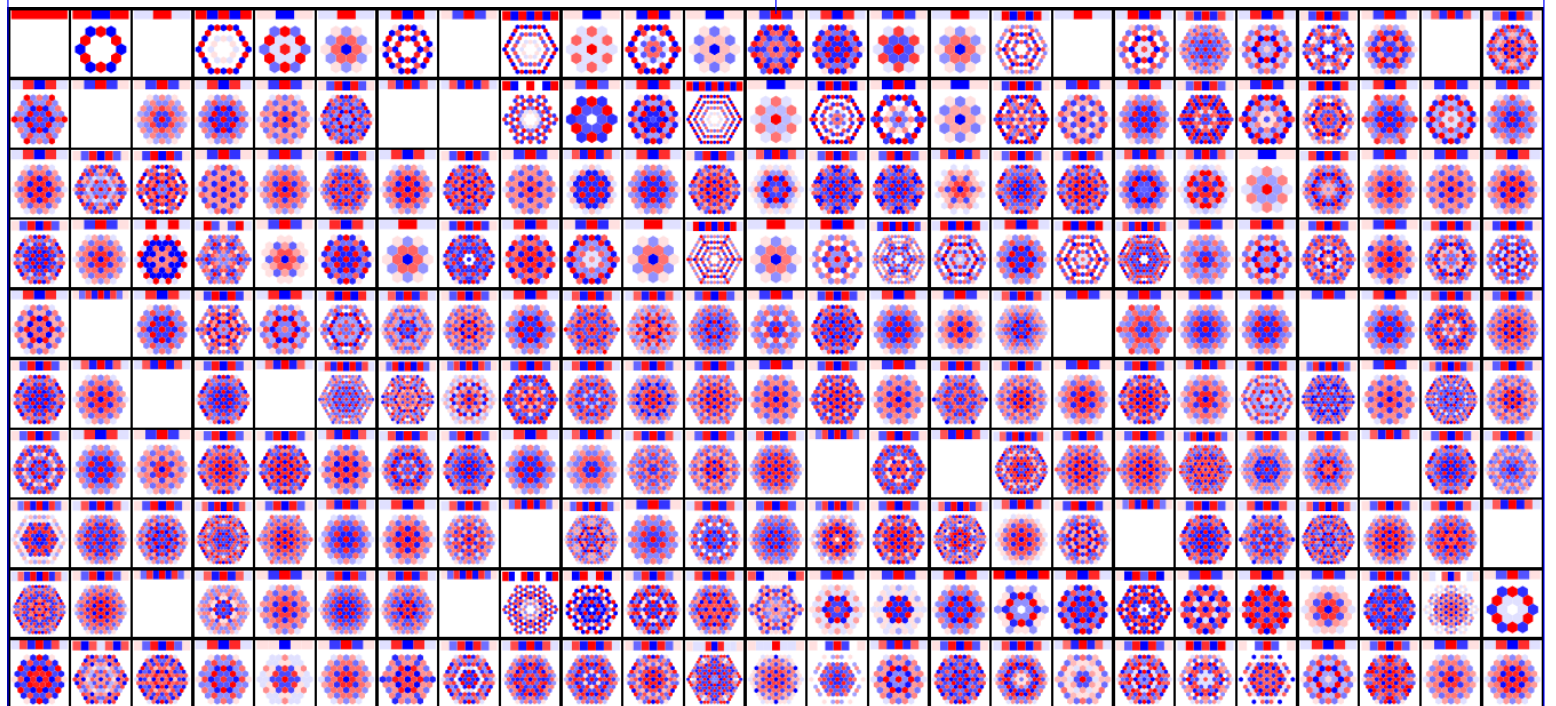
I wish I could give my talk in the language of the Kabbalah, but I ain't smart enough for that. So I'll highlight the Kabbalistic points that we're still missing, and then stick to the Talmud.

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Talmud vs Kabbalah

AI Overview

The Talmud is the foundational, comprehensive compilation of Jewish law (Halacha) and legal debates, focusing on the "how" of daily practice. Kabbalah is the esoteric, mystical tradition of Judaism, focusing on the inner meaning of the Torah, the nature of God, and spiritual universes. Both are considered part of the Oral Law tradition. Ohr Somayach +2



TBW.

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- [BN3] —, *Homework*, talk in Les Diablerets, January 2026. Handout and video at [ωεβ/HW](#).
- [BV1] —, R. van der Veen, *A Perturbed-Alexander Invariant*, *Quantum Topology* **15** (2024) 449–472, [ωεβ/APAI](#).
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- [Sch] S. Schaveling, *Expansions of Quantum Group Invariants*, Ph.D. thesis, Universiteit Leiden, September 2020, [ωεβ/Scha](#).