Pensieve header: Demo of NOE-0 and NOE-1t for LesDiablerets-1608. Follows pensieve://Projects/OneCo-1606/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\LesDiablerets-1608"];
```

NOE-0

0R

```
R⁺[0,i_,j_] := 𝔼[b_i c_j + b_i^-1 (e^b_i - 1) u_i w_j]; R⁻[0,i_,j_] := 𝔼[-b_i c_j + b_i^-1 (e^-b_i - 1) u_i w_j];
```

0Util

```
CF[ω_ . 𝔼[Q_]] := Simplify[ω] 𝔼[Simplify[Q]];
𝔼 /: 𝔼[Q1_] 𝔼[Q2_] := CF@𝔼[Q1 + Q2];
ω1_ . 𝔼[Q1_] ≡ ω2_ . 𝔼[Q2_] := Simplify[ω1 == ω2 ∧ Q1 == Q2];
```

0NO

```
N[u_i c_j→k_][ω_ . 𝔼[Q_]] := CF[
   ω 𝔼[e^-γ β u_k + γ c_k + (Q /. c_j | u_i → 0)]] /. {γ → ∂[c_j] Q, β → ∂[u_i] Q}];
N[w_i c_j→k_][ω_ . 𝔼[Q_]] := CF[
   ω 𝔼[e^γ α w_k + γ c_k + (Q /. c_j | w_i → 0)]] /. {γ → ∂[c_j] Q, α → ∂[w_i] Q}];
N[w_i u_j→k_][ω_ . 𝔼[Q_]] := CF[
   ν ω 𝔼[-b_k ν α β + ν β u_k + ν δ u_k w_k + ν α w_k + (Q /. w_i | u_j → 0)]] /. ν → (1 + b_k δ)^-1 /.
    {α → ∂[w_i] Q /. u_j → 0, β → ∂[u_j] Q /. w_i → 0, δ → ∂[w_i,u_j] Q}];
```

0m

```
m[i_,j_→k_][ω_ . 𝔼[Q_]] := CF[Module[{x},
   (ω 𝔼[Q] /. b_i|j → b_k // N[w_i c_j→x] // N[u_i c_x→x] // N[w_x u_j→x]) /. {c_i → c_k, w_j → w_k, y_x :> y_k}]]
```

T00

```
T[0,0] = R⁺[0,5,1] R⁺[0,2,4] R⁻[0,3,6]
```

T00

$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{\left(-1+e^{b_5}\right) u_5 w_1}{b_5} + \frac{\left(-1+e^{b_2}\right) u_2 w_4}{b_2} + \frac{\left(-1+e^{-b_3}\right) u_3 w_6}{b_3}\right]$$

T01

```
T[0,1] = T[0,0] // N[u_3 c_4→4]
```

T01

$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{\left(-1+e^{b_5}\right) u_5 w_1}{b_5} + \frac{\left(-1+e^{b_2}\right) u_2 w_4}{b_2} + \frac{e^{-b_2}\left(-1+e^{-b_3}\right) u_4 w_6}{b_3}\right]$$

T02

```
T[0,2] = T[0,1] // N[w_4 u_5→4]
```

T02

$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 + \frac{\left(-1+e^{b_5}\right)\left(-\left(-1+e^{b_2}\right) b_4 u_2 + b_2 u_4\right) w_1}{b_2 b_5} + \frac{\left(-1+e^{b_2}\right) u_2 w_4}{b_2} - \frac{b_3^2 c_6 + e^{-b_2-b_3}\left(-1+e^{b_3}\right) u_4 w_6}{b_3}\right]$$

T03

```
T[0,2] // N[w_1 u_2→1]
```

T03

$$\frac{1}{1-\frac{\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_4}{b_2 b_5}}\,\mathbb{E}\Big[\frac{1}{b_3\left(\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_4 - b_2 b_5\right)}$$
$$\left(b_3 b_5\left(\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_4 - b_2 b_5\right) c_1 + b_2 b_3\left(\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_4 - b_2 b_5\right) c_4 +\right.$$
$$\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_3 b_4 u_1 w_1 - \left(-1+e^{b_5}\right) b_2 b_3 u_4 w_1 - \left(-1+e^{b_2}\right) b_3 b_5 u_1 w_4 +$$
$$\left.\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_3 u_4 w_4 - \left(\left(-1+e^{b_2}\right)\left(-1+e^{b_5}\right) b_1 b_4 - b_2 b_5\right)\left(b_3^2 c_6 + e^{-b_2-b_3}\left(-1+e^{b_3}\right) u_4 w_6\right)\right)\Big]$$

T04

```
T_{0,0} // m_{1,2→1} // m_{3,4→3} // m_{3,5→3} // m_{3,6→3}
```

T04

$$\mathbb{E}\left[b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{\left(-1+e^{b_1}\right)\left(-1+e^{b_3}\right) u_1 w_1}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_1} - \frac{e^{-b_3}\left(-1+e^{b_1}\right)\left(b_3 u_1 - e^{b_3}\left(-1+e^{b_3}\right) b_1 u_3\right) w_3}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_1 b_3} + \frac{e^{-b_1}\left(-1+e^{b_3}\right) u_3 \left(-e^{b_1+b_3} w_1 + \left(e^{b_1}+e^{b_3}-e^{b_1+b_3}\right) w_3\right)}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_3}\right] \Big/ \left(1-\left(-1+e^{b_1}\right)\left(-1+e^{b_3}\right)\right)$$

0Q0

```
Q0 = 𝔼[Sum[f_i c_i, {i, 3}] + Sum[f_{i,j} u_i w_j, {i, 3}, {j, 3}]]
```

0Q0

$$\mathbb{E}\left[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1,1} + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + u_2 w_1 f_{2,1} + u_2 w_2 f_{2,2} + u_2 w_3 f_{2,3} + u_3 w_1 f_{3,1} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}\right]$$

0NODemo

```
Q0 // N_{w_1 u_2→3}
```

0NODemo

$$\frac{1}{1+b_3 f_{2,1}}\mathbb{E}\left[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + \frac{u_3 w_3 f_{2,1}}{1+b_3 f_{2,1}} + \frac{u_3\left(w_2 f_{2,2} + w_3 f_{2,3}\right)}{1+b_3 f_{2,1}} + \frac{w_3\left(u_1 f_{1,1} + u_3 f_{3,1}\right)}{1+b_3 f_{2,1}} - \frac{b_3\left(w_2 f_{2,2} + w_3 f_{2,3}\right)\left(u_1 f_{1,1} + u_3 f_{3,1}\right)}{1+b_3 f_{2,1}} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}\right]$$

0mDemo

```
Q0 // m_{1,2→1}
```

0mDemo

$$\frac{1}{1+e^{f_2} b_1 f_{2,1}}\mathbb{E}\left[c_1 f_1 + c_1 f_2 + c_3 f_3 + e^{-f_2} u_1\left(w_1 f_{1,2} + w_3 f_{1,3}\right) + \frac{e^{f_2} u_1 w_1 f_{2,1}}{1+e^{f_2} b_1 f_{2,1}} + \frac{u_1\left(w_1 f_{2,2} + w_3 f_{2,3}\right)}{1+e^{f_2} b_1 f_{2,1}} + \frac{w_1\left(u_1 f_{1,1} + e^{f_2} u_3 f_{3,1}\right)}{1+e^{f_2} b_1 f_{2,1}} - \frac{b_1\left(w_1 f_{2,2} + w_3 f_{2,3}\right)\left(u_1 f_{1,1} + e^{f_2} u_3 f_{3,1}\right)}{1+e^{f_2} b_1 f_{2,1}} + u_3 w_1 f_{3,2} + u_3 w_3 f_{3,3}\right]$$

0MetaAssoc

```
(Q0 // m_{1,2→1} // m_{1,3→1}) ≡ (Q0 // m_{2,3→2} // m_{1,2→1})
```

0MetaAssoc

```
True
```

0R3Left

```
t1 = R^+_{0,1,2} R^+_{0,3,4} R^+_{0,5,6} // m_{3,5→x} // m_{1,6→y} // m_{2,4→z}
```

0R3Left

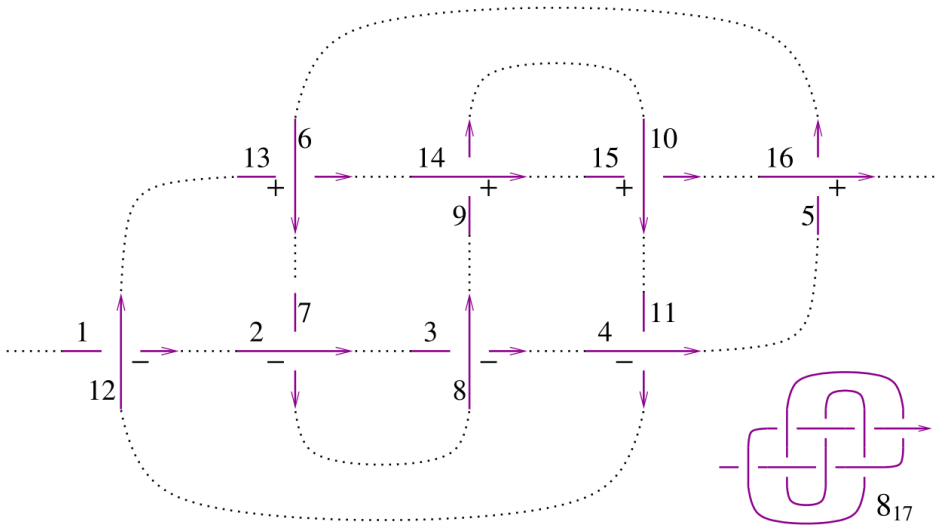
$$\mathbb{E}\left[b_x\left(c_y + c_z\right) + \frac{\left(-1+e^{b_x}\right) u_x\left(w_y + w_z\right)}{b_x} + \frac{b_y^2 c_z + \left(-1+e^{b_y}\right) u_y w_z}{b_y}\right]$$

0R3

```
t1 ≡ (R^+_{0,1,2} R^+_{0,3,4} R^+_{0,5,6} // m_{1,3→x} // m_{2,5→y} // m_{4,6→z})
```

0R3

```
True
```



0817

```
z1 = R⁻[0,12,1] R⁻[0,2,7] R⁻[0,8,3] R⁻[0,4,11] R⁺[0,16,5] R⁺[0,6,13] R⁺[0,14,9] R⁺[0,10,15];
Do[z1 = (z1 // m[1,n→1]) /. b_ → b, {n, 2, 16}];
{CF@z1, KnotData[{8, 17}, "AlexanderPolynomial"][t]}
```

0817

$$\left\{-\frac{e^{3b}\,\mathbb{E}[0]}{1-4e^{b}+8e^{2b}-11e^{3b}+8e^{4b}-4e^{5b}+e^{6b}},\ 11-\frac{1}{t^3}+\frac{4}{t^2}-\frac{8}{t}-8t+4t^2-t^3\right\}$$

NOE-1t

Logos

```
Λ[k_] := (1 - t_k) (α² β² + 4 α β δ μ + 2 δ² μ²) / 2 + 2 μ² (α β + δ μ) c_k - β (2 μ - 1) (α β + 2 δ μ) u_k +
   2 β δ μ² c_k u_k - β² δ (3 μ - 1) u_k² / 2 + α (α β + 2 δ μ) w_k + 2 α δ μ² c_k w_k - 2 (t_k - 1) δ² (α β + δ μ) u_k w_k +
   2 δ² μ² c_k u_k w_k - β δ² (2 μ - 1) u_k² w_k + α² δ (1 + μ) w_k² / 2 + α δ² u_k w_k² - (t_k - 1) δ⁴ u_k² w_k² / 2;
```

1DP

```
DP[x_→D_α, y_→D_β_][P_][f_] := (* means P[∂_α, ∂_β][f] *)
 Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[E[ω_, L_, Q_, P_]] := Expand /@ Together /@
    E[ω /. b_L :> Log[t_L], L, Q /. b_L :> Log[t_L], P /. b_L :> Log[t_L]];
E /: E[ω1_, L1_, Q1_, P1_] E[ω2_, L2_, Q2_, P2_] := CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2⁴ P1 + ω1⁴ P2];
```

1NOc

```
N[u_i, c_j_→k_][E[ω_, L_, Q_, P_]] := With[{q = e^(-γ) β u_k + γ c_k}, CF[
    E[ω, γ c_k + (L /. c_j → 0), ω e^(-γ) β u_k + (Q /. u_i → 0), e^(-q) DP[c_j→D_γ, u_i→D_β][P][e^q]] /. {γ → ∂_(c_j) L, β → ω^-1 ∂_(u_i) Q}]];
N[w_i, c_j_→k_][E[ω_, L_, Q_, P_]] := With[{q = e^γ α w_k + γ c_k}, CF[
    E[ω, γ c_k + (L /. c_j → 0), ω e^γ α w_k + (Q /. w_i → 0), e^(-q) DP[c_j→D_γ, w_i→D_α][P][e^q]] /. {γ → ∂_(c_j) L, α → ω^-1 ∂_(w_i) Q}]];
```

1NOuw

```
N[w_i, u_j_→k_][E[ω_, L_, Q_, P_]] := With[{q = (1 - t_k) μ^-1 α β + μ^-1 β u_k + μ^-1 δ u_k w_k + μ^-1 α w_k}, CF[
    E[μ ω, L, μ ω q + μ (Q /. w_i | u_j → 0), μ⁴ e^(-q) DP[w_i→D_α, u_j→D_β][P][e^q] + ω⁴ Λ[k]] /. μ → 1 + (t_k - 1) δ /.
     {α → ω^-1 (∂_(w_i) Q /. u_j → 0), β → ω^-1 (∂_(u_j) Q /. w_i → 0), δ → ω^-1 ∂_(w_i, u_j) Q}]];
```

1m

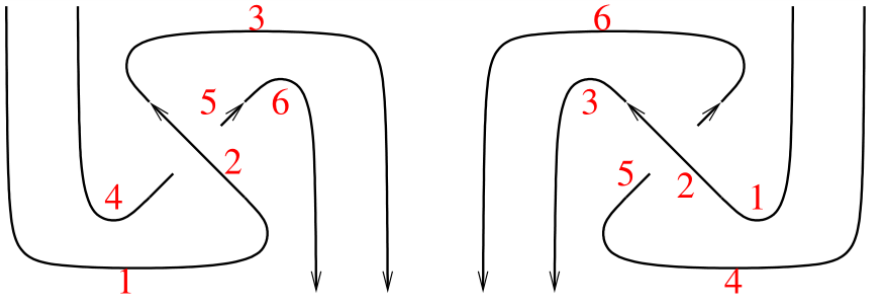
```
m[i_,j_→k_][Z_] := Module[{x, y, z},
  Z // N[w_i c_j→x] // N[w_x u_j→y] // ReplaceAll[{c_x|y → c_x, w_j → w_y}] // N[u_i c_x→x] // ReplaceAll[z_i|j|x|y → z_k] // CF]
```

1Gens

```
R⁺[i_,j_] := E[1, b_i c_j, u_i w_j, -c_i (t_i - 1)² / 2 - c_i² (t_i - 1)² / 2 + c_i c_j (t_j² - t_i - 2) / 2 - c_j u_i w_i / 2 + c_i (1 - t_i) u_i w_i -
    u_i² w_i² / 2 + u_i w_j + c_j t_i u_i w_j / 2 + c_i (t_i - 2) t_i u_i w_j + c_i (1 + t_j) u_j w_j / 2 + (t_i - 1) u_i² w_i w_j - (t_i - 2) t_i u_i² w_j² / 2];
R⁻[i_,j_] := E[1, -b_i c_j, -t_i^-1 u_i w_j, c_i (t_i - 1)² / 2 + c_i² (t_i - 1)² / 2 + c_i c_j (2 + t_i - t_j²) / 2 + c_j u_i w_i / 2 +
    c_i (t_i - 1) u_i w_i + u_i² w_i² / 2 + (1 - t_i^-1) u_i w_j / 2 + c_i (2 t_i - 5 + 3 t_i^-1) u_i w_j / 2 + c_j (t_i^-1 + 1 - t_i^-1 t_j²) u_i w_j / 2 -
    c_i (t_j + 1) u_j w_j / 2 + (2 - 3 t_i^-1) u_i² w_i w_j / 2 + (1 + 2 t_i^-2 - 3 t_i^-1) u_i² w_j² / 2 - t_i^-1 (1 + t_j) u_i u_j w_j² / 2];
ur_i := E[t_i^-1/4, 0, 0, c_i t_i / 4 + u_i w_i / 8];
nr_i := E[t_i^1/4, 0, 0, -c_i t_i³ / 4 - t_i² u_i w_i / 8];
ul_i := E[t_i^1/4, 0, 0, c_i t_i (4 + t_i) / 4 - t_i² u_i w_i / 8];
nl_i := E[t_i^-1/4, 0, 0, -c_i (1 + 4 t_i^-1) / 4 + u_i w_i / 8];
```

```
Series[Last@R⁺[1,2] /. {u_i :> ħ u_i, t_i :> e^(ħ b_i)}, {ħ, 0, 1}]
```

$$-c_1 c_2+\left(-\frac{1}{2}b_1 c_1 c_2+b_2 c_1 c_2-\frac{1}{2}c_2 u_1 w_1+u_1 w_2-c_1 u_1 w_2+\frac{1}{2}c_2 u_1 w_2+c_1 u_2 w_2\right)\hbar+O[\hbar]^2$$



1SwirlLeft

```
t2 = ur1 R-2,5 nr3 ur4 nr6 // m1,2→1 // m1,3→1 // m4,5→4 // m4,6→4
```

1SwirlLeft

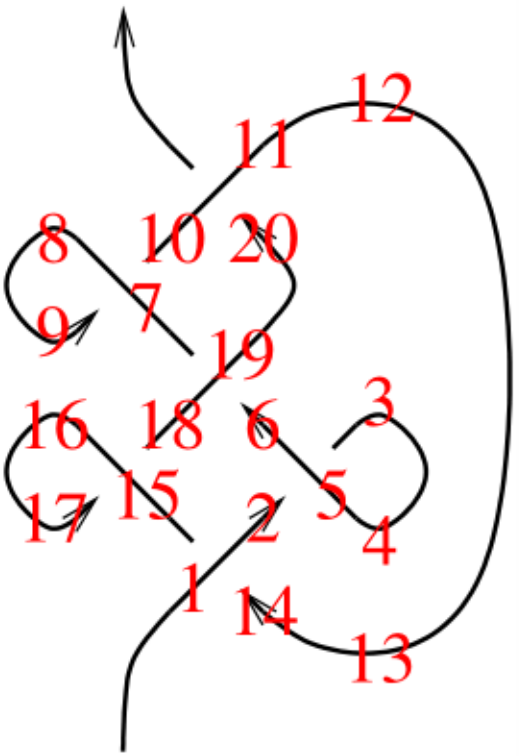
$$\mathbb{E}\Big[1,\ -b_1\,c_4,\ -\frac{u_1\,w_4}{t_1},\ \frac{c_1}{2}+\frac{c_1^2}{2}+c_1\,c_4-c_1\,t_1-c_1^2\,t_1+\frac{1}{2}\,c_1\,c_4\,t_1+\frac{1}{2}\,c_1\,t_1^2+\frac{1}{2}\,c_1^2\,t_1^2-\frac{1}{2}\,c_1\,c_4\,t_4^2-c_1\,u_1\,w_1+\frac{1}{2}\,c_4\,u_1\,w_1+$$
$$c_1\,t_1\,u_1\,w_1+\frac{1}{2}\,u_1^2\,w_1^2+\frac{3\,u_1\,w_4}{8}-\frac{5}{2}\,c_1\,u_1\,w_4+\frac{1}{2}\,c_4\,u_1\,w_4-\frac{u_1\,w_4}{2\,t_1}+\frac{3\,c_1\,u_1\,w_4}{2\,t_1}+\frac{c_4\,u_1\,w_4}{2\,t_1}-\frac{1}{8}\,t_1\,u_1\,w_4+c_1\,t_1\,u_1\,w_4+\frac{t_4\,u_1\,w_4}{8\,t_1}+$$
$$\frac{t_4^2\,u_1\,w_4}{8\,t_1}-\frac{c_4\,t_4^2\,u_1\,w_4}{2\,t_1}-\frac{1}{2}\,c_1\,u_4\,w_4-\frac{1}{2}\,c_1\,t_4\,u_4\,w_4+u_1^2\,w_1\,w_4-\frac{3\,u_1^2\,w_1\,w_4}{2\,t_1}+\frac{1}{2}\,u_1^2\,w_4^2+\frac{u_1^2\,w_4^2}{t_1^2}-\frac{3\,u_1^2\,w_4^2}{2\,t_1}-\frac{u_1\,u_4\,w_4^2}{2\,t_1}-\frac{t_4\,u_1\,u_4\,w_4^2}{2\,t_1}\Big]$$

1Swirl

```
t2 == (ul1 R-2,5 nl3 ul4 nl6 // m1,2→1 // m1,3→1 // m4,5→4 // m4,6→4)
```

1Swirl

```
True
```



131

```
z2 = R+1,14 R-5,2 nr3 ul4 R+19,6 R-7,10 nl8 ur9 R+11,20 nr12 ul13 R-15,18 nl16 ur17;
(Do[z2 = z2 // mi,k→1, {k, 2, 20}]; z2 = z2 /. a_1 :→ a)
```

131

$$\mathbb{E}\Big[-1+\frac{1}{t}+t,\ 0,\ 0,\ -16+\frac{9\,c}{2}-\frac{2\,c}{t^4}+\frac{1}{t^3}+\frac{11\,c}{2\,t^3}-\frac{4}{t^2}-\frac{8\,c}{t^2}+\frac{10}{t}+\frac{4\,c}{t}+18\,t-10\,c\,t-14\,t^2+8\,c\,t^2+7\,t^3-\frac{3\,c\,t^3}{2}-$$
$$2\,t^4-2\,c\,t^4+2\,c\,t^5-\frac{c\,t^6}{2}-4\,u\,w+\frac{2\,u\,w}{t^4}-\frac{7\,u\,w}{2\,t^3}+\frac{9\,u\,w}{2\,t^2}+\frac{u\,w}{2\,t}+6\,t\,u\,w-2\,t^2\,u\,w-\frac{1}{2}\,t^3\,u\,w+\frac{3}{2}\,t^4\,u\,w-\frac{1}{2}\,t^5\,u\,w\Big]$$

131a

```
FromCoefficientRules[CoefficientRules[z2[[4]], {c, u, w}] /. {(e_ → a_) :> (e → Simplify[a])}, {c, u, w}]
```

131a

$$-\frac{\left(1-t+t^2\right)^2\left(-1+2t-3t^2+2t^3\right)}{t^3}-\frac{c\left(1-t+t^2\right)^3\left(4+t-5t^2-t^3+t^4\right)}{2t^4}-\frac{\left(1-t+t^2\right)^3\left(-4-5t+t^3\right)uw}{2t^4}$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]

SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["ct-def"] = {PageWidth → 6/0.66};
Options[CellExport] = {
   PageWidth → 4/0.66, CellFilter → Identity, ExportDirectory → "Snips",
   ExportBaseFilename → Automatic, ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
CellExport[tag_String, opts___Rule] := CellExport[
   NotebookGet[EvaluationNotebook[]],
   tag, opts
  ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
   {cells, cell, filename, format},
   filename = FileNameJoin[{
      OptionValue[ExportDirectory] /. Automatic → Directory[],
      OptionValue[ExportBaseFilename] /. Automatic → tag
     }];
   format = OptionValue[ExportFormat];
   cells = OptionValue[CellFilter][Cases[
      nb, c_Cell /; FreeQ[List @@ c, Cell] && ! FreeQ[c, CellTags → tag],
      Infinity
     ]];
   If[! OptionValue[Split],
    If[Length[cells] ≥ 1,
     If[Length[cells] == 1,
      cells = Join[First[cells],
        Cell[PageWidth → 1.2 × 72 OptionValue[PageWidth], Background → {White, Opacity[0]}]],
      cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
     ];
     ConditionalExport[
      filename <> format, cells,
      ImageResolution → 300,
      OptionValue[ExportOptions]
```

```
      ]
     ],
     k = 0;
     Table[
      ++k;
      ConditionalExport[
       filename <> "-" <> ToString[k] <> format,
       Append[cell, PageWidth → 72 OptionValue[PageWidth]],
       ImageResolution → 300,
       OptionValue[ExportOptions]
      ],
      {cell, cells}
     ]
    ]
   ];

ExportCells := (
    nb = NotebookGet[EvaluationNotebook[]];
    tags = Cases[nb, (CellTags → tag_String) :> tag, Infinity] // Union;
    Print[tags];
    CellExport /@ tags;
    Print["Done."]
   );
```

ExportCells

```
{0817, 0m, 0mDemo, 0MetaAssoc, 0NO, 0NODemo, 0Q0, 0R, 0R3, 0R3Left, 0Util, 131,
 131a, 1DP, 1Gens, 1m, 1NOc, 1NOuw, 1Swirl, 1SwirlLeft, 1Util, Logos, T00, T01, T02, T03, T04}

Exporting Snips\1m.pdf...

Done.
```