Acronym: AQDW

Title: The Alexander Polynomial is a Quantum Invariant in a Different Way.

Abstract: [the last paragraph of the sidebar].

Sidebar: On a chat window here I saw a comment “Alexander is the quantum gl(1|1) invariant”. I have an opinion about this, and I’d like to share it. First, some stories.

I left the wonderful subject of Categorification nearly 15 years ago. It got crowded, lots of very smart people had things to say, and I feared I will have nothing to add. Also, clearly the next step was to categorify all other “quantum invariants”. Except it was not clear what “categorify” means. Worse, I felt that I (perhaps “we all”) didn’t understand “quantum invariants” well enough to try to categorify them, whatever that may mean.

I still feel that way! I learned a lot since 2006, yet I’m still not comfortable with quantum algebra, quantum groups, and quantum invariants. I still don’t feel that I know what God had in mind when She created this topic.

Yet I’m not here to rant about my philosophical quandaries, but only about things that I learned about the Alexander polynomial after 2006.

Yes, the Alexander polynomial fits within the Dogma, “one invariant for every Lie algebra and representation” (it’s gl(1|1), I hear). But it’s better to think of it as a quantum invariant arising by other means, outside the Dogma.

It comes from (or in) practically any non-Abelian Lie algebra. Foremost from the not-even-semi-simple 2D ax+b algebra. You get a polynomially-sized extension to tangles using some lovely formulas (can you categorify them?). It generalizes to higher dimensions and it has an organized family of siblings. (There are some questions too, beyond categorification).

I note the spectacular existing categorification of Alexander by Ozsváth and Szabó. The theorems are proven and a lot they say, the programs run and fast they run. Yet if that’s where the story ends, She has abandoned us. Or at least abandoned me: a simpleton will never be able to catch up.

If you care only about categorification, the take-home from my talk will be a challenge: Categorify what I believe is the best Alexander invariant for tangles.

Plan:

1. (2m) Thanks, technicalities.
2. (4m) Read the sidebar.
3. (4m) Quantum invariants in an algebra and the read-out issue.
4. (2m) The Dogma and the exp-issue.
5. (5m) For ax+b, get Gaussians! (these are easily computable as we shall see),
6. (3m) In general, get “docile perturbed Gaussians”; the meaning of \eps (still efficiently computable!).
7. (8m) The “Gold Standard” theorem.
8. (8m) Ending discussion.
9. (24m) Full computability.

Notes: Landscape handout to fit in screens? Make a Star Wars opening roll (web-search “star wars intro creator”).