Pensieve header: Demo of NOE-0 and NOE-1t for Indiana-1611. Follows pensieve://Projects/OneCo-1606/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Indiana-1611"];
```

NOE-0

0R

```
R0+,i_,j_ := 𝔼[bi cj + bi^-1 (e^bi - 1) ui wj]; R0-,i_,j_ := 𝔼[-bi cj + bi^-1 (e^-bi - 1) ui wj];
```

0Util

```
CF[ω_ . 𝔼[Q_]] := Simplify[ω] 𝔼[Simplify[Q]];
𝔼 /: 𝔼[Q1_] 𝔼[Q2_] := CF@𝔼[Q1 + Q2];
ω1_ . 𝔼[Q1_] ≡ ω2_ . 𝔼[Q2_] := Simplify[ω1 == ω2 ∧ Q1 == Q2];
```

0NO

```
Nui_ cj_→k_[ω_ . 𝔼[Q_]] := CF[
   ω 𝔼[e^-γ β uk + γ ck + (Q /. cj | ui → 0)]] /. {γ → ∂cj Q, β → ∂ui Q}];
Nwi_ cj_→k_[ω_ . 𝔼[Q_]] := CF[
   ω 𝔼[e^γ α wk + γ ck + (Q /. cj | wi → 0)]] /. {γ → ∂cj Q, α → ∂wi Q}];
Nwi_ uj_→k_[ω_ . 𝔼[Q_]] := CF[
   ν ω 𝔼[-bk ν α β + ν β uk + ν δ uk wk + ν α wk + (Q /. wi | uj → 0)]] /. ν → (1 + bk δ)^-1 /.
    {α → ∂wi Q /. uj → 0, β → ∂uj Q /. wi → 0, δ → ∂wi,uj Q}];
```

0m

```
mi_,j_→k_[ω_ . 𝔼[Q_]] := CF[Module[{x},
   (ω 𝔼[Q] /. bi|j → bk // Nwi cj→x // Nui cx→x // Nwx uj→x) /. {ci → ck, wj → wk, y_x :> yk}]]
```

T0

```
T0 = R0+,5,1 R0+,2,4 R0-,3,6
```

T0

$$\mathbb{E}\left[b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{\left(-1+e^{b_5}\right) u_5 w_1}{b_5} + \frac{\left(-1+e^{b_2}\right) u_2 w_4}{b_2} + \frac{\left(-1+e^{-b_3}\right) u_3 w_6}{b_3}\right]$$

ZT0

```
T0 // m1,2→1 // m3,4→3 // m3,5→3 // m3,6→3
```

ZT0

$$\mathbb{E}\left[b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{\left(-1+e^{b_1}\right)\left(-1+e^{b_3}\right) u_1 w_1}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_1} - \frac{e^{-b_3}\left(-1+e^{b_1}\right)\left(b_3 u_1 - e^{b_3}\left(-1+e^{b_3}\right) b_1 u_3\right) w_3}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_1 b_3} + \frac{e^{-b_1}\left(-1+e^{b_3}\right) u_3\left(-e^{b_1+b_3} w_1 + \left(e^{b_1}+e^{b_3}-e^{b_1+b_3}\right) w_3\right)}{\left(-e^{b_1}-e^{b_3}+e^{b_1+b_3}\right) b_3}\right] \Big/ \left(1 - \left(-1+e^{b_1}\right)\left(-1+e^{b_3}\right)\right)$$

0Q0

```
Q0 = 𝔼[Sum[fi ci, {i, 3}] + Sum[fi,j ui wj, {i, 3}, {j, 3}]]
```

0Q0

$$\mathbb{E}\left[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1,1} + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + u_2 w_1 f_{2,1} + u_2 w_2 f_{2,2} + u_2 w_3 f_{2,3} + u_3 w_1 f_{3,1} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}\right]$$

0NODemo

```
Q0 // Nw1 u2→3
```

0NODemo

$$\frac{1}{1+b_3 f_{2,1}}\mathbb{E}\left[c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_2 f_{1,2} + u_1 w_3 f_{1,3} + \frac{u_3 w_3 f_{2,1}}{1+b_3 f_{2,1}} + \frac{u_3\left(w_2 f_{2,2} + w_3 f_{2,3}\right)}{1+b_3 f_{2,1}} + \frac{w_3\left(u_1 f_{1,1} + u_3 f_{3,1}\right)}{1+b_3 f_{2,1}} - \frac{b_3\left(w_2 f_{2,2} + w_3 f_{2,3}\right)\left(u_1 f_{1,1} + u_3 f_{3,1}\right)}{1+b_3 f_{2,1}} + u_3 w_2 f_{3,2} + u_3 w_3 f_{3,3}\right]$$

0mDemo

```
Q0 // m1,2→1
```

0mDemo

$$\frac{1}{1+e^{f_2}\,b_1\,f_{2,1}}\mathbb{E}\left[c_1\,f_1+c_1\,f_2+c_3\,f_3+e^{-f_2}\,u_1\,\left(w_1\,f_{1,2}+w_3\,f_{1,3}\right)+\frac{e^{f_2}\,u_1\,w_1\,f_{2,1}}{1+e^{f_2}\,b_1\,f_{2,1}}+\frac{u_1\,\left(w_1\,f_{2,2}+w_3\,f_{2,3}\right)}{1+e^{f_2}\,b_1\,f_{2,1}}+\right.$$
$$\left.\frac{w_1\,\left(u_1\,f_{1,1}+e^{f_2}\,u_3\,f_{3,1}\right)}{1+e^{f_2}\,b_1\,f_{2,1}}-\frac{b_1\,\left(w_1\,f_{2,2}+w_3\,f_{2,3}\right)\,\left(u_1\,f_{1,1}+e^{f_2}\,u_3\,f_{3,1}\right)}{1+e^{f_2}\,b_1\,f_{2,1}}+u_3\,w_1\,f_{3,2}+u_3\,w_3\,f_{3,3}\right]$$

0MetaAssoc

```
(Q0 // m1,2→1 // m1,3→1) ≡ (Q0 // m2,3→2 // m1,2→1)
```

0MetaAssoc

```
True
```

0R3Left

```
t1 = R+0,1,2 R+0,3,4 R+0,5,6 // m3,5→x // m1,6→y // m2,4→z
```

0R3Left

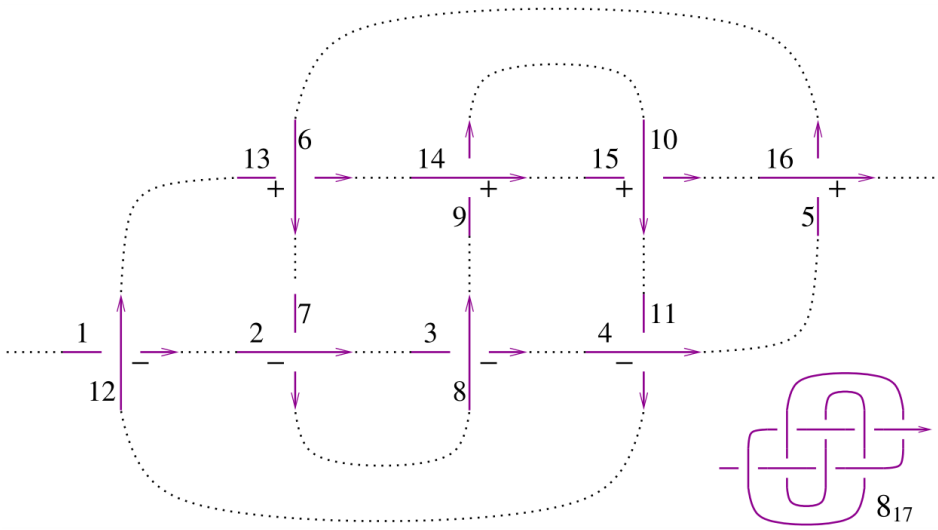
$$\mathbb{E}\left[b_x\,\left(c_y+c_z\right)+\frac{\left(-1+e^{b_x}\right)\,u_x\,\left(w_y+w_z\right)}{b_x}+\frac{b_y^2\,c_z+\left(-1+e^{b_y}\right)\,u_y\,w_z}{b_y}\right]$$

0R3

```
t1 ≡ (R+0,1,2 R+0,3,4 R+0,5,6 // m1,3→x // m2,5→y // m4,6→z)
```

0R3

```
True
```



0817

```
z1 = R-0,12,1 R-0,2,7 R-0,8,3 R-0,4,11 R+0,16,5 R+0,6,13 R+0,14,9 R+0,10,15;
Do[z1 = (z1 // m1,n→1) /. b_ → b, {n, 2, 16}];
{CF@z1, KnotData[{8, 17}, "AlexanderPolynomial"][t]}
```

0817

$$\left\{-\frac{e^{3\,b}\,\mathbb{E}[0]}{1-4\,e^{b}+8\,e^{2\,b}-11\,e^{3\,b}+8\,e^{4\,b}-4\,e^{5\,b}+e^{6\,b}},\ 11-\frac{1}{t^3}+\frac{4}{t^2}-\frac{8}{t}-8\,t+4\,t^2-t^3\right\}$$

NOE-It

Logos

```
Λ[k_] := (1 - tk) (α^2 β^2 + 4 α β δ μ + 2 δ^2 μ^2) / 2 + 2 μ^2 (α β + δ μ) ck - β (2 μ - 1) (α β + 2 δ μ) uk +
   2 β δ μ^2 ck uk - β^2 δ (3 μ - 1) uk^2 / 2 + α (α β + 2 δ μ) wk + 2 α δ μ^2 ck wk - 2 (tk - 1) δ^2 (α β + δ μ) uk wk +
   2 δ^2 μ^2 ck uk wk - β δ^2 (2 μ - 1) uk^2 wk + α^2 δ (1 + μ) wk^2 / 2 + α δ^2 uk wk^2 - (tk - 1) δ^4 uk^2 wk^2 / 2;
```

1DP

```
DPx_→Dα,y_→Dβ[P_][f_] := (* means P[∂α,∂β][f] *)
 Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[𝔼[ω_, L_, Q_, P_]] := Expand /@ Together /@
    𝔼[ω /. b_L_ :> Log[t_L], L, Q /. b_L_ :> Log[t_L], P /. b_L_ :> Log[t_L]];
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] := CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

1NOc

```
N[u_i_ c_j_→k_][𝔼[ω_, L_, Q_, P_]] := With[{q = e^-γ β u_k + γ c_k}, CF[
    𝔼[ω, γ c_k + (L /. c_j → 0), ω e^-γ β u_k + (Q /. u_i → 0), e^-q DP[c_j→D_γ, u_i→D_β][P][e^q]] /. {γ → ∂_c_j L, β → ω^-1 ∂_u_i Q}]];
N[w_i_ c_j_→k_][𝔼[ω_, L_, Q_, P_]] := With[{q = e^γ α w_k + γ c_k}, CF[
    𝔼[ω, γ c_k + (L /. c_j → 0), ω e^γ α w_k + (Q /. w_i → 0), e^-q DP[c_j→D_γ, w_i→D_α][P][e^q]] /. {γ → ∂_c_j L, α → ω^-1 ∂_w_i Q}]];
```

1NOuw

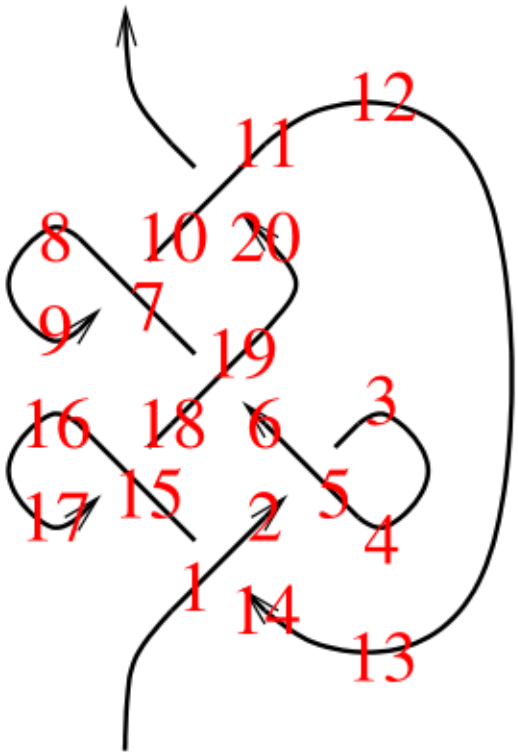
```
N[w_i_ u_j_→k_][𝔼[ω_, L_, Q_, P_]] := With[{q = (1 - t_k) μ^-1 α β + μ^-1 β u_k + μ^-1 δ u_k w_k + μ^-1 α w_k}, CF[
    𝔼[μ ω, L, μ ω q + μ (Q /. w_i | u_j → 0), μ^4 e^-q DP[w_i→D_α, u_j→D_β][P][e^q] + ω^4 Λ[k]] /. μ → 1 + (t_k - 1) δ /.
     {α → ω^-1 (∂_w_i Q /. u_j → 0), β → ω^-1 (∂_u_j Q /. w_i → 0), δ → ω^-1 ∂_w_i,u_j Q}]];
```

1m

```
m_i_,j_→k_[Z_] := Module[{x, y, z},
  Z // N[w_i c_j→x] // N[w_x u_j→y] // ReplaceAll[{c_x|y → c_x, w_j → w_y}] // N[u_i c_x→x] // ReplaceAll[z_i|j|x|y → z_k] // CF]
```

1Gens

```
R+_i_,j_ := 𝔼[1, b_i c_j, u_i w_j, -c_i (t_i - 1)^2 / 2 - c_i^2 (t_i - 1)^2 / 2 + c_i c_j (t_j^2 - t_i - 2) / 2 - c_j u_i w_i / 2 + c_i (1 - t_i) u_i w_i -
    u_i^2 w_i^2 / 2 + u_i w_j + c_j t_i u_i w_j / 2 + c_i (t_i - 2) t_i u_i w_j + c_i (1 + t_j) u_j w_j / 2 + (t_i - 1) u_i^2 w_i w_j - (t_i - 2) t_i u_i^2 w_j^2 / 2];
R-_i_,j_ := 𝔼[1, -b_i c_j, -t_i^-1 u_i w_j, c_i (t_i - 1)^2 / 2 + c_i^2 (t_i - 1)^2 / 2 + c_i c_j (2 + t_i - t_j^2) / 2 + c_j u_i w_i / 2 +
    c_i (t_i - 1) u_i w_i + u_i^2 w_i^2 / 2 + (1 - t_i^-1) u_i w_j / 2 + c_i (2 t_i - 5 + 3 t_i^-1) u_i w_j / 2 + c_j (t_i^-1 + 1 - t_i^-1 t_j^2) u_i w_j / 2 -
    c_i (t_j + 1) u_j w_j / 2 + (2 - 3 t_i^-1) u_i^2 w_i w_j / 2 + (1 + 2 t_i^-2 - 3 t_i^-1) u_i^2 w_j^2 / 2 - t_i^-1 (1 + t_j) u_i u_j w_j^2 / 2];
ur_i_ := 𝔼[t_i^-1/4, 0, 0, c_i t_i / 4 + u_i w_i / 8];
nr_i_ := 𝔼[t_i^1/4, 0, 0, -c_i t_i^3 / 4 - t_i^2 u_i w_i / 8];
ul_i_ := 𝔼[t_i^1/4, 0, 0, c_i t_i (4 + t_i) / 4 - t_i^2 u_i w_i / 8];
nl_i_ := 𝔼[t_i^-1/4, 0, 0, -c_i (1 + 4 t_i^-1) / 4 + u_i w_i / 8];
```



131

```
z2 = R+1,14 R-5,2 nr3 ul4 R+19,6 R-7,10 nl8 ur9 R+11,20 nr12 ul13 R-15,18 nl16 ur17;
(Do[z2 = z2 // m1,k→1, {k, 2, 20}]; z2 = z2 /. a_1 :→ a)
```

131

$$\mathbb{E}\Big[-1+\frac{1}{t}+t,\ 0,\ 0,\ -16+\frac{9\,c}{2}-\frac{2\,c}{t^4}+\frac{1}{t^3}+\frac{11\,c}{2\,t^3}-\frac{4}{t^2}-\frac{8\,c}{t^2}+\frac{10}{t}+\frac{4\,c}{t}+18\,t-10\,c\,t-14\,t^2+8\,c\,t^2+7\,t^3-\frac{3\,c\,t^3}{2}-$$
$$2\,t^4-2\,c\,t^4+2\,c\,t^5-\frac{c\,t^6}{2}-4\,u\,w+\frac{2\,u\,w}{t^4}-\frac{7\,u\,w}{2\,t^3}+\frac{9\,u\,w}{2\,t^2}+\frac{u\,w}{2\,t}+6\,t\,u\,w-2\,t^2\,u\,w-\frac{1}{2}\,t^3\,u\,w+\frac{3}{2}\,t^4\,u\,w-\frac{1}{2}\,t^5\,u\,w\Big]$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```
ConditionalExport[fname_String, rest___] := Module[{temp, exists},
  temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
  exists = FileExistsQ[fname];
  Export[temp, rest];
  If[exists && FileByteCount[fname] === FileByteCount[temp],
   DeleteFile[temp],
   (* else *) Print["Exporting " <> fname <> "..."];
   If[exists, DeleteFile[fname]];
   RenameFile[temp, fname]
  ];
  fname
 ]

SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["131"] = {PageWidth → 3.2/0.66};
Options[CellExport] = {
   PageWidth → 4/0.66, CellFilter → Identity, ExportDirectory → "Snips",
   ExportBaseFilename → Automatic, ExportFormat → ".pdf", ExportOptions → {}, Split → False
  };
CellExport[tag_String, opts___Rule] := CellExport[
   NotebookGet[EvaluationNotebook[]],
   tag, opts
  ];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
   {cells, cell, filename, format},
   filename = FileNameJoin[{
      OptionValue[ExportDirectory] /. Automatic → Directory[],
      OptionValue[ExportBaseFilename] /. Automatic → tag
     }];
   format = OptionValue[ExportFormat];
   cells = OptionValue[CellFilter][Cases[
      nb, c_Cell /; FreeQ[List @@ c, Cell] && ! FreeQ[c, CellTags → tag],
      Infinity
     ]];
   If[! OptionValue[Split],
    If[Length[cells] ≥ 1,
     If[Length[cells] == 1,
      cells = Join[First[cells],
        Cell[PageWidth → 1.2 × 72 OptionValue[PageWidth], Background → {White, Opacity[0]}]],
      cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
     ];
     ConditionalExport[
```

```
       filename <> format, cells,
       ImageResolution → 300,
       OptionValue[ExportOptions]
      ]
     ],
     k = 0;
     Table[
      ++k;
      ConditionalExport[
       filename <> "-" <> ToString[k] <> format,
       Append[cell, PageWidth → 72 OptionValue[PageWidth]],
       ImageResolution → 300,
       OptionValue[ExportOptions]
      ],
      {cell, cells}
     ]
    ]
   ];

ExportCells := (
    nb = NotebookGet[EvaluationNotebook[]];
    tags = Cases[nb, (CellTags → tag_String) :↦ tag, Infinity] // Union;
    Print[tags];
    CellExport /@ tags;
    Print["Done."]
   );
```

ExportCells

```
{0817, 0m, 0mDemo, 0MetaAssoc, 0NO, 0NODemo, 0Q0, 0R, 0R3,
 0R3Left, 0Util, 131, 1DP, 1Gens, 1m, 1NOc, 1NOuw, 1Util, Logos, T0, ZT0}

Exporting Snips\131.pdf...

Done.
```