Pensieve header: The Objects, from pensieve://Projects/SL2Portfolio2/.

Program

The Objects

Program

Symmetric Algebra Objects

Program

```
sm_{i_,j_→k_} := 𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j), y_k (η_i + η_j) + x_k (ξ_i + ξ_j), 1];
sΥ_{i_→j_,k_,l_,m_} := 𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l, η_i y_j + ξ_i x_m, 1];
sΔ_{i_→j_,k_} := 𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k), η_i (y_j + y_k) + ξ_i (x_j + x_k), 1];
sS_{i_} := 𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i, -η_i y_i - ξ_i x_i, 1];
sϵ_{i_} := 𝔼_{{}→{i}}[0, 0, 1]; sη_{i_} := 𝔼_{{i}→{}}[0, 0, 1];
sσ_{i_→j_} := 𝔼_{{i}→{j,k}}[β_i b_j + τ_i t_j + α_i a_j, η_i y_j + ξ_i x_j, 1];
```

Program

Booting Up QU

Program

```
Define[aσ_{i→j} = 𝔼_{{i}→{j}}[a_j α_i, x_j ξ_i, 1], bσ_{i→j} = 𝔼_{{i}→{j}}[b_j β_i, y_j η_i, 1]]
```

Program

```
Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k, (𝒜_j^{-1} ξ_i + ξ_j) x_k, 1]_{$k},
  bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k, (η_i + η_j) y_k, ⅇ^{(ⅇ^{-ϵ β_i}-1) η_j y_k}]_{$k}]
```

Program

```
Define[R_{i,j} = 𝔼_{{}→{i,j}}[ℏ a_j b_i, ℏ x_j y_i, ⅇ^(Sum_{k=2}^{$k+1} ((1 - ⅇ^{γ ϵ ℏ})^k (ℏ y_i x_j)^k) / (k (1 - ⅇ^{k γ ϵ ℏ})))]_{$k},
  R̄_{i,j} = CF@𝔼_{{}→{i,j}}[-ℏ a_j b_i, -ℏ x_j y_i / B_i, 1 + If[$k == 0, 0, (R̄_{{i,j},$k-1})_{$k}[3] -
       (((R̄_{{i,j},0})_{$k} R_{1,2} (R̄_{{3,4},$k-1})_{$k}) // (bm_{i,1→i} am_{j,2→j}) // (bm_{i,3→i} am_{j,4→j}))[3]]],
  P_{i,j} = 𝔼_{{i,j}→{}}[β_i α_j / ℏ, η_i ξ_j / ℏ, 1 + If[$k == 0, 0, (P_{{i,j},$k-1})_{$k}[3] -
      (R_{1,2} // ((P_{{1,j},0})_{$k} (P_{{i,2},$k-1})_{$k}))[3]]]]
```

Program

In[∘]:=

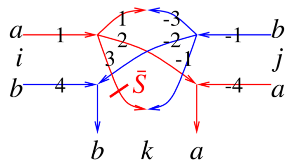
```
Define[aS_j = R̄_{i,j} ~ B_i ~ P_{i,j},
  aS̄_i = 𝔼_{{i}→{i}}[-a_i α_i, -x_i 𝒜_i ξ_i, 1 + If[$k == 0, 0, (aS̄_{{i},$k-1})_{$k}[3] -
      ((aS̄_{{i},0})_{$k} ~ B_i ~ aS_i ~ B_i ~ (aS̄_{{i},$k-1})_{$k})[3]]]]
```

Program

In[∘]:=

```
Define[bS_i = R_{i,1} ~ B_1 ~ aS_1 ~ B_1 ~ P_{i,1},
  bS̄_i = R_{i,1} ~ B_1 ~ aS̄_1 ~ B_1 ~ P_{i,1},
  aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
  bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
```

Program

The Drinfel'd double: 

Program

$$\text{Define}\Big[\ \text{dm}_{i,j\to k} = \Big(\big(\text{s}\Upsilon_{i\to 4,4,1,1} \,//\, \text{a}\Delta_{1\to 1,2} \,//\, \text{a}\Delta_{2\to 2,3} \,//\, \overline{\text{aS}}_3\big)\ \big(\text{s}\Upsilon_{j\to -1,-1,-4,-4} \,//\, \text{b}\Delta_{-1\to -1,-2} \,//\, \text{b}\Delta_{-2\to -2,-3}\big)\Big) \,//\, \big(P_{-1,3}\, P_{-3,1}\, \text{am}_{2,-4\to k}\, \text{bm}_{4,-2\to k}\big)\Big]$$

Program

$$\text{Define}\big[\text{d}\sigma_{i\to j} = \text{a}\sigma_{i\to j}\, \text{b}\sigma_{i\to j},$$
$$\text{d}\epsilon_i = \text{s}\epsilon_i,\ \text{d}\eta_i = \text{s}\eta_i,$$
$$\text{dS}_i = \text{s}\Upsilon_{i\to 1,1,2,2} \,//\, \big(\overline{\text{bS}}_1\, \text{aS}_2\big) \,//\, \text{dm}_{2,1\to i},$$
$$\overline{\text{dS}}_i = \text{s}\Upsilon_{i\to 1,1,2,2} \,//\, \big(\text{bS}_1\, \overline{\text{aS}}_2\big) \,//\, \text{dm}_{2,1\to i},$$
$$\text{d}\Delta_{i\to j,k} = \big(\text{b}\Delta_{i\to 3,1}\, \text{a}\Delta_{i\to 2,4}\big) \,//\, \big(\text{dm}_{3,4\to k}\, \text{dm}_{1,2\to j}\big)\big]$$

Program

In[ ]:=

$$\text{Define}\Big[C_i = \mathbb{E}_{\{\}\to\{i\}}\big[0,\ 0,\ B_i^{1/2}\, e^{-\hbar\,\epsilon\, a_i/2}\big]_{\$k},$$
$$\overline{C}_i = \mathbb{E}_{\{\}\to\{i\}}\big[0,\ 0,\ B_i^{-1/2}\, e^{\hbar\,\epsilon\, a_i/2}\big]_{\$k},$$
$$\text{Kink}_i = \big(R_{1,3}\, \overline{C}_2\big) \,//\, \text{dm}_{1,2\to 1} \,//\, \text{dm}_{1,3\to i},$$
$$\overline{\text{Kink}}_i = \big(\overline{R}_{1,3}\, C_2\big) \,//\, \text{dm}_{1,2\to 1} \,//\, \text{dm}_{1,3\to i}\Big]$$

Program

Note. $t == \epsilon a - \gamma b$ and $b == -t/\gamma + \epsilon a/\gamma$.

Program

In[ ]:=

$$\text{Define}\Big[\text{b2t}_i = \mathbb{E}_{\{i\}\to\{i\}}\big[\alpha_i\, a_i - \beta_i\, t_i \big/ \gamma,\ \xi_i\, x_i + \eta_i\, y_i,\ e^{\epsilon\,\beta_i\, a_i/\gamma}\big]_{\$k},$$
$$\text{t2b}_i = \mathbb{E}_{\{i\}\to\{i\}}\big[\alpha_i\, a_i - \tau_i\, \gamma\, b_i,\ \xi_i\, x_i + \eta_i\, y_i,\ e^{\epsilon\,\tau_i\, a_i}\big]_{\$k}\Big]$$

Program

The CU Definitions

Program

$$\text{Define}\Big[\text{cm}_{i,j\to k} = \text{CF}@\mathbb{E}_{\{i,j\}\to\{k\}}\Big[$$
$$a_k\,(\alpha_i + \alpha_j) + b_k\,(\beta_i + \beta_j),$$
$$y_k\left(\eta_i + \frac{\eta_j}{\mathcal{A}_i}\right) + \gamma\, b_k\, \eta_j\, \xi_i + x_k\left(\frac{\xi_i}{\mathcal{A}_j} + \xi_j\right),$$
$$e^{y_k\,\eta_j\left(\frac{e^{-\epsilon\beta_i}}{\mathcal{A}_i + \gamma\,\epsilon\,\mathcal{A}_i\,\eta_j\,\xi_i} - \frac{1}{\mathcal{A}_i}\right) + \xi_i\left(x_k\left(\frac{e^{-\epsilon\beta_j}}{\mathcal{A}_j + \gamma\,\epsilon\,\mathcal{A}_j\,\eta_j\,\xi_i} - \frac{1}{\mathcal{A}_j}\right) - \gamma\, b_k\,\eta_j\right)}\ \big(1 + \gamma\,\epsilon\,\eta_j\,\xi_i\big)^{\frac{a_k}{\gamma} + \frac{b_k}{\epsilon}}\Big]_{\$k}\Big]$$

Program

$$\text{Define}\big[\text{c}\sigma_{i\to j} = \text{s}\sigma_{i,j} \,/.\, \tau_i \to 0,$$
$$\text{c}\epsilon_i = \text{s}\epsilon_i,\ \text{c}\eta_i = \text{s}\eta_i,$$
$$\text{c}\Delta_{i\to j,k} = \text{s}\Delta_{i\to j,k},$$
$$\text{cS}_i = \text{sS}_i \,//\, \text{s}\Upsilon_{i\to 1,2,3,4} \,//\, \text{cm}_{4,3\to i} \,//\, \text{cm}_{i,2\to i} \,//\, \text{cm}_{i,1\to i}\big];$$

Program

The Knot Tensors

Program

In[∘]:=

```
Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} → t,
  \overline{kR}_{i,j} = \overline{R}_{i,j} // (b2t_i b2t_j) /. {t_{i|j} → t, T_{i|j} → T},
  km_{i,j→k} = (t2b_i t2b_j) // dm_{i,j→k} // b2t_k /. {t_k → t, T_k → T, τ_{i|j} → 0},
  kC_i = C_i // b2t_i /. T_i → T,
  \overline{kC}_i = \overline{C}_i // b2t_i /. T_i → T,
  kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
  \overline{kKink}_i = \overline{Kink}_i // b2t_i /. {t_i → t, T_i → T}]
```