

Kas[K_, ω_] :=

Module[{u, v, XingsByArmpits, bends, faces, p, A, is},

u = Re[ω^{1/2}]; v = Re[ω];

XingsByArmpits =

List@@PD[K] /. x : X[i_, j_, k_, l_] =>

If[PositiveQ[x], X₊[-i, j, k, -l], X₋[-j, k, l, -i]]];

bends = Times@@XingsByArmpits /.

_ [X] [a_, b_, c_, d_] => p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};

faces = bends //. p_{x_,y_} p_{y_,z_} => p_{x,y,z};

A = Table[0, Length@faces, Length@faces];

Do[is = Position[faces, #][[1, 1]] & /@ List@@x;

A[[is, is]] += If[Head[x] === X₊,

$$\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}, - \begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}],$$

{x, XingsByArmpits}];

(MatrixSignature[A] - Writhe[K]) / 2];