

Kas $[K_, \omega_] :=$

Module $\{u, v, \text{XingsByArmpits}, \text{bends}, \text{faces}, p, A, \text{is}\},$

$u = \text{Re}[\omega^{1/2}]; v = \text{Re}[\omega];$

$\text{XingsByArmpits} =$

List@@PD[K] /. $x : X[i_, j_, k_, l_] \Rightarrow$

If[PositiveQ[x], $X_+[-i, j, k, -l], X_-[-j, k, l, -i]$];

$\text{bends} = \text{Times} @@ \text{XingsByArmpits} /.$

$_ [X][a_, b_, c_, d_] \Rightarrow p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c}$

$\text{faces} = \text{bends} // . p_{x_, y_} p_{y_, z_} \Rightarrow p_{x, y, z};$

$A = \text{Table}[0, \text{Length}@\text{faces}, \text{Length}@\text{faces}];$

Do[is = Position[faces, #][[1, 1]] & /@ List@@x;

$A[[is, is]] += \text{If}[\text{Head}[x] == X_+,$

$$\left(\begin{matrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{matrix} \right), - \left(\begin{matrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{matrix} \right) \Big],$$

$\{x, \text{XingsByArmpits}\}];$

$(\text{MatrixSignature}[A] - \text{Writhe}[K]) / 2];$