

Pensieve header: Knot Signatures as in KnotTheory` and as in Kashaev. Continues Kashaev.nb at pensieve://People/Abbasi/.

tex

```
\begin{multicols}{2}
```

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```
In[ ]:= Once[<< KnotTheory`]
```

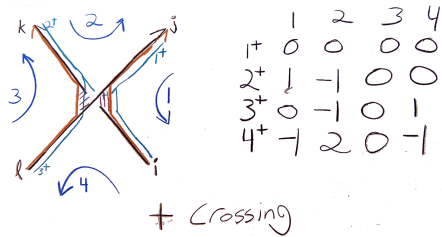
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Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

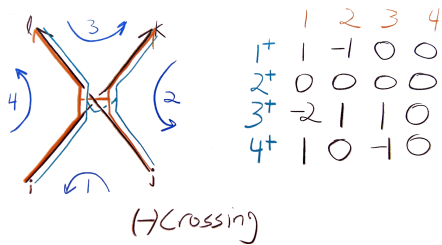
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```
In[ ]:= MatrixSignature[A_] := Total[Sign[Select[Eigenvalues[A], Abs[#] > 10-12 &]]];
Writhe[K_] := Sum[If[PositiveQ[x], 1, -1], {x, List@@PD@K}];
```

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```
\end{multicols}
\vskip -3mm \hrule \vskip -7mm
\begin{multicols}{2}
```

exec

```
nb2tex$PDFWidth = 1.23 N[284/72];
```

tex

```
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}
```

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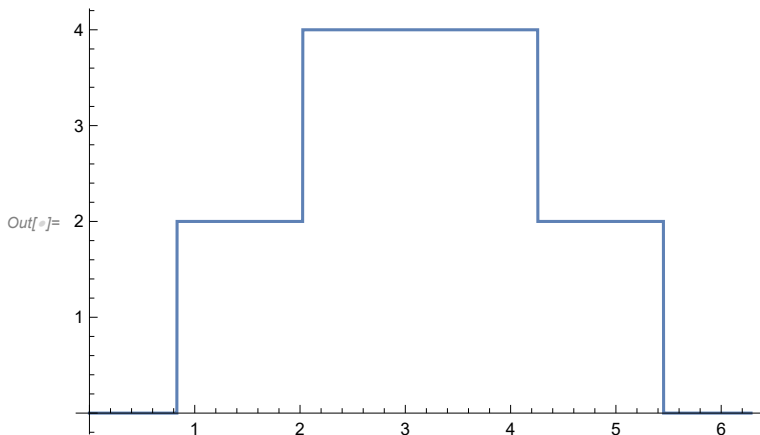
## The Bedlewo program

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```
In[ ]:= Bed[K_, ω_] := Module [{t, r, XingsByArmpits, bends, faces, p, A, is},
  t = 1 - ω; r = t + t*;
  XingsByArmpits = List @@ PD[K] /.
    x : X[i_, j_, k_, l_] => If[PositiveQ[x], X, [-i, j, k, -l], X, [-j, k, l, -i]];
  bends = Times @@ XingsByArmpits /. _[X][a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends /. px_,y_ py_,z_ => px,y,z;
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
  A[[is, is]] += If[Head[x] === X,
    
$$\begin{pmatrix} -r & -t & 2t & t^* \\ -t^* & 0 & t^* & 0 \\ 2t^* & t & -r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}, \begin{pmatrix} r & -t & -2t^* & t^* \\ -t^* & 0 & t^* & 0 \\ -2t & t & r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}],$$

    {x, XingsByArmpits}];
  MatrixSignature[A];
```

```
In[ ]:= Plot[Bed[K = Knot[8, 2], ei t], {t, 0, 2 π}]
PositiveQ /@ (List @@ PD[K])
```

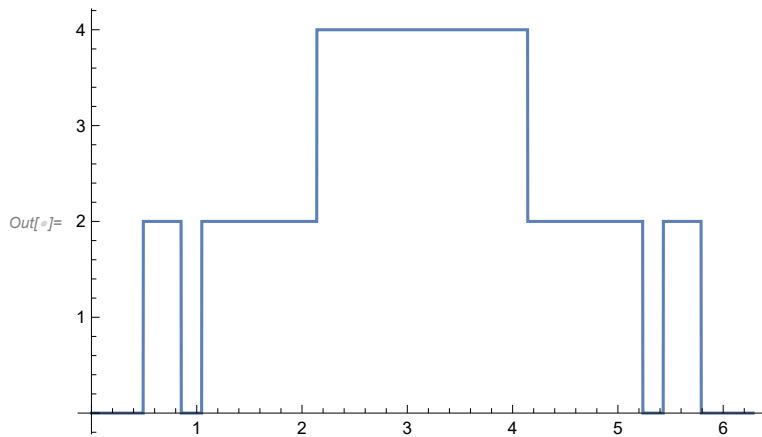


Out[ ]:= {False, False, True, True, False, False, False, False}

```
In[ ]:= Draw[Knot[8, 2]]
```

Out[ ]:= **Draw**[**Knot**[8, 2]]

```
In[ ]:= Plot [Bed [Knot@"K12a422", ei t], {t, 0, 2 π}]
```



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```
\columnbreak
```

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## The Kashaev Program

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```
In[ ]:= Kas[K_, ω_] := Module[{u, v, XingsByArmpits, bends, faces, p, A, is},
  u = Re[ω1/2]; v = Re[ω];
  XingsByArmpits = List@@PD[K] /.
    x : X[i_, j_, k_, L_] => If[PositiveQ[x], X+[-i, j, k, -L], X-[-j, k, L, -i]];
  bends = Times@@XingsByArmpits /. _[X][a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends /. px-,y- py-,z- => px,y,z;
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List@@x;
    A[[is, is]] += If[Head[x] === X+,
      
$$\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}, - \begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}],
    {x, XingsByArmpits}];
  (MatrixSignature[A] - Writhe[K]) / 2];$$

```

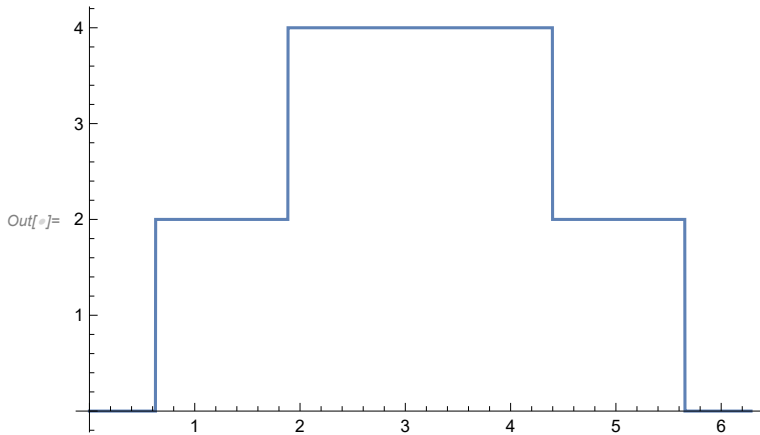
exec

```
nb2tex$PDFWidth = 1.1 N[284/72];
```

tex

```
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
```

```
In[ ]:= Plot[Kas[Knot[5, 1], ei t], {t, 0, 2 π}]
```



tex

```
\end{multicols}
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\begin{multicols}{2}
```

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## Comparisons

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```
In[ ]:= Sum[ω = ei RandomReal[{0, 2 π}]; Bed[K, ω] == Kas[K, ω], {10}, {K, AllKnots[{3, 10}]}]
```

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```
Out[ ]:= 2490 True
```

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```
\end{multicols}
```

```
In[ ]:= Table[Bed[K, i], {K, AllKnots[{3, 5}]}]
```

```
Out[ ]:= {2, 0, 2, 2}
```

```
In[ ]:= Table[KnotSignature[K], {K, AllKnots[{3, 5}]}]
```

```
Out[ ]:= {-2, 0, -4, -2}
```

```
In[ ]:= Total@Table[Bed[K, -1] == -KnotSignature[K], {K, AllKnots[{3, 10}]}]
```

```
Out[ ]:= 249 True
```