by successive approximations presents no problems. For this we introduce the following modification $GRT(k)$ of the group $GT(k)$. We denote by $GRT_1(k)$ the set of all $g \in Fr_k(A, B)$ such that

$$g(B, A) = g(A, B)^{-1}, \tag{5.12}$$

$$g(C, A)g(B, C)g(A, B) = 1 \quad \text{for } A + B + C = 0, \tag{5.13}$$

$$A + g(A, B)^{-1}Bg(A, B) + g(A, C)^{-1}Cg(A, C) = 0 \quad \text{for } A + B + C = 0, \tag{5.14}$$

$$g(X^{12}, X^{23} + X^{24})g(X^{13} + X^{23}, X^{34}) = g(X^{23}, X^{34})g(X^{12} + X^{13}, X^{24} + X^{34})g(X^{12}, X^{23}), \tag{5.15}$$

where the X^{ij} satisfy (5.1). $GRT_1(k)$ is a group with the operation

$$(g_1 \circ g_2)(A, B) = g_1(g_2(A, B)Ag_2(A, B)^{-1}, B) \cdot g_2(A, B). \tag{5.16}$$

On $GRT_1(k)$ there is an action of k^*, given by $\tilde{g}(A, B) = g(c^{-1}A, c^{-1}B)$, $c \in k^*$. The semidirect product of k^* and $GRT_1(k)$ we denote by $GRT(k)$. The Lie algebra $\mathfrak{grt}_1(k)$ of the group $GRT_1(k)$ consists of the series $\psi \in \mathfrak{fr}_k(A, B)$ such that

$$\psi(B, A) = -\psi(A, B), \tag{5.17}$$

$$\psi(C, A) + \psi(B, C) + \psi(A, B) = 0 \quad \text{for } A + B + C = 0, \tag{5.18}$$

$$[B, \psi(A, B)] + [C, \psi(A, C)] = 0 \quad \text{for } A + B + C = 0, \tag{5.19}$$

$$\psi(X^{12}, X^{23} + X^{24}) + \psi(X^{13} + X^{23}, X^{34}) = \psi(X^{23}, X^{34}) + \psi(X^{12} + X^{13}, X^{24} + X^{34}) + \psi(X^{12}, X^{23}), \tag{5.20}$$

where the X^{ij} satisfy (5.1). A commutator $\langle\ ,\ \rangle$ in $\mathfrak{grt}_1(k)$ is of the form

$$\langle\psi_1, \psi_2\rangle = [\psi_1, \psi_2] + D_{\psi_2}(\psi_1) - D_{\psi_1}(\psi_2), \tag{5.21}$$

where $[\psi_1, \psi_2]$ is the commutator in $\mathfrak{fr}_k(A, B)$ and D_ψ is the derivation of $\mathfrak{fr}_k(A, B)$ given by $D_\psi(A) = [\psi, A]$, $D_\psi(B) = 0$. The algebra $\mathfrak{grt}_1(k)$ is

Proposition 5.1. *The action of* $GT(k)$ *on* $M(k)$ *is free and transitive.*

Proof. If $(\mu, \varphi) \in M(k)$ and $(\overline{\mu}, \overline{\varphi}) \in M(k)$, then there is exactly one f such that $\overline{\varphi}(A, B) = f(\varphi(A, B)e^A\varphi(A, B)^{-1}, e^B) \cdot \varphi(A, B)$. We need to show that $(\lambda, f) \in GT(k)$, where $\lambda = \overline{\mu}/\mu$. We prove (4.10). Let G_n be the semidirect product of S_n and $\exp \mathfrak{a}_n^k$. Consider the homomorphism $B_n \to G_n$ that takes σ_i into

$$\varphi(X^{1i} + \cdots + X^{i-1,i}, X^{i,i+1})^{-1}\sigma^{i,i+1}e^{\mu X^{i,i+1}/2}\varphi(X^{1i} + \cdots + X^{i-1,i}, X^{i,i+1}),$$

where $\sigma^{ij} \in S_n$ transposes i and j. It induces a homomorphism $K_n \to \exp \mathfrak{a}_n^k$, and therefore a homomorphism α_n: $K_n(k) \to \exp \mathfrak{a}_n^k$, where $K_n(k)$ is the k-pro-unipotent completion of K_n. It is easily shown that the left- and right-hand sides of (4.10) have the same images in $\exp \mathfrak{a}_4^k$. It remains to prove that α_n is an isomorphism. The algebra Lie $K_n(k)$ is topologically generated by the elements ξ_{ij}, $1 \le i < j \le n$, with defining relations obtained from (4.7)–(4.9) by substituting $x_{ij} = \exp \xi_{ij}$. The principal parts of these relations are the same as in (5.1), while $(\alpha_n)_*(\xi_{ij}) = \mu X^{ij} +$ {lower terms}, where $(\alpha_n)_*$: Lie $K_n(k) \to \mathfrak{a}_n^k$ is induced by the homomorphism α_n. Therefore α_n is an isomorphism, i.e., (4.10) is proved. (4.3) is obvious. To prove (4.4), we can interpret it in terms of K_3 and argue as in the proof of (4.10), or, what is equivalent, make the substitution

$$X_1 = e^A, \quad X_2 = e^{-A/2}\varphi(B, A)e^B\varphi(B, A)^{-1}e^{A/2}, \quad X_3 = \varphi(C, A)e^C\varphi(C, A)^{-1}, \tag{5.4}$$

where $A + B + C = 0$. ●