

$$[g, e] = \alpha e \Rightarrow e^{\beta} e e^{-\beta} = e^{\beta} e \Rightarrow$$

$$G e = e^{\beta} e G \Rightarrow [G, e] = (e^{\beta} - 1) e G$$

$$\text{Also, } [g, e] = -\alpha e \Rightarrow e^{-\beta} e e^{\beta} = e^{-\beta} e$$

$$\Rightarrow e G = G e^{-\beta} e \Rightarrow [e, G] = (e^{-\beta} - 1) G e$$

In PBW algebras, would it be better to have an 'imperative' description, instead of a relational ("bracket") description?

$$G = e^g, H = e^h$$

$$e^{\alpha h} = G^{\alpha} H = H^{\alpha} G$$

$$e^{\alpha g h} = G^{\alpha} H^{\alpha} = H^{\alpha} G^{\alpha}$$

$$\langle e^{\alpha g h} \rangle = \sum \frac{\alpha^n}{n!} \langle G^n, H^n \rangle =$$

$$\sum \frac{\alpha^n}{n!} \langle G^n, \Delta^n(H) \rangle =$$

$$= \sum \frac{\alpha^n}{n!} n! \langle G, H \rangle^n = \frac{1}{1 - \alpha^2}$$

$$(\psi, a) \mapsto sw(\psi, a) = \sum \psi_i b_i;$$

$$\langle sw(a, \psi), (c, \lambda) \rangle \quad \text{rough}$$

$$= \langle \psi(s(a_1) \subset a_2) \cdot (\psi_1 \lambda(s\psi_2)) \rangle(a)$$

$$= \langle \psi_1, s(a_1) \rangle \langle \psi_2, c_2 \rangle \langle \psi_3, a_3 \rangle$$

$$\cdot \langle \psi_1, a_1 \rangle \langle \lambda, a_2 \rangle \langle s\psi_2, a_3 \rangle$$

$$sw(a, \psi) := \langle sa_1, \psi_1 \rangle \psi_2 a_2 \langle a_3, \psi_3 \rangle$$

Eval against c :

$$\rightarrow \langle sa_1, \psi_1 \rangle \langle c, \psi_2 \rangle \langle a_3, \psi_3 \rangle a_2$$

$$= \langle sa_1 c a_3, \psi \rangle a_2$$

\sim roughly, ψ is acted on by a .