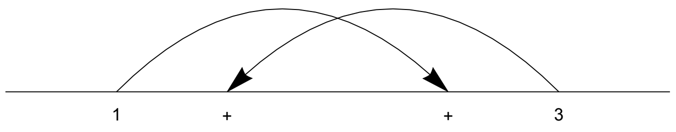
Pensieve header: A program to enumerate w-knots.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-03"]
```

```
C:\drorbn\AcademicPensieve\2015-03
```

```
A_List \ B_List := Complement[A, B];

Draw[w_wLDiag | w_wCDiag] := Module[{n, w1},
   n = Length[w];
   w1 = Abs /@ w;
   Graphics[{
     Line[{{0, 0}, {n + 1, 0}}],
     Table[
      {
       Arrow[BezierCurve[
          {{w1[[j]] - 0.5, 0}, {(w1[[j]] + j - 0.5) / 2, 0.5 Abs[j - w1[[j]] + 0.5]}, {j, 0}}]],
       Text[If[w[[j]] > 0, "+", "-"], {j, -0.1}],
       Text[w1[[j]], {w1[[j]] - 0.5, -0.1}]
      },
      {j, n}
     ]
    }]
  ];
Draw[expr_] := expr /. w_wLDiag | w_wCDiag :→ Draw[w]

Draw[wLDiag[3, 1]]
```



```
AllLinearDiagrams[n_] := Flatten@Table[
   wLDiag @@@ Tuples[Range[k + 1] ⋃ (-Range[k + 1]), k],
   {k, 0, n}
  ]
```

```
AllLinearDiagrams[2]
```

```
{wLDiag[], wLDiag[-2], wLDiag[-1], wLDiag[1], wLDiag[2], wLDiag[-3, -3],
 wLDiag[-3, -2], wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[-3, 2], wLDiag[-3, 3],
 wLDiag[-2, -3], wLDiag[-2, -2], wLDiag[-2, -1], wLDiag[-2, 1], wLDiag[-2, 2],
 wLDiag[-2, 3], wLDiag[-1, -3], wLDiag[-1, -2], wLDiag[-1, -1], wLDiag[-1, 1],
 wLDiag[-1, 2], wLDiag[-1, 3], wLDiag[1, -3], wLDiag[1, -2], wLDiag[1, -1],
 wLDiag[1, 1], wLDiag[1, 2], wLDiag[1, 3], wLDiag[2, -3], wLDiag[2, -2],
 wLDiag[2, -1], wLDiag[2, 1], wLDiag[2, 2], wLDiag[2, 3], wLDiag[3, -3],
 wLDiag[3, -2], wLDiag[3, -1], wLDiag[3, 1], wLDiag[3, 2], wLDiag[3, 3]}
```

```
wCDiag /: RotateLeft[w_wCDiag] := Module[{n},
  n = Length[w];
  wCDiag @@ (RotateLeft[List @@ w] /. j_Integer :> Which[
       j == 1, n,
       j == -1, -n,
       j > 1, j - 1,
       j < -1, j + 1
      ])
 ]
```

```
RotateLeft[wCDiag[-3, 1, 3, -2]]
```

```
wCDiag[4, 2, -1, -2]
```

```
RotateToMinimal[w_wCDiag] := RotateToMinimal[w] = Module[
    {bestw = w, rotatedw = RotateLeft[w]},
    While[rotatedw =!= w,
     bestw = First[Sort[{bestw, rotatedw}]];
     rotatedw = RotateLeft[rotatedw]
    ];
    bestw
   ];
```

```
wDiag[5, 2, -1, -2] // RotateToMinimal
```

```
wDiag[-5, -1, 4, 1]
```

```
wCDiag[w_wLDiag] := Module[{n},
  n = Length[w];
  RotateToMinimal[wCDiag @@ w /. {n + 1 → 1, -n - 1 → -1}]
 ]
```

```
AllCircularDiagrams[n_] :=
 AllCircularDiagrams[n] = Union[RotateToMinimal /@ Flatten@Table[
      wCDiag @@@ Tuples[Range[k] ⋃ (-Range[k]), k],
      {k, 0, n}
     ]]
```

```
AllCircularDiagrams[2]
```

```
{wCDiag[], wCDiag[-1], wCDiag[1], wCDiag[-2, -2],
 wCDiag[-2, -1], wCDiag[-2, 1], wCDiag[-2, 2], wCDiag[-1, -2],
 wCDiag[-1, 1], wCDiag[-1, 2], wCDiag[1, 1], wCDiag[1, 2], wCDiag[2, 1]}
```

```
RemoveR1[w_wLDiag] := RemoveR1[w] = Module[{j, k = 0},
   Do[If[MemberQ[{j, j + 1}, Abs[w[[j]]]], k = j], {j, Length[w]}];
   If[k == 0, w,
    Delete[w, k] /. j_Integer /; Abs[j] > k :> Sign[j] (Abs[j] - 1)
   ]
  ]
```

```
RemoveR1[wLDiag[-4, 1, 3, -4]]
```

```
wLDiag[-4, 1, 3]
```

```
RemoveR1 /@ AllLinearDiagrams[2]
```

```
{wLDiag[], wLDiag[], wLDiag[], wLDiag[], wLDiag[], wLDiag[-2], wLDiag[-2],
 wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[-2], wLDiag[-2], wLDiag[-2], wLDiag[-2],
 wLDiag[-1], wLDiag[1], wLDiag[-2], wLDiag[-2], wLDiag[-1], wLDiag[-1], wLDiag[-1],
 wLDiag[1], wLDiag[-1], wLDiag[-1], wLDiag[1], wLDiag[1], wLDiag[-1], wLDiag[1],
 wLDiag[1], wLDiag[1], wLDiag[2], wLDiag[2], wLDiag[-1], wLDiag[1], wLDiag[2],
 wLDiag[2], wLDiag[2], wLDiag[2], wLDiag[3, -1], wLDiag[3, 1], wLDiag[2], wLDiag[2]}
```

```
RemoveR1[wCDiag[]] = wCDiag[];
RemoveR1[w_wCDiag] := RemoveR1[w] = Module[{n, j, k = 0},
   n = Length[w];
   Do[If[MemberQ[{j, j + 1}, Abs[w[[j]]]], k = j], {j, n - 1}];
   If[k != 0,
    Delete[w, k] /. j_Integer /; Abs[j] > k :> Sign[j] (Abs[j] - 1),
    (*else*) If[! MemberQ[{1, n}, Abs[Last[w]]], w,
     Drop[w, -1] /. {n → 1, -n → -1}]
   ]
  ]
```

```
RemoveR1 /@ AllCircularDiagrams[2]
```

```
{wCDiag[], wCDiag[], wCDiag[], wCDiag[-1], wCDiag[-1], wCDiag[1], wCDiag[1],
 wCDiag[-1], wCDiag[1], wCDiag[1], wCDiag[1], wCDiag[1], wCDiag[1]}
```

```
RemoveR1s[w_wLDiag | w_wCDiag] := RemoveR1s[w] = FixedPoint[RemoveR1, w]
```

```
RemoveR1s /@ AllLinearDiagrams[2] // Union
```

```
{wLDiag[], wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[3, -1], wLDiag[3, 1]}
```

```
RemoveR1s /@ AllCircularDiagrams[4] // Union
```

```
{wCDiag[], wCDiag[-3, -1, -2], wCDiag[-3, -1, 2], wCDiag[-3, 1, -2],
 wCDiag[-3, 1, 2], wCDiag[3, 1, 2], wCDiag[-4, -4, -2, -3], wCDiag[-4, -4, -2, -2],
 wCDiag[-4, -4, -2, 2], wCDiag[-4, -4, -2, 3], wCDiag[-4, -4, -1, -3],
 wCDiag[-4, -4, -1, -2], wCDiag[-4, -4, -1, 2], wCDiag[-4, -4, -1, 3],
 wCDiag[-4, -4, 1, -3], wCDiag[-4, -4, 1, -2], wCDiag[-4, -4, 1, 2],
 wCDiag[-4, -4, 1, 3], wCDiag[-4, -4, 2, -3], wCDiag[-4, -4, 2, -2],
 wCDiag[-4, -4, 2, 2], wCDiag[-4, -4, 2, 3], wCDiag[-4, -1, -2, -3],
 wCDiag[-4, -1, -2, 2], wCDiag[-4, -1, -2, 3], wCDiag[-4, -1, 1, -2],
 wCDiag[-4, -1, 1, 2], wCDiag[-4, -1, 1, 3], wCDiag[-4, -1, 2, -2],
 wCDiag[-4, -1, 2, 2], wCDiag[-4, -1, 2, 3], wCDiag[-4, 1, -2, 2],
 wCDiag[-4, 1, -2, 3], wCDiag[-4, 1, -1, -2], wCDiag[-4, 1, -1, 2],
 wCDiag[-4, 1, -1, 3], wCDiag[-4, 1, 1, -2], wCDiag[-4, 1, 1, 2],
 wCDiag[-4, 1, 1, 3], wCDiag[-4, 1, 2, -2], wCDiag[-4, 1, 2, 2], wCDiag[-4, 1, 2, 3],
 wCDiag[-4, 4, -2, 2], wCDiag[-4, 4, -1, -2], wCDiag[-4, 4, -1, 2],
 wCDiag[-4, 4, -1, 3], wCDiag[-4, 4, 1, -2], wCDiag[-4, 4, 1, 2],
 wCDiag[-4, 4, 1, 3], wCDiag[-4, 4, 2, -2], wCDiag[-4, 4, 2, 2], wCDiag[-4, 4, 2, 3],
 wCDiag[-3, -4, -1, -2], wCDiag[-3, -4, -1, 2], wCDiag[-3, -4, -1, 3],
 wCDiag[-3, -4, 1, 2], wCDiag[-3, -4, 1, 3], wCDiag[-3, -4, 2, 2],
 wCDiag[-3, -4, 2, 3], wCDiag[-3, 1, -1, 2], wCDiag[-3, 1, -1, 3],
 wCDiag[-3, 1, 1, 2], wCDiag[-3, 1, 1, 3], wCDiag[-3, 1, 2, 2], wCDiag[-3, 1, 2, 3],
 wCDiag[-3, 4, -1, 2], wCDiag[-3, 4, 1, 2], wCDiag[-3, 4, 1, 3],
 wCDiag[-3, 4, 2, 2], wCDiag[-3, 4, 2, 3], wCDiag[3, 1, 1, 2], wCDiag[3, 1, 1, 3],
 wCDiag[3, 1, 2, 2], wCDiag[3, 1, 2, 3], wCDiag[3, 4, 1, 2], wCDiag[4, 1, 2, 3]}
```

```
RemoveR2[w_wLDiag] := RemoveR2[w] = Module[{j, k = 0},
   Do[If[w[[j]] + w[[j + 1]] == 0 && ! MemberQ[Abs[List @@ w], j + 1], k = j],
    {j, Length[w] - 1}];
   If[k == 0, w,
    Delete[w, {{k}, {k + 1}}] /. j_Integer /; Abs[j] > k :> Sign[j] (Abs[j] - 2)
   ]
  ]
```

```
wLDiag[2, -2] // RemoveR2
```

```
wLDiag[2, -2]
```

```
RemoveR2 /@ AllLinearDiagrams[2]
```

```
{wLDiag[], wLDiag[-2], wLDiag[-1], wLDiag[1], wLDiag[2], wLDiag[-3, -3],
 wLDiag[-3, -2], wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[-3, 2], wLDiag[],
 wLDiag[-2, -3], wLDiag[-2, -2], wLDiag[-2, -1], wLDiag[-2, 1], wLDiag[-2, 2],
 wLDiag[-2, 3], wLDiag[-1, -3], wLDiag[-1, -2], wLDiag[-1, -1], wLDiag[],
 wLDiag[-1, 2], wLDiag[-1, 3], wLDiag[1, -3], wLDiag[1, -2], wLDiag[],
 wLDiag[1, 1], wLDiag[1, 2], wLDiag[1, 3], wLDiag[2, -3], wLDiag[2, -2],
 wLDiag[2, -1], wLDiag[2, 1], wLDiag[2, 2], wLDiag[2, 3], wLDiag[],
 wLDiag[3, -2], wLDiag[3, -1], wLDiag[3, 1], wLDiag[3, 2], wLDiag[3, 3]}
```

```
AllLinearDiagrams[2]
```

```
{wLDiag[], wLDiag[-2], wLDiag[-1], wLDiag[1], wLDiag[2], wLDiag[-3, -3],
 wLDiag[-3, -2], wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[-3, 2], wLDiag[-3, 3],
 wLDiag[-2, -3], wLDiag[-2, -2], wLDiag[-2, -1], wLDiag[-2, 1], wLDiag[-2, 2],
 wLDiag[-2, 3], wLDiag[-1, -3], wLDiag[-1, -2], wLDiag[-1, -1], wLDiag[-1, 1],
 wLDiag[-1, 2], wLDiag[-1, 3], wLDiag[1, -3], wLDiag[1, -2], wLDiag[1, -1],
 wLDiag[1, 1], wLDiag[1, 2], wLDiag[1, 3], wLDiag[2, -3], wLDiag[2, -2],
 wLDiag[2, -1], wLDiag[2, 1], wLDiag[2, 2], wLDiag[2, 3], wLDiag[3, -3],
 wLDiag[3, -2], wLDiag[3, -1], wLDiag[3, 1], wLDiag[3, 2], wLDiag[3, 3]}
```

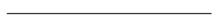
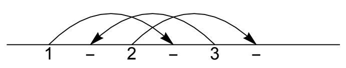
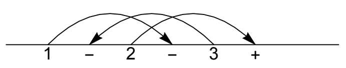
```
Select[AllLinearDiagrams[2], (# =!= RemoveR2[#]) &]
```

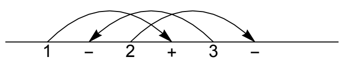
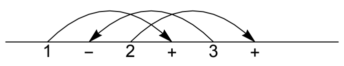
```
{wLDiag[-3, 3], wLDiag[-1, 1], wLDiag[1, -1], wLDiag[3, -3]}
```

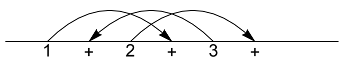
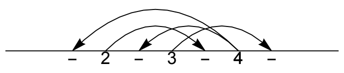
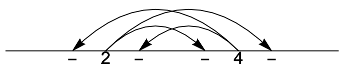
```
RemoveR2[w_wCDiag] /; Length[w] < 2 := w;
RemoveR2[w_wCDiag] := RemoveR2[w] = Module[{n, j, k = 0},
   n = Length[w];
   Do[If[w[[j]] + w[[j + 1]] == 0 && ! MemberQ[Abs[List @@ w], j + 1], k = j], {j, n - 1}];
   If[k ≠ 0,
    Delete[w, {{k}, {k + 1}}] /.
       j_Integer /; Abs[j] > k :> Sign[j] (Abs[j] - 2) /. {n - 1 → 1, 1 - n → -1},
    If[w[[1]] + w[[n]] == 0 && ! MemberQ[Abs[List @@ w], 1],
     w[[2 ;; n - 1]] /. j_Integer :> Sign[j] (Abs[j] - 1) /. {n - 1 → 1, 1 - n → -1},
     (*else*) w
    ]
   ]
  ]
```

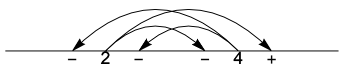
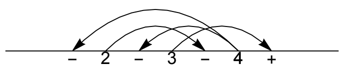
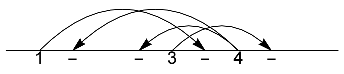
```
RemoveR12s[w_wLDiag | w_wCDiag] :=
 RemoveR12s[w] = FixedPoint[RemoveR2[RemoveR1[#]] &, w]
```

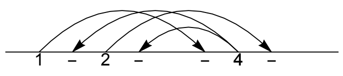
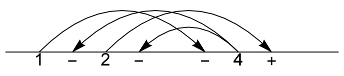
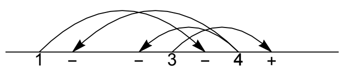
```
Union[RemoveR12s /@ AllCircularDiagrams[4]] // Draw
```

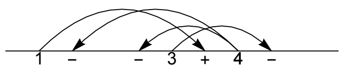
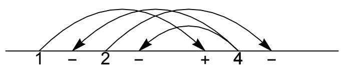
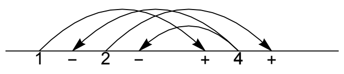
{ , 
1 – 2 – 3 –
, 
1 – 2 – 3 +
,

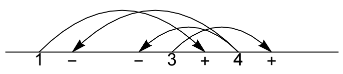
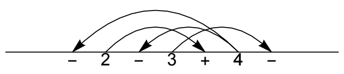
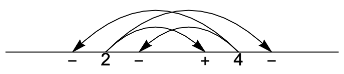

1 – 2 + 3 –
, 
1 – 2 + 3 +
,

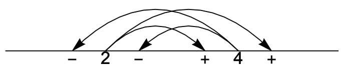
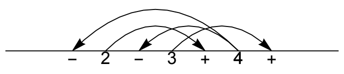
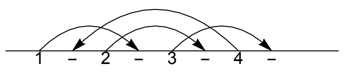

1 + 2 + 3 +
, 
– 2 – 3 – 4 –
, 
– 2 – – 4 –
,

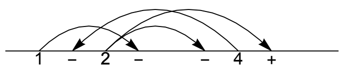
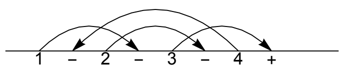
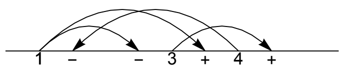

– 2 – – 4 +
, 
– 2 – 3 – 4 +
, 
1 – – 3 – 4 –
,

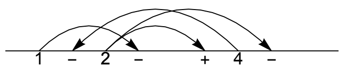
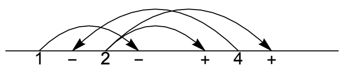
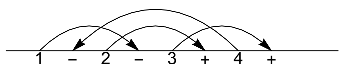

1 – 2 – – 4 –
, 
1 – 2 – – 4 +
, 
1 – – 3 – 4 +
,

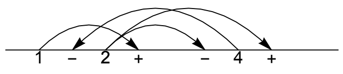
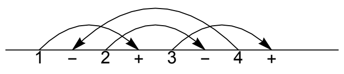
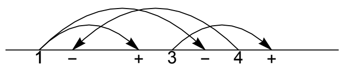

1 – – 3 + 4 –
, 
1 – 2 – + 4 –
, 
1 – 2 – + 4 +
,

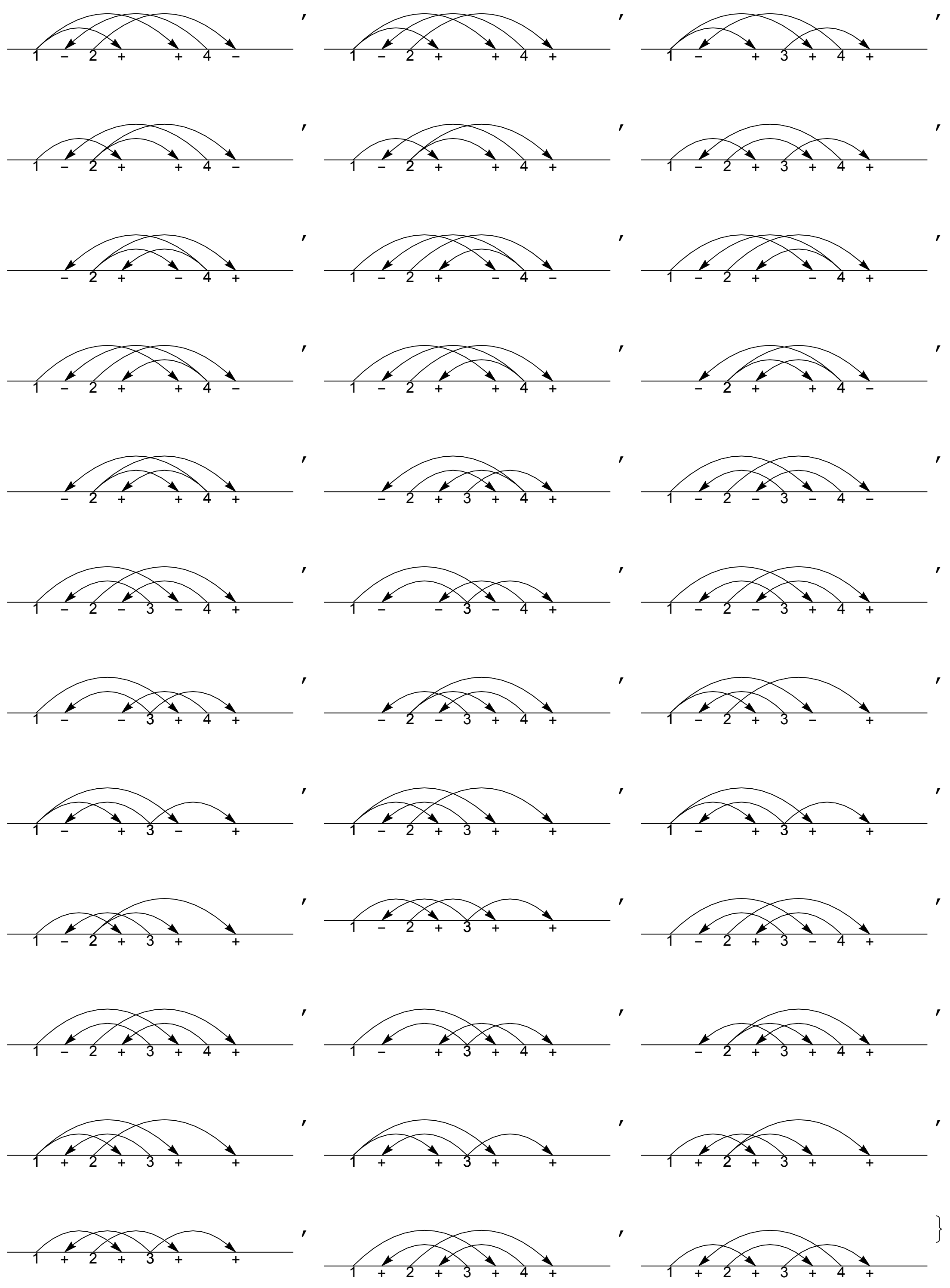

1 – – 3 + 4 +
, 
– 2 – 3 + 4 –
, 
– 2 – + 4 –
,


– 2 – + 4 +
, 
– 2 – 3 + 4 +
, 
1 – 2 – 3 – 4 –
,

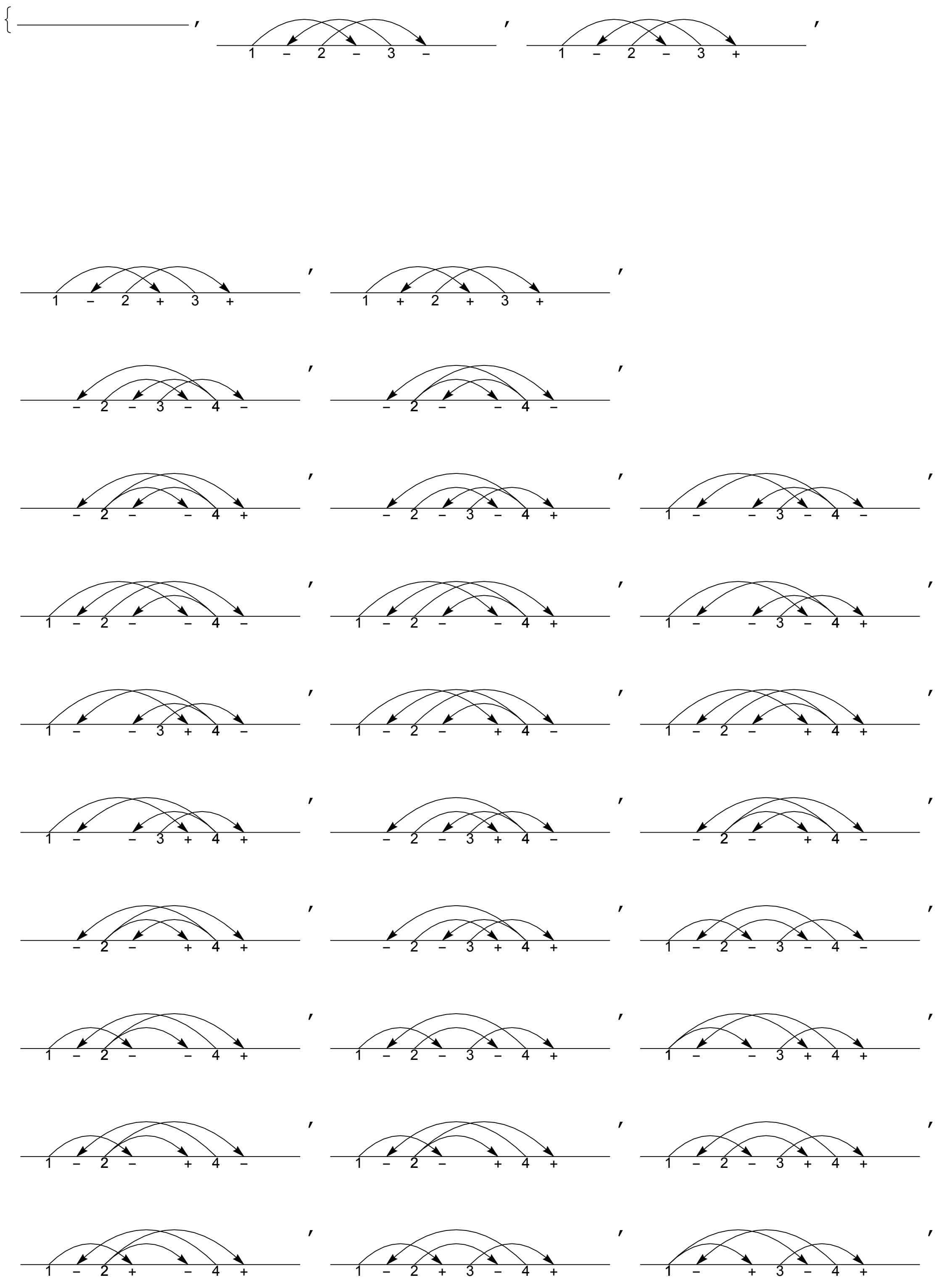

1 – 2 – – 4 +
, 
1 – 2 – 3 – 4 +
, 
1 – – 3 + 4 +
,

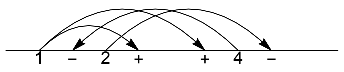
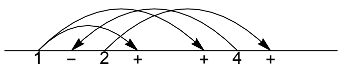
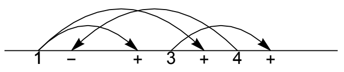

1 – 2 – + 4 –
, 
1 – 2 – + 4 +
, 
1 – 2 – 3 + 4 +
,

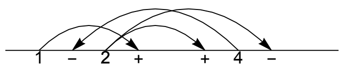
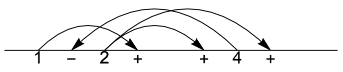
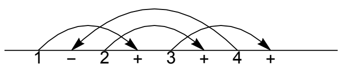

1 – 2 + – 4 +
, 
1 – 2 + 3 – 4 +
, 
1 – + 3 – 4 +
,

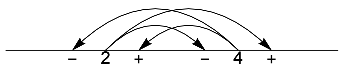
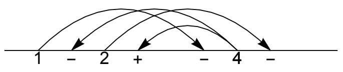
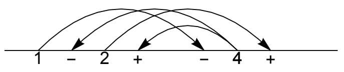


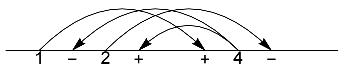
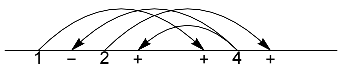
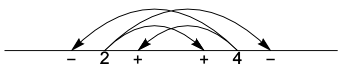
```
RF[w_wCDiag] := RF[w] = RotateToMinimal[RemoveR12s[w]];
RF[w_wLDiag] := RemoveR12s[w];

Union[RF /@ AllCircularDiagrams[4]] // Draw
```

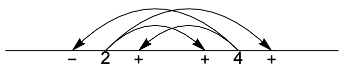
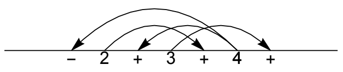
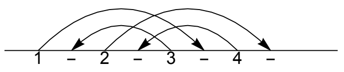

1 – 2 – 3 –
1 – 2 – 3 +
1 – 2 + 3 +
1 + 2 + 3 +
– 2 – 3 – 4 –
– 2 – – 4 –
– 2 – – 4 +
– 2 – 3 – 4 +
1 – – 3 – 4 –
1 – 2 – – 4 –
1 – 2 – – 4 +
1 – – 3 – 4 +
1 – – 3 + 4 –
1 – 2 – + 4 –
1 – 2 – + 4 +
1 – – 3 + 4 +
– 2 – 3 + 4 –
– 2 – + 4 –
– 2 – + 4 +
– 2 – 3 + 4 +
1 – 2 – 3 – 4 –
1 – 2 – – 4 +
1 – 2 – 3 – 4 +
1 – – 3 + 4 +
1 – 2 – + 4 –
1 – 2 – + 4 +
1 – 2 – 3 + 4 +
1 – 2 + – 4 +
1 – 2 + 3 – 4 +
1 – + 3 – 4 +

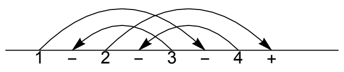
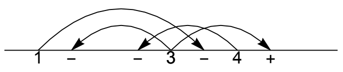
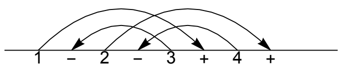

1 − 2 + + 4 −
,

1 − 2 + + 4 +
,

1 − + 3 + 4 +
,

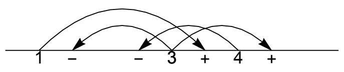
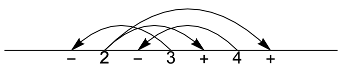
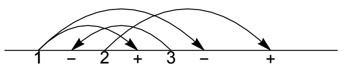

1 − 2 + + 4 −
,

1 − 2 + + 4 +
,

1 − 2 + 3 + 4 +
,

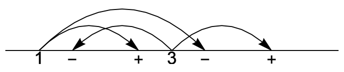
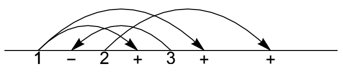
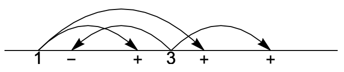

− 2 + − 4 +
,

1 − 2 + − 4 −
,

1 − 2 + − 4 +
,

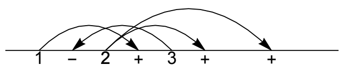
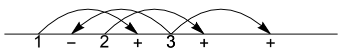
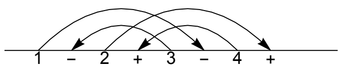

1 − 2 + + 4 −
,

1 − 2 + + 4 +
,

− 2 + + 4 −
,

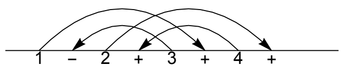
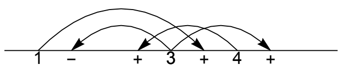
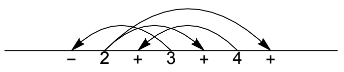

− 2 + + 4 +
,

− 2 + 3 + 4 +
,

1 − 2 − 3 − 4 −
,

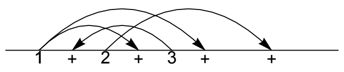
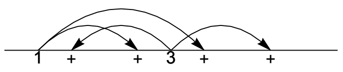
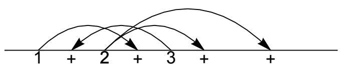

1 − 2 − 3 − 4 +
,

1 − − 3 − 4 +
,

1 − 2 − 3 + 4 +
,

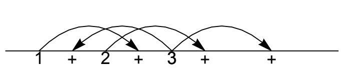
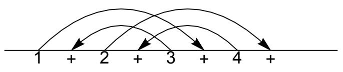
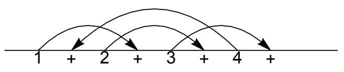

1 − − 3 + 4 +
,

− 2 − 3 + 4 +
,

1 − 2 + 3 − +
,


1 − + 3 − +
,

1 − 2 + 3 + +
,

1 − + 3 + +
,

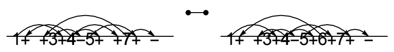

1 − 2 + 3 + +
,

1 − 2 + 3 + +
,

1 − 2 + 3 − 4 +
,


1 − 2 + 3 + 4 +
,

1 − + 3 + 4 +
,

− 2 + 3 + 4 +
,

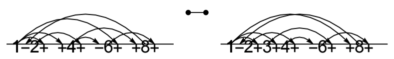

1 + 2 + 3 + +
,

1 + + 3 + +
,

1 + 2 + 3 + +
,


1 + 2 + 3 + +
,

1 + 2 + 3 + 4 +
,

1 + 2 + 3 + 4 +
}

```
wLDiag /:
  Resolve[wLDiag[R3[top_, mid_, bot_, s1_, s2_, s3_], ts__]] := UndirectedEdge[
   RF@ReplacePart[wLDiag@ts, {bot + (1 - s3) / 2 → s2 s3 top,
      bot + (1 + s3) / 2 → s1 s3 (mid + 1), mid → s2 top}],
   RF@ReplacePart[wLDiag@ts, {bot + (1 - s3) / 2 → s1 s3 mid,
      bot + (1 + s3) / 2 → s2 s3 top, mid → s2 top}]
  ];
wCDiag /: Resolve[wCDiag[R3[top_, mid_, bot_, s1_, s2_, s3_], ts__]] :=
 (RF[wCDiag[#]]) & /@ Resolve[wLDiag[R3[top, mid, bot, s1, s2, s3], ts]]

Resolve@wLDiag[R3[4, 6, 1, 1, 1, 1], 0, 0, +1, -3, +4, 0, +5, -7] // Draw
```



```
Resolve@wCDiag[R3[4, 6, 1, 1, 1, 1], 0, 0, +1, -3, +4, 0, +5, -7] // Draw
```

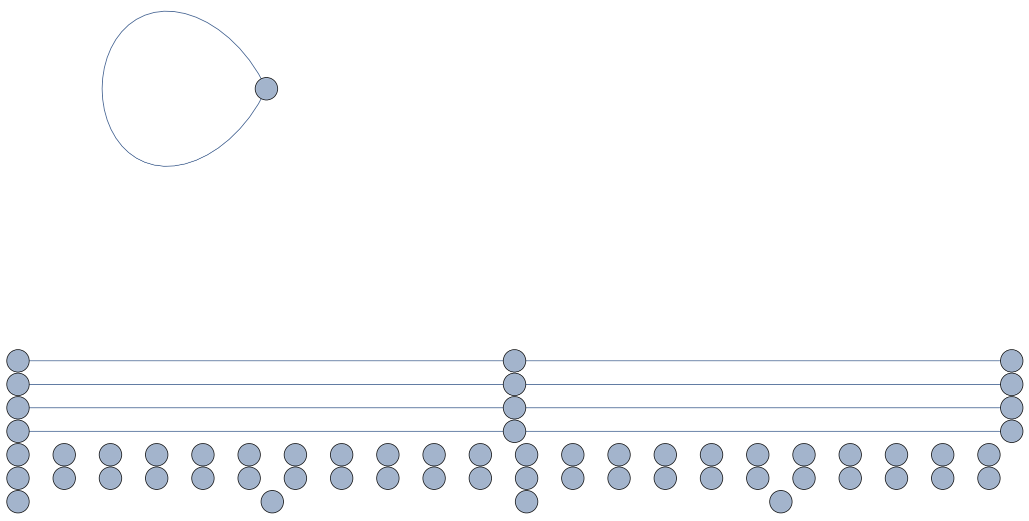


```
AllLinearR3s[n_] /; n < 3 := {};
AllLinearR3s[n_] := Flatten@Table[
    Prepend[
       ReplacePart[wLDiag @@ Table[0, {n}],
        Thread[Range[n] \ {bot, bot + 1, mid} → #]],
       R3[top, mid, bot, s1, s2, s3]
      ] & /@ Tuples[Range[-n - 1, n + 1] \ {-bot - 1, 0, bot + 1}, n - 3],
    {bot, Range[n - 1]},
    {mid, Range[n] \ {bot, bot + 1}}, {top, Range[n + 1] \ {bot + 1}},
    {s1, {-1, 1}}, {s2, {-1, 1}}, {s3, {-1, 1}}
   ];
AllCircularR3s[n_] /; n < 3 := {};
AllCircularR3s[n_] := Flatten@Table[
    Prepend[
       ReplacePart[wCDiag @@ Table[0, {n}], Thread[Range[n] \ {1, 2, mid} → #]],
       R3[top, mid, 1, s1, s2, s3]
      ] & /@ Tuples[Range[-n, n] \ {-2, 0, 2}, n - 3],
    {mid, Range[n] \ {1, 2}}, {top, Range[n] \ {2}},
    {s1, {-1, 1}}, {s2, {-1, 1}}, {s3, {-1, 1}}
   ];
```

```
Union[RF /@ AllLinearDiagrams[4]]
```

```
{wLDiag[], wLDiag[-3, -1], wLDiag[-3, 1], wLDiag[3, -1], wLDiag[3, 1],
 wLDiag[-4, -4, -2], wLDiag[-4, -4, -1], ⋯ 1163 ⋯, wLDiag[5, 5, 2, 3],
 wLDiag[5, 5, 5, -3], wLDiag[5, 5, 5, -2], wLDiag[5, 5, 5, -1],
 wLDiag[5, 5, 5, 1], wLDiag[5, 5, 5, 2], wLDiag[5, 5, 5, 3]}

large output   show less   show more   show all   set size limit...
```

```
n = 3;
vs = Union[RF /@ AllLinearDiagrams[n]];
es = Union[Resolve /@ AllLinearR3s[n]] /. Thread[vs → Range[Length@vs]];
g = Graph[Range[Length@vs], es]
```



```
ConnectedComponents[g]
```

```
{{58, 4, 39}, {3, 28, 9}, {61, 5, 44}, {23, 2, 6}, {15}, {16},
 {33}, {25}, {7}, {34}, {59}, {29}, {40}, {48}, {20}, {37}, {50}, {43},
 {22}, {46}, {32}, {41}, {17}, {53}, {35}, {60}, {51}, {13}, {19},
 {27}, {30}, {42}, {45}, {8}, {11}, {14}, {56}, {26}, {47}, {57}, {10},
 {12}, {49}, {18}, {24}, {36}, {52}, {54}, {21}, {31}, {1}, {55}, {38}}
```

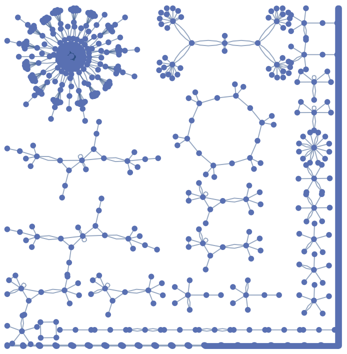
```
vs⟦Flatten@ConnectedComponents[g]⟧
```

```
{wLDiag[4, 4, -2], wLDiag[3, -1], wLDiag[3, -1, -1], wLDiag[-3, 1], wLDiag[-3, 1, 1],
 wLDiag[-4, -4, 2], wLDiag[4, 4, 2], wLDiag[3, 1], wLDiag[3, 1, 1],
 wLDiag[-3, -1, -1], wLDiag[-3, -1], wLDiag[-4, -4, -2], wLDiag[-4, 1, 2],
 wLDiag[-4, 4, -2], wLDiag[-3, 4, 2], wLDiag[-3, -1, 2], wLDiag[-4, -4, -1],
 wLDiag[3, -4, -2], wLDiag[4, 4, -1], wLDiag[-3, 1, 2], wLDiag[3, -1, 1],
 wLDiag[3, 4, 1], wLDiag[-3, -4, 1], wLDiag[3, -4, 2], wLDiag[4, -4, -2],
 wLDiag[3, 1, -1], wLDiag[-3, -1, -2], wLDiag[3, 4, -2], wLDiag[-3, 4, 1],
 wLDiag[3, -1, 2], wLDiag[-4, 4, 2], wLDiag[4, -1, -1], wLDiag[3, -4, -1],
 wLDiag[4, 4, 1], wLDiag[4, -4, 2], wLDiag[-4, 1, -2], wLDiag[-3, -4, -1],
 wLDiag[-3, 1, -1], wLDiag[-3, 4, -2], wLDiag[3, 1, -2], wLDiag[3, 1, 2],
 wLDiag[-4, -4, 1], wLDiag[-4, -1, -1], wLDiag[-4, 1, 1], wLDiag[4, 1, 1],
 wLDiag[-3, 1, -2], wLDiag[3, 4, -1], wLDiag[4, 1, 2], wLDiag[-4, -1, -2],
 wLDiag[-4, -1, 2], wLDiag[3, 4, 2], wLDiag[-3, -4, -2], wLDiag[-3, -1, 1],
 wLDiag[3, -4, 1], wLDiag[4, -1, -2], wLDiag[4, -1, 2], wLDiag[-3, -4, 2],
 wLDiag[-3, 4, -1], wLDiag[], wLDiag[4, 1, -2], wLDiag[3, -1, -2]}
```

```
wCDiag /@ vs⟦Flatten@ConnectedComponents[g]⟧ // Union
```

```
{wCDiag[], wCDiag[-2, -2], wCDiag[-2, 2], wCDiag[-1, 1],
 wCDiag[1, 1], wCDiag[-3, -3, -3], wCDiag[-3, -3, -2], wCDiag[-3, -3, 2],
 wCDiag[-3, -3, 3], wCDiag[-3, -2, -2], wCDiag[-3, -2, 2],
 wCDiag[-3, -1, -2], wCDiag[-3, -1, 1], wCDiag[-3, -1, 2], wCDiag[-3, -1, 3],
 wCDiag[-3, 1, -3], wCDiag[-3, 1, -1], wCDiag[-3, 1, 1], wCDiag[-3, 1, 2],
 wCDiag[-3, 1, 3], wCDiag[-3, 2, 2], wCDiag[-3, 3, -3], wCDiag[-3, 3, 2],
 wCDiag[-2, 1, 2], wCDiag[-2, 2, 2], wCDiag[-1, 1, 1], wCDiag[-1, 1, 2],
 wCDiag[1, 1, 1], wCDiag[1, 1, 2], wCDiag[2, 1, 2], wCDiag[3, 1, 2]}
```

```
Timing[n = 5;
 vs = Union[RF /@ AllCircularDiagrams[n]];
 es = Union[Resolve /@ AllLinearR3s[n] /. w_wLDiag ⧴ RF[wCDiag[w]]] /.
   Thread[vs → Range[Length@vs]];
 g = Graph[Range[Length@vs], es]
]
```

{29.624590,  }

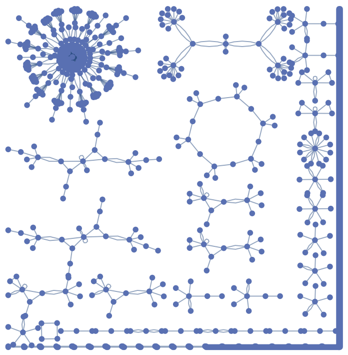
```
vs⟦Flatten@ConnectedComponents[g]⟧ // Length
```

```
67
```

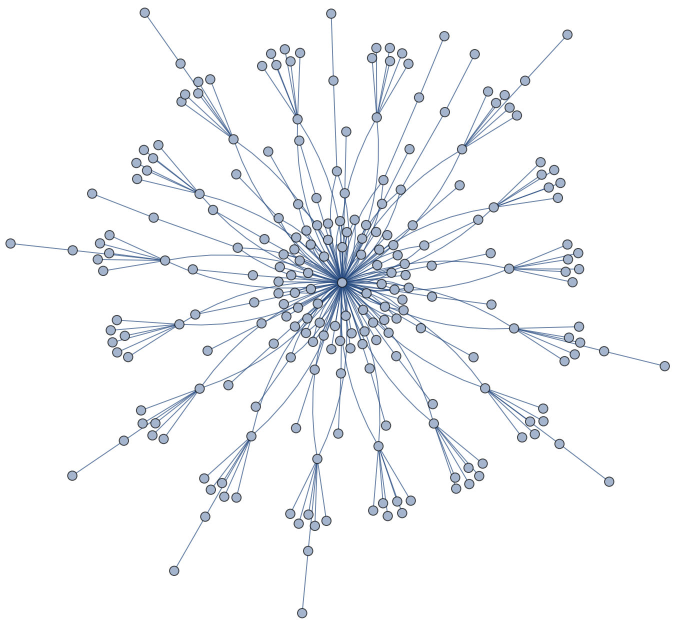
```
Select[vs⟦Flatten@ConnectedComponents[g]⟧, Length[#] == 3 &] // Length
```

```
4
```

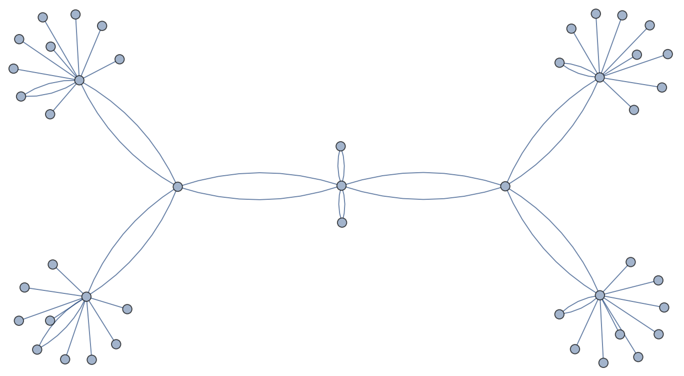
```
Timing[n = 5;
 vs = Union[RF /@ AllCircularDiagrams[n]];
 es = Union[Resolve /@ AllCircularR3s[n]] /. Thread[vs → Range[Length@vs]];
 g = Graph[Range[Length@vs], es]
]
```

```
{22.198942,
```

```
}
```

```
cc = ConnectedComponents[g];
Subgraph[g, cc⟦1⟧]
```



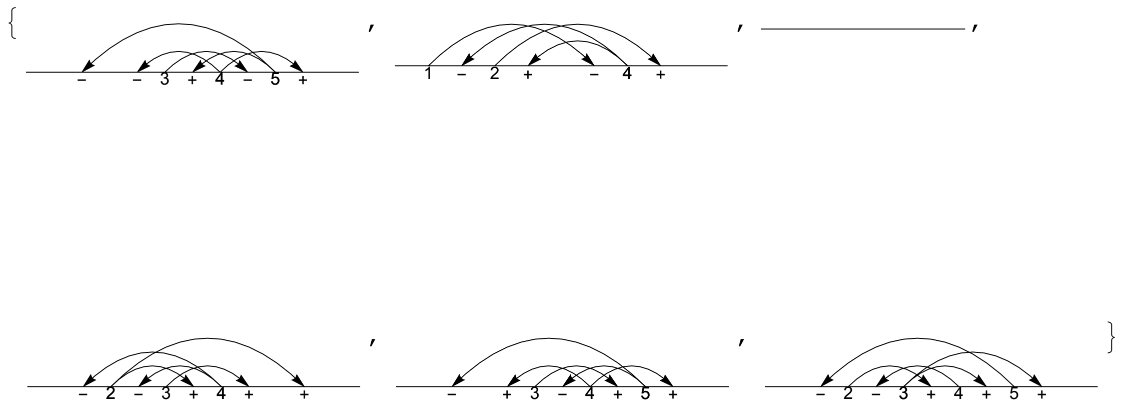
```
Subgraph[g, cc⟦2⟧]
```



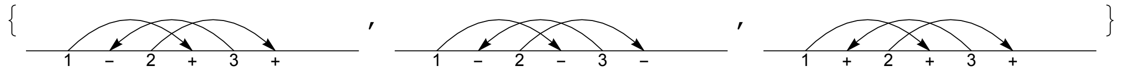
```
Length@cc⟦1⟧
```

```
237
```

```
vs⟦#⟧ & /@ FindShortestPath[g, cc⟦1, 1⟧, cc⟦1, 237⟧] // Draw
```



```
Select[Table[
   First@MinimalBy[vs⟦#⟧ & /@ c, Length],
   {c, cc}
  ], Length[#] == 3 &] // Draw
```



```
Select[Table[
   First@MinimalBy[vs⟦#⟧ & /@ c, Length],
   {c, cc}
  ], Length[#] == 4 &] // Length
```

```
25
```