\[ \eta = \sum C_i \alpha_{iy} \quad \lambda(\xi) = \frac{e^\xi - 1}{\xi} \Rightarrow \]

\[ j(\eta) = \sum C_i \alpha_{iy} \quad j(\eta) \alpha_{iy} = \sum C_i \alpha_{iy} \]

\[ e^\eta - 1 = \sum C_i \alpha_{iy} \quad \alpha = j(\eta) \alpha \]

\[ \eta = \log (1 + \sum C_i \alpha_{iy}) \quad \text{so} \quad j(\eta) = \frac{\sum C_i \alpha_{iy}}{\log (1 + \sum C_i \alpha_{iy})} \]

In this language,

\[ \beta_{iz} = \frac{1}{j(\xi + \eta)} \left( j(\xi) \alpha_{ix} + e^\xi j(\eta) \alpha_{iy} \right) \]

becomes

\[ \tilde{\beta}_{iz} = \tilde{\alpha}_{ix} + (1 + \sum C_j \alpha_{ix}) \tilde{\alpha}_{iy} \]

\[ = \tilde{\alpha}_{ix} + \tilde{\alpha}_{iy} + (\sum C_j \tilde{\alpha}) \tilde{\alpha}_{iy} \]

It would be nice to verify associativity here directly.

The simplification here is potentially so major it cannot be ignored.