

Pensieve Header: Alexander blobs Results.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2011-08\\w-Computations\\Recycling"];  
<< "AlexanderBlobs-Program-110820.m"
```

```
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]]
```

```
-Diag[h[1], ar[2, 3]] + Diag[h[2], ar[1, 3]]
```

```
b[r[1, 2], r[1, 3]] + b[r[1, 2], r[2, 3]] + b[r[1, 3], r[2, 3]]
```

```
0
```

```
ModDegree[4, R[1, 2]]
```

```
Diag[1] + Diag[1, ar[1, 2]] +
```

```
 $\frac{1}{2}$  Diag[1, ar[1, 2], ar[1, 2]] +  $\frac{1}{6}$  Diag[1, ar[1, 2], ar[1, 2], ar[1, 2]]
```

```
ModDegree[7, R[1, 2] ** R[1, 3] ** R[2, 3] - R[2, 3] ** R[1, 3] ** R[1, 2]]
```

```
0
```

```
v[0] = 0;
```

```
d = 1;
```

```
ModDegree[d + 1,
```

```
Print[
```

```
 $v[d] = v[d - 1] - \frac{1}{2}$  Diag[1, ar[1, 2]]
```

```
];
```

```
V[d] = DExp[v[d]];
```

```
V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
```

```
]
```

```
 $-\frac{1}{2}$  Diag[1, ar[1, 2]]
```

```
0
```

```
d = 2;
```

```
ModDegree[d + 1,
```

```
Print[
```

```
 $v[d] = v[d - 1] + (1 / 24)$  Diag[h[1], ar[1, 2]] +  $(1 / 12)$  Diag[h[2], ar[1, 2]]
```

```
];
```

```
V[d] = DExp[v[d]];
```

```
V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
```

```
]
```

```
 $-\frac{1}{2}$  Diag[1, ar[1, 2]] +  $\frac{1}{24}$  Diag[h[1], ar[1, 2]] +  $\frac{1}{12}$  Diag[h[2], ar[1, 2]]
```

```
0
```

```

d = 3;
ModDegree[d + 1,
  Print[
    v[d] = v[d - 1]
  ];
  V[d] = DExp[v[d]];
  V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]]$$

0

d = 4;
ModDegree[d + 1,
  Print[
    c[d, 0] = -1/720; c[d, 1] = -1/480; c[d, 2] = -1/720; c[d, 3] = -1/2880;
    v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 1 - k), ar[1, 2]], {k, 0, d - 1}]
  ];
  V[d] = DExp[v[d]];
  V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} +$$


$$\frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] -$$


$$\frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]]$$

0

d = 5;
ModDegree[d + 1,
  Print[
    c[5, _] = 0;
    v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 1 - k), ar[1, 2]], {k, 0, d - 1}]
  ];
  V[d] = DExp[v[d]];
  V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

$$-\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} +$$


$$\frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] -$$


$$\frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]]$$

0

```

```

d = 6;
ModDegree[d + 1,
Print[
  c[6, 0] = 1 / 30 240; c[6, 1] = 1 / 12 096; c[6, 2] = 1 / 9072;
  c[6, 3] = 1 / 12 096; c[6, 4] = 1 / 30 240; c[6, 5] = 1 / 181 440;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 1 - k), ar[1, 2]], {k, 0, d - 1}]
];
V[d] = DExp[v[d]];
V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

```

$$\begin{aligned}
& -\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} + \\
& \frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440} + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240} - \frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096} - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9072} + \frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096} + \frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240}
\end{aligned}$$

0

```
ModDegree[7, V[6] ** Adjoint[V[6]]]
```

```
Diag[1]
```

```

d = 7;
ModDegree[d + 1,
Print[
  c[7, _] = 0;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 1 - k), ar[1, 2]], {k, 0, d - 1}]
];
V[d] = DExp[v[d]];
V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

```

$$\begin{aligned}
& -\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} + \\
& \frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440} + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240} - \frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096} - \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]] + \\
& \frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9072} + \frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096} + \frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240}
\end{aligned}$$

0

```

d = 8;
ModDegree[d + 1,
Print[
  c[8, 0] = -1 / 1 209 600; c[8, 1] = -1 / 345 600;
  c[8, 2] = -1 / 172 800; c[8, 3] = -1 / 138 240; c[8, 4] = -1 / 172 800;
  c[8, 5] = -1 / 345 600; c[8, 6] = -1 / 1 209 600; c[8, 7] = -1 / 9 676 800;
  v[d] = v[d - 1] + Sum[c[d, k] Diag[h[1]^k * h[2]^(d - 1 - k), ar[1, 2]], {k, 0, d - 1}]
];
V[d] = DExp[v[d]];
V[d] ** DExp[r[1, 3] + r[2, 3]] - R[1, 3] ** R[2, 3] ** V[d]
]

```

$$\begin{aligned}
& -\frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[1, 2]] - \frac{\text{Diag}[h[1]^3, \text{ar}[1, 2]]}{2880} + \\
& \frac{\text{Diag}[h[1]^5, \text{ar}[1, 2]]}{181\,440} - \frac{\text{Diag}[h[1]^7, \text{ar}[1, 2]]}{9\,676\,800} + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 2]] - \\
& \frac{1}{720} \text{Diag}[h[1]^2 h[2], \text{ar}[1, 2]] + \frac{\text{Diag}[h[1]^4 h[2], \text{ar}[1, 2]]}{30\,240} - \frac{\text{Diag}[h[1]^6 h[2], \text{ar}[1, 2]]}{1\,209\,600} - \\
& \frac{1}{480} \text{Diag}[h[1] h[2]^2, \text{ar}[1, 2]] + \frac{\text{Diag}[h[1]^3 h[2]^2, \text{ar}[1, 2]]}{12\,096} - \frac{\text{Diag}[h[1]^5 h[2]^2, \text{ar}[1, 2]]}{345\,600} - \\
& \frac{1}{720} \text{Diag}[h[2]^3, \text{ar}[1, 2]] + \frac{\text{Diag}[h[1]^2 h[2]^3, \text{ar}[1, 2]]}{9\,072} - \frac{\text{Diag}[h[1]^4 h[2]^3, \text{ar}[1, 2]]}{172\,800} + \\
& \frac{\text{Diag}[h[1] h[2]^4, \text{ar}[1, 2]]}{12\,096} - \frac{\text{Diag}[h[1]^3 h[2]^4, \text{ar}[1, 2]]}{138\,240} + \frac{\text{Diag}[h[2]^5, \text{ar}[1, 2]]}{30\,240} - \\
& \frac{\text{Diag}[h[1]^2 h[2]^5, \text{ar}[1, 2]]}{172\,800} - \frac{\text{Diag}[h[1] h[2]^6, \text{ar}[1, 2]]}{345\,600} - \frac{\text{Diag}[h[2]^7, \text{ar}[1, 2]]}{1\,209\,600}
\end{aligned}$$

0

```
ModDegree[9, V[8] ** Adjoint[V[8]]]
```

```
Diag[1]
```

```
v[8] /. Diag[hs_, ar[1, 2]] -> hs /. {h[1] -> h1, h[2] -> h2}
```

$$\begin{aligned}
& -\frac{1}{2} + \frac{h1}{24} - \frac{h1^3}{2880} + \frac{h1^5}{181\,440} - \frac{h1^7}{9\,676\,800} + \frac{h2}{12} - \frac{h1^2 h2}{720} + \frac{h1^4 h2}{30\,240} - \frac{h1^6 h2}{1\,209\,600} - \frac{h1 h2^2}{480} + \frac{h1^3 h2^2}{12\,096} - \\
& \frac{h1^5 h2^2}{345\,600} - \frac{h2^3}{720} + \frac{h1^2 h2^3}{9\,072} - \frac{h1^4 h2^3}{172\,800} + \frac{h1 h2^4}{12\,096} - \frac{h1^3 h2^4}{138\,240} + \frac{h2^5}{30\,240} - \frac{h1^2 h2^5}{172\,800} - \frac{h1 h2^6}{345\,600} - \frac{h2^7}{1\,209\,600}
\end{aligned}$$

```
-1 209 600 * 8 * Table[c[8, k], {k, 0, 7}]
```

```
{8, 28, 56, 70, 56, 28, 8, 1}
```

```
Table[Binomial[8, k], {k, 1, 8}]
```

```
{8, 28, 56, 70, 56, 28, 8, 1}
```

```
Clear[Phi];
```

```
Phi[d_] := ModDegree[d + 1,
```

```
  V[d] = DExp[v[d]];
  Phi[d] = PutOn[{{1, 2}, 3}, Adjoint[V[d]]] **
```

```
  Adjoint[V[d]] ** PutOn[{2, 3}, V[d]] ** PutOn[{1, {2, 3}}, V[d]]
```

```
]

```

ModDegree[3, V[3]]

$$\begin{aligned} & \text{Diag}[1] - \frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[\text{h}[1], \text{ar}[1, 2]] + \frac{1}{12} \text{Diag}[\text{h}[2], \text{ar}[1, 2]] + \\ & \frac{1}{8} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[\text{h}[1], \text{ar}[1, 2], \text{ar}[1, 2]] - \\ & \frac{1}{24} \text{Diag}[\text{h}[2], \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] \end{aligned}$$

V[2]

$$\begin{aligned} & \text{Diag}[1] - \frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[\text{h}[1], \text{ar}[1, 2]] + \\ & \frac{1}{12} \text{Diag}[\text{h}[2], \text{ar}[1, 2]] + \frac{1}{8} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] \end{aligned}$$

Phi[2]

$$\text{Diag}[1] + \frac{1}{12} \text{Diag}[\text{h}[1], \text{ar}[2, 3]] - \frac{1}{12} \text{Diag}[\text{h}[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[\text{h}[3], \text{ar}[1, 2]]$$

Pentagon[d_] := ModDegree[d+1,

Phi[d] ** PutOn[{1, {2, 3}, 4}, Phi[d]] ** PutOn[{2, 3, 4}, Phi[d]] -

PutOn[{1, 2}, 3, 4], Phi[d]] ** PutOn[{1, 2, {3, 4}}, Phi[d]]

]

Pentagon[2]

0

V[3]

$$\begin{aligned} & \text{Diag}[1] - \frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{24} \text{Diag}[\text{h}[1], \text{ar}[1, 2]] + \frac{1}{12} \text{Diag}[\text{h}[2], \text{ar}[1, 2]] + \\ & \frac{1}{8} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[\text{h}[1], \text{ar}[1, 2], \text{ar}[1, 2]] - \\ & \frac{1}{24} \text{Diag}[\text{h}[2], \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] \end{aligned}$$

Phi[3]

$$\text{Diag}[1] + \frac{1}{12} \text{Diag}[\text{h}[1], \text{ar}[2, 3]] - \frac{1}{12} \text{Diag}[\text{h}[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[\text{h}[3], \text{ar}[1, 2]]$$

Pentagon[3]

0

Pentagon[4]

0

Pentagon[5]

\$Aborted

Pentagon[3]

0

b[Pentagon[3], PutOn[{{1, 2, 3, 4}, 5}, Diag[1, ar[1, 2]]]]

0

b[Pentagon[3], PutOn[{{1, 2, 3, 4}, 5}, Diag[1, ar[2, 1]]]]

0

Phi[3]

$$\text{Diag}[1] + \frac{1}{12} \text{Diag}[h[1], \text{ar}[2, 3]] - \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[h[3], \text{ar}[1, 2]]$$

b[Phi[3], PutOn[{{1, 2, 3}, 4}, Diag[1, ar[2, 1]]]]

0

b[Phi[3], PutOn[{{1, 2, 3}, 4}, Diag[1, ar[1, 2]]]]

0

Phi[2]

$$\text{Diag}[1] + \frac{1}{12} \text{Diag}[h[1], \text{ar}[2, 3]] - \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[h[3], \text{ar}[1, 2]]$$

b[Phi[2], PutOn[{{1, 2, 3}, 4}, Diag[1, ar[1, 2]]]]

0

RR[d_] := ModDegree[d + 1, DExp[1 / 2 Diag[1, ar[1, 2]] + 1 / 2 Diag[1, ar[2, 1]]]];

RR[3]

$$\begin{aligned} & \text{Diag}[1] + \frac{1}{2} \text{Diag}[1, \text{ar}[1, 2]] + \frac{1}{2} \text{Diag}[1, \text{ar}[2, 1]] + \frac{1}{8} \text{Diag}[h[1], \text{ar}[2, 1]] + \\ & \frac{1}{8} \text{Diag}[h[1], \text{ar}[2, 2]] + \frac{1}{48} \text{Diag}[h[1]^2, \text{ar}[2, 1]] + \frac{1}{48} \text{Diag}[h[1]^2, \text{ar}[2, 2]] - \\ & \frac{1}{8} \text{Diag}[h[2], \text{ar}[1, 1]] - \frac{1}{8} \text{Diag}[h[2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[h[1] h[2], \text{ar}[1, 1]] - \\ & \frac{1}{48} \text{Diag}[h[1] h[2], \text{ar}[1, 2]] - \frac{1}{48} \text{Diag}[h[1] h[2], \text{ar}[2, 1]] - \frac{1}{48} \text{Diag}[h[1] h[2], \text{ar}[2, 2]] + \\ & \frac{1}{48} \text{Diag}[h[2]^2, \text{ar}[1, 1]] + \frac{1}{48} \text{Diag}[h[2]^2, \text{ar}[1, 2]] + \frac{1}{8} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2]] + \\ & \frac{1}{4} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{8} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1]] + \\ & \frac{1}{16} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 1]] + \frac{1}{16} \text{Diag}[h[1], \text{ar}[1, 2], \text{ar}[2, 2]] + \\ & \frac{1}{16} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 1]] + \frac{1}{16} \text{Diag}[h[1], \text{ar}[2, 1], \text{ar}[2, 2]] - \\ & \frac{1}{16} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[1, 2]] - \frac{1}{16} \text{Diag}[h[2], \text{ar}[1, 1], \text{ar}[2, 1]] - \\ & \frac{1}{16} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[1, 2]] - \frac{1}{16} \text{Diag}[h[2], \text{ar}[1, 2], \text{ar}[2, 1]] + \\ & \frac{1}{48} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[1, 2]] + \frac{1}{16} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[1, 2], \text{ar}[2, 1]] + \\ & \frac{1}{16} \text{Diag}[1, \text{ar}[1, 2], \text{ar}[2, 1], \text{ar}[2, 1]] + \frac{1}{48} \text{Diag}[1, \text{ar}[2, 1], \text{ar}[2, 1], \text{ar}[2, 1]] \end{aligned}$$

```

Hexagon[d_] := ModDegree[d + 1,
  PutOn[{{1, 2}, 3}, RR[d]] - Phi[d] ** PutOn[{2, 3}, RR[d]] **
  PutOn[{1, 3, 2}, DInvert[Phi[d]]] ** PutOn[{1, 3}, RR[d]] ** PutOn[{3, 1, 2}, Phi[d]]
]

```

```
Phi[2]
```

$$\text{Diag}[1] + \frac{1}{12} \text{Diag}[h[1], \text{ar}[2, 3]] - \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[h[3], \text{ar}[1, 2]]$$

```
Y = b[Diag[1, ar[1, 2]] + Diag[1, ar[2, 1]], Diag[1, ar[2, 3]] + Diag[1, ar[3, 2]]]
```

```
-Diag[h[1], ar[2, 3]] + Diag[h[1], ar[3, 2]] + Diag[h[2], ar[1, 3]] -
  Diag[h[2], ar[3, 1]] - Diag[h[3], ar[1, 2]] + Diag[h[3], ar[2, 1]]
```

```
Y + b[Diag[1, ar[1, 2]] + Diag[1, ar[2, 1]], Diag[1, ar[1, 3]] + Diag[1, ar[3, 1]]]
```

```
0
```

```
PutOn[{2, 3, 1}, Phi[2]] - Phi[2]
```

$$\frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[3, 1]] -$$

$$\frac{1}{12} \text{Diag}[h[3], \text{ar}[1, 2]] - \frac{1}{12} \text{Diag}[h[3], \text{ar}[2, 1]]$$

```
PutOn[{2, 1, 3}, Phi[2]]
```

$$\text{Diag}[1] - \frac{1}{12} \text{Diag}[h[1], \text{ar}[2, 3]] + \frac{1}{12} \text{Diag}[h[2], \text{ar}[1, 3]] + \frac{1}{12} \text{Diag}[h[3], \text{ar}[2, 1]]$$

```
b[Y, PutOn[{{1, 2, 3}, 4}, Diag[1, ar[1, 2]]]]
```

```
0
```

```
b[Y, PutOn[{{1, 2, 3}, 4}, Diag[1, ar[2, 1]]]]
```

```
0
```

```
Hexagon[2]
```

$$\frac{1}{24} \text{Diag}[h[1], \text{ar}[2, 3]] + \frac{1}{24} \text{Diag}[h[1], \text{ar}[3, 2]] + \frac{1}{24} \text{Diag}[h[2], \text{ar}[1, 3]] +$$

$$\frac{1}{24} \text{Diag}[h[2], \text{ar}[3, 1]] - \frac{1}{8} \text{Diag}[h[3], \text{ar}[1, 2]] - \frac{1}{8} \text{Diag}[h[3], \text{ar}[2, 1]]$$