

Pensieve header: Testing if vertical flip is a symmetry of SolKV.

```

SetDirectory["C:/drorbn/AcademicPensieve/Projects/WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V0 = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
R4Eqn[V_] = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn[V_] = (V ** (V // dA) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn[V_] = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
  (Cap (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs[x] = 1/4; βs[y] = 0;
SeriesSolve[{α, β, γ, κ}, (ħ-1 R4Eqn[V0]) && UnitarityEqn[V0] && CapEqn[V0]];
V1 = ((V0 // dA) ** Rs[x, y]) // dS;
V2 = (V0 // dA) // dS;
{V0, κ, V1, V2}

FreeLie` implements / extends
{*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, EulerE, Exp,
  Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ℓ, Λ, σ, ħ, ←, ↗}.

FreeLie` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg,
  dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.

AwCalculus` is in the public domain; yet Dror Bar-Natan
  is committed to support it within reason until July 15, 2022.

Arbitrarily setting {κs[x] → 0}.

Arbitrarily setting {αs[x, y, y] → 0}.

```

$$\left\{ \text{Es} \left[ \left\langle x \rightarrow \text{LS} \left[ -\frac{\overline{y}}{4}, -\frac{\overline{xy}}{96}, \frac{1}{128} \overline{xx\overline{y}}, \dots \right], y \rightarrow \text{LS} \left[ \frac{\overline{x}}{4}, \frac{\overline{xy}}{96}, \frac{1}{128} \overline{x\overline{xy}} - \frac{1}{128} \overline{x\overline{y}y}, \dots \right] \right\rangle, \right. \\ \left. \text{CWS} \left[ 0, -\frac{\overline{xy}}{48}, \frac{\overline{xx\overline{y}}}{512} + \frac{\overline{xy\overline{y}}}{512}, \dots \right], \text{CWS} \left[ 0, -\frac{\overline{xx}}{96}, \frac{\overline{xxx}}{1536}, \dots \right], \text{Es} \left[ \right. \right. \\ \left. \left. \left\langle x \rightarrow \text{LS} \left[ \frac{\overline{y}}{4}, -\frac{\overline{xy}}{96}, -\frac{1}{128} \overline{x\overline{xy}}, \dots \right], y \rightarrow \text{LS} \left[ \frac{3\overline{x}}{4}, -\frac{23\overline{xy}}{96}, -\frac{3}{128} \overline{x\overline{xy}} + \frac{5}{128} \overline{x\overline{y}y}, \dots \right] \right\rangle, \right. \right. \\ \left. \left. \text{CWS} \left[ 0, -\frac{\overline{xy}}{48}, -\frac{\overline{xx\overline{y}}}{512} - \frac{\overline{xy\overline{y}}}{512}, \dots \right], \right. \right. \\ \left. \left. \text{Es} \left[ \left\langle x \rightarrow \text{LS} \left[ \frac{\overline{y}}{4}, -\frac{\overline{xy}}{96}, -\frac{1}{128} \overline{x\overline{xy}}, \dots \right], y \rightarrow \text{LS} \left[ -\frac{\overline{x}}{4}, \frac{\overline{xy}}{96}, -\frac{1}{128} \overline{x\overline{xy}} + \frac{1}{128} \overline{x\overline{y}y}, \dots \right] \right\rangle, \right. \right. \\ \left. \left. \text{CWS} \left[ 0, -\frac{\overline{xy}}{48}, -\frac{\overline{xx\overline{y}}}{512} - \frac{\overline{xy\overline{y}}}{512}, \dots \right] \right] \right\}$$

**R4Eqn[V1]@{7}**

Arbitrarily setting {as[x, x, x, y, y] → 0}.

Arbitrarily setting {as[x, x, x, x, x, y, y] → 0}.

BS[8 True, ...]

**R4Eqn[V2]@{3}**

$$\text{BS} \left[ 2 \text{ True}, 0 = -\overline{xy}, 0 = -\overline{xy} \ \&\& \ 0 = \frac{1}{4} \overline{x\overline{xy}} + \frac{1}{4} \overline{x\overline{y}y}, \dots \right]$$

**UnitarityEqn[V1]@{7}**

BS[8 True, ...]

**CapEqn[V1]@{3}**

$$\text{BS} \left[ 3 \text{ True}, \frac{\overline{xxx}}{1536} + \frac{\overline{xx\overline{y}}}{256} + \frac{\overline{xy\overline{y}}}{256} + \frac{\overline{yyy}}{1536} = \frac{\overline{xxx}}{1536} + \frac{\overline{yyy}}{1536}, \dots \right]$$

**κ@{10}**

Arbitrarily setting {as[x, x, x, x, y, x, y, y] → 0}.

Arbitrarily setting {as[x, x, x, x, x, x, x, y, y] → 0}.

Arbitrarily setting {as[x, x, x, x, x, x, y, x, y, y] → 0}.

$$\text{CWS} \left[ 0, -\frac{\overline{xx}}{96}, \frac{\overline{xxx}}{1536}, \frac{\overline{xxxx}}{11520}, -\frac{13 \overline{xxxxx}}{491520}, -\frac{\overline{xxxxxx}}{725760}, \right. \\ \left. \frac{233 \overline{xxxxxxx}}{330301440}, \frac{\overline{xxxxxxxx}}{38707200}, -\frac{21163 \overline{xxxxxxxxx}}{1141521776640}, -\frac{\overline{xxxxxxxxxx}}{1916006400}, \dots \right]$$

```
κ1 = CWS[{x}, κ1s]; Cap1 = Es[⟨x → LS[0]⟩, κ1];
SeriesSolve[κ1, ((V1 ** (Cap1 // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
(Cap1 (Cap1 // dσ[x, y]) // dc[x] // dc[y]))
];
```

```
κ1@{8}
```

```
Arbitrarily setting {κ1s[x] → 0}.
```

```
CWS[0, - $\frac{\overline{xx}}$ , - $\frac{\overline{xxx}}$ ,  $\frac{\overline{xxxx}}$ ,  $\frac{13 \overline{xxxxx}}$ , - $\frac{\overline{xxxxxxx}}$ , - $\frac{233 \overline{xxxxxxxx}}$ ,  $\frac{\overline{xxxxxxxxx}}$ , ...]
```

```
(V1 ≡ (Rs[x, y] ** (V0 // dA // dS)))@{7}
```

```
BS[8 True, ...]
```

```
{TimeUsed[], MaxMemoryUsed[]}
```

```
{2650.99, 3 936 649 024}
```