

Pensieve header: Debugging α Map.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
$SeriesShowDegree = 4;
```

FreeLie` implements / extends

```
{*, +, **, $SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords,
AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence,
BracketForm, BS, CC, Crop, CW, CWS, CWSeries, D, Deg, DegreeScale,
DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA,
LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism,
New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
t, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ℓ, Λ, σ, ħ, ⌊, ⌋}.
```

FreeLie` is in the public domain. Dror Bar-Natan

is committed to support it within reason until July 15, 2022.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm,

dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan

is committed to support it within reason until July 15, 2022.

```
Φs[2, 1] = Φs[3, 1] = Φs[3, 2] = 0; Φs[3, 1, 2] = 1/24; Φ = DKS[3, Φs];
SeriesSolve[Φ, (Φσ[3,2,1] ≡ -Φ) ∧ (Φ ** Φσ[1,23,4] ** Φσ[2,3,4] ≡ Φσ[12,3,4] ** Φσ[1,2,34])]];
Φ@{6}
```

SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {Φs[3, 1, 1, 2] → 0}.

SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {Φs[3, 1, 1, 1, 1, 2] → 0}.

$$\text{DKS}\left[0, \frac{1}{24} t_{13} t_{23}, 0, -\frac{7 t_{13} t_{23} t_{23} t_{23}}{5760} + \frac{7 t_{13} t_{13} t_{23} t_{23}}{5760} - \frac{t_{13} t_{13} t_{13} t_{23}}{1440}, 0, \right. \\ \left. \frac{31 t_{13} t_{23} t_{23} t_{23} t_{23} t_{23}}{967680} - \frac{157 t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}{1935360} - \frac{31 t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}{387072} - \right. \\ \left. \frac{31 t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}{483840} + \frac{11 t_{13} t_{13} t_{13} t_{23} t_{13} t_{23}}{290304} + \frac{31 t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}{725760} + \right. \\ \left. \frac{83 t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}{967680} - \frac{13 t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}{241920} + \frac{t_{13} t_{13} t_{13} t_{13} t_{13} t_{23}}{60480}, \dots\right]$$

```

αMap[S_List][dks_DKSeries] := Sum[
  LieMorphism[
    Table[
      LW@S[[i]] → AngleBracket@@Table[
        S[[j]] → LS[Switch[j,
          i, LW@S[[k]],
          k, LW@S[[i]],
          _, 0
        ]],
      {j, Length[S]}],
    {i, k - 1}
  ],
  AngleBracket@@S, tb
][dks_S[[k]],
{k, 2, Length[S]}
]

```

```

Ψ = Γ[El[αMap[{1, 2, 3}][Φ], CWS[0]]]

```

$$\begin{aligned}
 \text{Es} \left[\left(1 \rightarrow \text{LS} \left[0, \frac{\overline{23}}{24}, 0, -\frac{\overline{1123}}{1440} + \frac{\overline{71223}}{5760} + \frac{\overline{1233}}{5760} - \frac{\overline{72223}}{5760} + \frac{\overline{72233}}{5760} + \right. \right. \\
 \left. \left. \frac{1}{480} \frac{\overline{1213}}{\overline{1213}} - \frac{\overline{1323}}{1920} + \frac{1}{640} \frac{\overline{123}2}{\overline{123}2} - \frac{\overline{1322}}{1152} - \frac{\overline{1332}}{1152} - \frac{\overline{2333}}{1440}, \dots \right], \right. \\
 2 \rightarrow \text{LS} \left[0, -\frac{\overline{13}}{24}, 0, \frac{\overline{1113}}{1440} - \frac{\overline{1123}}{1152} + \frac{\overline{71223}}{1920} - \frac{1}{480} \frac{\overline{1132}}{\overline{1132}} - \frac{\overline{1133}}{5760} + \frac{\overline{1233}}{1152} + \right. \\
 \left. \frac{7\overline{1213}}{5760} + \frac{19\overline{1323}}{5760} + \frac{7\overline{123}2}{1920} + \frac{7\overline{1322}}{5760} + \frac{7\overline{1332}}{5760} + \frac{\overline{1333}}{1440}, \dots \right], \\
 3 \rightarrow \text{LS} \left[0, \frac{\overline{12}}{24}, 0, -\frac{\overline{1112}}{1440} + \frac{\overline{1123}}{5760} + \frac{\overline{71223}}{5760} + \frac{7\overline{1122}}{5760} - \frac{\overline{1132}}{1440} - \frac{\overline{1233}}{1440} + \frac{\overline{1213}}{5760} + \right. \\
 \left. \frac{\overline{1323}}{1440} - \frac{\overline{123}2}{1152} - \frac{7\overline{1222}}{5760} - \frac{7\overline{1322}}{5760} - \frac{\overline{1332}}{1440}, \dots \right] \Big), \text{CWS}[0, 0, 0, 0, \dots] \Big]
 \end{aligned}$$

$$(\Psi ** \Psi^{\sigma[1,23,4]} ** \Psi^{\sigma[2,3,4]} \equiv \Psi^{\sigma[12,3,4]} ** \Psi^{\sigma[1,2,34]}) @ \{8\}$$

SeriesSolve::ArbitrarilySetting : In degree 7 arbitrarily setting {Φs[3, 1, 1, 1, 1, 1, 2] → 0}.

SeriesSolve::ArbitrarilySetting : In degree 8 arbitrarily setting {Φs[3, 1, 1, 1, 1, 1, 2, 1, 2] → 0}.

BS[9 True, ...]

S = {1, 2, 3}; dks = Φ;

k = 2

2

dks_x

LS[0, 0, 0, 0, ...]

```

Table[
  LW@S[[i]] → AngleBracket@@ Table[
    S[[j]] → LS[Switch[j,
      i, LW@S[[k]],
      k, LW@S[[i]],
      _, 0
    ]],
    {j, Length[S]}],
  {i, k - 1}
]
{LW[1] → ⟨1 → LS[2̄, 0, 0, 0, ...], 2 → LS[1̄, 0, 0, 0, ...], 3 → LS[0, 0, 0, 0, ...]}

AngleBracket@@Take[S, k]
⟨1, 2⟩

LieMorphism[
  Table[
    LW@S[[i]] → AngleBracket@@
      Table[S[[j]] → LS[Switch[j, i, LW@S[[k]], k, LW@S[[i]], _, 0]], {j, k}],
    {i, k - 1}],
  AngleBracket@@Take[S, k], tb
]
LieMorphism[LieMorphism$347]

LieMorphism[
  Table[
    LW@S[[i]] → AngleBracket@@
      Table[S[[j]] → LS[Switch[j, i, LW@S[[k]], k, LW@S[[i]], _, 0]], {j, k}],
    {i, k - 1}],
  AngleBracket@@Take[S, k], tb
][dksk]
⟨1 → LS[0, 0, 0, 0, ...], 2 → LS[0, 0, 0, 0, ...]⟩

```