

# RESPONSE TO THE REFEREE'S REPORT ON "FINITE TYPE INVARIANTS OF W-KNOTTED OBJECTS I: BRAIDS, KNOTS AND THE ALEXANDER POLYNOMIAL"

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First of all thank you very much for reading the paper so thoroughly, and for your corrections and suggestions.

We have addressed all of the editing suggestions and questions in the referee report. Below we give a list of those points where we differ from your suggestions, as well as any points where the changes require an explanation:

**Some of the punctuation remarks:** We agreed with most of these, and asked and followed the advice of an "English expert" on the ones where we disagreed.

**4:** The statement "algebraically interesting" refers to "...automorphisms of a free group ..." and we believe it is appropriate. True, a few sentences below, when Lie algebras come into play, things are even more "algebraically interesting", but there's nothing wrong with that.

**13:** This is where we first introduce the word "usual" in place of "ordinary", so we think it's worth leaving in there for clarity.

**Referencing Subsections:** We think it is ok to reference subsections as "Section X.Y". It makes the text more readable by reducing the number of syllables, and it does not cause confusion.

**37:** Since this fact easy to find on Wikipedia, we think it's ok not to go into the details.

**40:** It's really  $n$  in this case.

**Relation notation:** We only used parentheses when first introducing an abbreviation, as in "This is called the Overcrossings Commute (OC) relation". Aside from that we never use parentheses.

**48:** The point here is to show that they are equivalent.

**British spelling VS American spelling:** We use consistent Canadian spelling, and would like to stick with it if ok with the editor.

**62 and several others:** We admit that Abelian is less often capitalized than not, but we think dead people still deserve to have their names capitalized.

**69:** We added a sentence to draw the reader's attention to the geometric explanation immediately after.

**81:** It seems that the result of Bardakov that was originally quoted here was wrong. We reworded the Problem 2.9 accordingly and removed the reference to Bardakov.

**82:** This is just a couple of facts aimed at braid theorists. We think readers not familiar with the notions will skip this, which is ok.

**115,116:** They already are.

**119:** It is a Hopf algebra, but we prefer not to talk about it as it plays no role.

**124:** We modified said paragraph in a minor way, but found no reasonable way to highlight it further without contradicting the general style of Section 2.5.

- 126:** “well known” is not an adjective here.
- 136:** We added a reference to 2.5.1.2 where we recall the definition.
- 148:** This is not a section title.
- 157:** No mirror relations...
- 190,200:** We think it is ok to use quotation marks in more than one way (for example, for direct quotes as well as notions that we have not defined). We don’t think this interferes with clarity.
- 192:** We do...
- 198:** This is a short combinatorial exercise (the sum of vertex degrees double-counts the edges of the graph). We’d rather not include it.
- 199:** We use the over arrow to distinguish arrow diagram relations from their classical (chord diagram) counterparts when they otherwise have the same name.
- 227:** We think a picture here is not worthwhile, the reader will gain more from drawing their own picture based on the words.
- 232:** We can’t move the section up because  $\mathcal{A}^{wt}$  is a pre-requisite, but we did add some pointers earlier on.
- 236:** No, this is the co-adjoint action.
- 245:** We use greek letters to denote elements of the dual.
- 253:** We think this is ok, none of the other  $w$ ’s are maps, so it should not be confusing.
- 260:** We failed to find a definitive reference, yet to the best of our knowledge, said operator is “Euler” and not “Cauchy-Euler”. Wikipedia thinks that “Cauchy-Euler” is a bit more general — supposedly it is  $p(x)\frac{\partial}{\partial x}$  where  $p$  is an arbitrary polynomial and not merely  $p(x) = x$ .
- 266:** We’ve made some minor modifications. Yet to a large extent the topic is hard and we are not sure how to make it significantly easier.
- 270:** The dot is there to distinguish multiplication from the action by  $E$ .
- 271:** This really is easy to check (a quick verification for  $f = x^n$ ).
- 276:** We chose this notation because it is suggestive of how we are going to apply this.
- 282:** We’ve made a minor modification. Yet to a large extent the topic is hard and we are not sure how to make it significantly easier.
- 286:** A few words were added at the end of Definition 3.30.
- 287:** We believe this point is addressed by the newly-added footnote 32.
- 289,290,291:** We believe this is understood at this point, and there is some merit in making sentences shorter and more readable.
- 292:** This is always an integer: for odd  $m$ ,  $(m - 1)$  is even.
- 309:** We think it is said at a fairly prominent place already?
- General remarks:** We disagree with the referee regarding the first two general remarks, yet if the editor concurs with the referee, we will be happy to make the relevant changes. Otherwise we’ve done our best to follow the rest of the general remarks.