\[
\begin{bmatrix}
(a_1, \ldots, a_n), (b_1, \ldots, b_n)
\end{bmatrix}
\]

\[
= \begin{pmatrix}
(a_1, b_1, a_1b_2, \ldots, a_1b_n), & (0)
\end{pmatrix} - \begin{pmatrix}
(0, 0, \ldots, 0)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
0, a_1b_2 - b_1a_2, a_1b_3 - b_1a_3, \ldots
\end{pmatrix}
\]

So if \((x_i)\) is the dual basis to \((a_i)\), we have

\[
[x_i, x_j] = \delta_{ij} x_j - \delta_{ij} x_i
\]

seems useless

By Milnor–Moore and given that \(A^\text{ad}\) is graded, it is enough to show that \([D_{x_i}, D_{x_j}]\) are lin-indep and that for any \(i > 2\), \(w_i\) is non-zero. All this can be done using the \(ax+b\) algebra.