

Degree 2

February-26-09
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$$\begin{array}{c|cccccc}
 & \begin{array}{c} \text{Diagram 1} \\ i \quad j \quad k \end{array} & + & \begin{array}{c} \text{Diagram 2} \\ i \quad j \quad k \end{array} & + & \begin{array}{c} \text{Diagram 3} \\ i \quad j \quad k \end{array} & = \\
 \begin{array}{c} \text{Diagram 4} \\ i \quad k \quad j \end{array} & D_3 & & \begin{array}{c} \text{Diagram 5} \\ i \quad j \quad k \end{array} & D_9 & \begin{array}{c} \text{Diagram 6} \\ i \quad j \quad k \end{array} & D_3 \\
 \begin{array}{c} \text{Diagram 7} \\ i \quad k \quad j \end{array} & 6 & & \begin{array}{c} \text{Diagram 8} \\ i \quad j \quad k \end{array} & 11 & \begin{array}{c} \text{Diagram 9} \\ i \quad j \quad k \end{array} & 3 \\
 \begin{array}{c} \text{Diagram 10} \\ j \quad i \quad k \end{array} & 10 & & \begin{array}{c} \text{Diagram 11} \\ i \quad j \quad k \end{array} & 2 & \begin{array}{c} \text{Diagram 12} \\ i \quad j \quad k \end{array} & 5 \\
 \begin{array}{c} \text{Diagram 13} \\ j \quad k \quad i \end{array} & 4 & & \begin{array}{c} \text{Diagram 14} \\ i \quad j \quad k \end{array} & 7 & \begin{array}{c} \text{Diagram 15} \\ i \quad j \quad k \end{array} & 1 \\
 \begin{array}{c} \text{Diagram 16} \\ k \quad i \quad j \end{array} & 2 & & \begin{array}{c} \text{Diagram 17} \\ i \quad j \quad k \end{array} & 7 & \begin{array}{c} \text{Diagram 18} \\ i \quad j \quad k \end{array} & 2 \\
 \begin{array}{c} \text{Diagram 19} \\ k \quad j \quad i \end{array} & 8 & & \begin{array}{c} \text{Diagram 20} \\ i \quad j \quad k \end{array} & 4 & \begin{array}{c} \text{Diagram 21} \\ i \quad j \quad k \end{array} & 12
 \end{array}$$

$$\begin{array}{llll}
 D_1 = & \text{Diagram 1} & D_4 = & \text{Diagram 4} \\
 D_2 = & \text{Diagram 2} & D_5 = & \text{Diagram 5} \\
 D_3 = & \text{Diagram 3} & D_6 = & \text{Diagram 6}
 \end{array}
 \quad
 \begin{array}{llll}
 D_7 = & \text{Diagram 7} & D_{10} = & \text{Diagram 10} \\
 D_8 = & \text{Diagram 8} & D_{11} = & \text{Diagram 11} \\
 D_9 = & \text{Diagram 9} & D_{12} = & \text{Diagram 12}
 \end{array}$$

$$\begin{array}{cccccccccccc}
 1342 & 2431 & 1324 & 3142 & 1432 & 1423 & 2341 & 3241 & 1234 & 2134 & 1243 & 2143 \\
 0 & 0 & 1 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
 \cancel{0} & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \\
 \cancel{0} & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 \\
 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 \\
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
 \end{array}$$

See also

http://katlas.math.toronto.edu/drorbn/AcademicPensieve/2009-01/one/Degree_2_for_Virtual_Knots.pdf

line. The ordering (ijk) becomes the relation $D_3 + D_9 + \cancel{D}_2 = D_6 + \cancel{D}_4 + D_6$. Likewise, $(ikj) \mapsto D_6 + \cancel{D}_4 + D_{11} = D_3 + D_5 + \cancel{D}_4$, $(jik) \mapsto D_{10} + \cancel{D}_8 + D_6 = \cancel{D}_2 + D_5 + D_3$, $(jki) \mapsto D_4 + D_7 + \cancel{D}_8 = D_8 + \cancel{D}_1 + D_{11}$, $(kij) \mapsto \cancel{D}_2 + D_7 + D_4 = D_{10} + \cancel{D}_2 + D_8$, and $(kji) \mapsto D_8 + \cancel{D}_4 + D_8 = \cancel{D}_4 + D_{12} + D_4$. After some linear algebra, we find that $\{D_1, D_2, D_6, D_8, D_9, D_{11}, D_{12}\}$ form a basis of $\mathcal{G}_2\mathcal{A}^-(\uparrow)$, and that the remaining diagrams reduce to the basis as follows: $D_2 \cancel{\downarrow} 2D_6 - D_9$, $D_4 \cancel{\downarrow} 2D_8 - D_{12}$, $D_5 = \cancel{D}_6 + D_{11} - D_8$, $D_7 \cancel{\downarrow} D_{11} + D_{12} - D_8$, and $D_{10} \cancel{\downarrow} D_{11} - D_8$.

$$\cancel{D}_9 + D_{11} - D_8$$

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$$\begin{array}{l}
 D_3 = 2D_6 - D_9 \xrightarrow{\text{some}} D_3 = -D_5 + D_6 + D_1 \\
 D_3 = D_6 - D_1, -D_5 \xrightarrow{\text{some}} D_4 = -D_7 + D_{11} + D_9 \\
 D_4 = 2D_1 - D_{12} \xrightarrow{\text{some}} D_4 = -D_7 + D_{10} + D_9
 \end{array}$$

$$\begin{aligned} D_7 &= 2D_8 - D_{12} & 2D_8 - D_{12} &= -D_7 + D_{11} + D_8 \\ \Rightarrow D_7 &= (11) + (12) - (8) \end{aligned}$$

$$\begin{aligned} D_3 &= 2D_6 - D_9 = -D_5 + D_6 + D_{11} \\ \Rightarrow D_5 &= D_9 - D_6 + D_{11} \end{aligned}$$